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THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the council-books and journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries, till the Forty-seventh Volume: the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March, 1752. And the grounds of their choice are, and will continue to

be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgment of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report, and even to certify in the public news-papers, that they have met with the highest applause and approbation. And therefore it is hoped, that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

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PHILOSOPHICAL

TRANSACTIONS.

I. *On the effects of temperature on the intensity of magnetic forces; and on the diurnal variation of the terrestrial magnetic intensity.* *By* SAMUEL HUNTER CHRISTIE, *Esq. M. A. of Trinity College, Cambridge, Fellow of the Cambridge Philosophical Society: of the Royal Military Academy.* *Communicated by the President*

Read June 17, 1824.

IN the paper on the diurnal deviations of the horizontal needle when under the influence of magnets, which the President did me the honour to present, I stated that these deviations were partly the effects of changes that took place in the temperature of the magnets; and that although the conclusions which I drew from the observations respecting the increase and decrease of the terrestrial magnetic forces during the day would not be materially affected, it was my intention to undertake a series of experiments for the purpose of determining the precise effects of changes of temperature in the magnets, so as to be able to free the observations entirely from such effects.

These experiments were immediately made; but I was induced from some effects which I observed, to carry them to

a greater extent, in the scale of temperature, than was necessary for the object which I had at first in view. In consequence of this, and the length of the calculations into which I have been obliged to enter, the accomplishment of my purpose was delayed for a considerable time, and continued indisposition has since prevented me, until now, completing the arrangement of the tables of results.

In the present paper, I propose to detail the experiments which I made in order to determine the effect of changes of temperature on the forces of the magnets, to the extent to which I observed their temperature to vary, during my observations on the diurnal changes in the direction of the needle, when under their influence; to apply the results which I obtained to the correction of the observations themselves, thereby accounting for the apparent anomalies noticed by Mr. Barlow and myself, in the observations made in doors and in the open air; and by means of these corrected observations, to point out the diurnal variations in the terrestrial magnetic intensity.

It had been my intention to determine purely from observation the portion of the arc of deviation due to the changes which I noticed in the temperature of the magnets; but I found that this depended so much on the situation of the point at which the needle was held in equilibrio by the terrestrial forces and those of the magnets, that it would hardly be possible to determine how much of this portion was due to the extent of the change of temperature, or the degree of temperature where the change took place, and how much to the azimuth of the needle, when affected by this change. I was therefore under the necessity of having recourse to

theory, and adopted the simplest, and that which is most generally received, viz. that the forces which two magnets exert upon one another may be referred to two centres or poles in each, near their respective ends; and that for either pole in one of the magnets, one pole of the other magnet is urged towards it, and the other from it, by forces varying inversely as the squares of their respective distances from that pole. Of the correctness of this theory of the action of one magnet upon another, the conclusions which I have obtained have given me no reason to doubt.

In the observations on the diurnal changes in the positions of the points of equilibrium at which the pole of the needle was retained by the joint action of two magnets and the terrestrial magnetism, where I noted the changes that took place in the temperature of the magnets, to which observations I have alluded near the conclusion of my former paper, two magnets, as in several of the preceding observations, were placed, with their axes in the magnetic meridian, on the same horizontal table as the compass, at equal distances from the centre of the needle, one towards the north, the other towards the south, the north pole of each magnet being towards the north; and their distances from the centre were such, that the points of equilibrium were nearly 180°, or south, N. 80° E. and N. 80° W. To determine here the changes that would take place in the situation of these points from changes in the force of the magnets, arising from a variation of their temperature, it was first necessary to determine the changes in the forces themselves, arising from certain variations of the temperature of the magnets, by observing the corresponding changes in the direction of the needle.

To obtain the equation requisite for this purpose, take the centre of the needle as the origin of the rectangular co-ordinates, the axis of the x's being in the magnetic meridian. Let x and y be the co-ordinates to the south pole of the needle, x being measured towards the north, and y towards the west: also, let r be the distance of either pole of the needle from its centre; ρ the distance of the poles of each of the magnets from their respective centres; and R the distance between the centre of the needle and the centre of either magnet. For the sake of expressing clearly and concisely the distances between the poles of the needle and those of the magnets, we will indicate these points as follow:

s, the *south* pole of the needle; that is, the pole which, when the needle is freely suspended, points towards the *north*;

n, the *north* pole of the needle;

σ_N, the *south* pole of the magnet which is to the *north* of the needle; that is, its pole *nearest* to the centre of the needle;

ν_N, the *north* pole of the same magnet, or its pole which is *furthest* from the needle's centre;

σ_S, the *south* pole of the *south* magnet, or that pole which is *furthest* from the centre of the needle;

ν_S, the *north* pole of the same magnet, or that pole which is *nearest* to the centre of the needle.

Now resolve the terrestrial magnetic force acting on the north arm of the needle, in the line of the dip, into two; one horizontal or in the direction x, and the other vertical: and let the horizontal force be M. Also, let the force with which a pole of the needle is repelled from the pole of the same name of either magnet, or attracted towards that of a con-

trary name at the unity of distance, be F: then the forces acting on the south pole of the needle will be,

M in the direction x;

$-\frac{F}{(s\sigma_N)^2}$, in the direction $s\sigma_N$; $\frac{F}{(s\nu_N)^2}$, in the direction $s\nu_N$;

$-\frac{F}{(s\sigma_S)^2}$, in the direction $s\sigma_S$; $\frac{F}{(s\nu_S)^2}$, in the direction $s\nu_S$.

The north pole of the needle will be urged by forces equal and parallel to these, but in contrary directions; so that in investigating the conditions of equilibrium of the needle, we may consider only the equilibrium of the south pole urged by forces double of the preceding, and constrained to move in a circle; and it is evident that the equation of equilibrium will be the same, whether we take these forces, or the doubles of them.

Resolving these forces into others in the directions x and y, calling X the sum of all the forces in the direction x, and Y the sum of all the forces in the direction y, we shall have,

$$X = M - F\left\{\frac{R-\rho-x}{(s\sigma_N)^3} - \frac{R+\rho-x}{(s\nu_N)^3} - \frac{R+\rho+x}{(s\sigma_S)^3} + \frac{R-\rho+x}{(s\nu_S)^3}\right\}$$

$$Y = F.\left\{\frac{y}{(s\sigma_N)^3} - \frac{y}{(s\nu_N)^3} + \frac{y}{(s\sigma_S)^3} - \frac{y}{(s\nu_S)^3}\right\}$$

The general equation of equilibrium for a point acted upon by forces in the same plane, and constrained to move in a curve whose equation is $L = o$, is

$$X\,dx + Y\,dy + \lambda\,dL = o. \qquad (1)$$

From this we obtain

$$X + \lambda.\frac{dL}{dx} = o, \text{ and } Y + \lambda\frac{dL}{dy} = o;$$

whence $$X.\frac{dL}{dy} - Y.\frac{dL}{dx} = o \qquad (2)$$

The equation $L = o$ is in this case,

$$x^2 + y^2 - r^2 = o;$$

and consequently $\frac{dL}{dx} = 2x, \frac{dL}{dy} = 2y.$

The equation (2) therefore becomes

$$Xy - Yx = o.$$

Substituting in this equation the values previously found for X and Y, and dividing by y, we obtain

$$M - F.\left[(R-\rho).\left\{\frac{1}{(s\sigma_N)^3} + \frac{1}{(s\nu_S)^3}\right\} - (R+\rho).\left\{\frac{1}{(s\nu_N)^3} + \frac{1}{(s\sigma_S)^3}\right\}\right] = o \ldots\ldots. (A)$$

Let φ be the angle which the axis of the needle makes with the meridian, or the azimuth of the point of equilibrium, and we shall have,

$$(s\sigma_N)^2 = (R-\rho)^2 + r^2 - 2r(R-\rho)\cos.\varphi; \quad (s\nu_S)^2 = (R-\rho)^2 + r^2 + 2r(R-\rho).\cos.\varphi:$$
$$(s\nu_N)^2 = (R+\rho)^2 + r^2 - 2r(R+\rho)\cos.\varphi; \quad (s\sigma_S)^2 = (R+\rho)^2 + r^2 + 2r(R+\rho)\ \cos.\varphi.$$

Substituting these values in the equation (A), it becomes,

$$M - F.\left\{\begin{array}{l} \dfrac{R-\rho}{\left\{(R-\rho)+r^2\right\}^{\frac{3}{2}}}.\left\{\dfrac{1}{\left\{1-\dfrac{2r(R-\rho)}{(R-\rho)^2+r^2}.\cos.\varphi\right\}^{\frac{3}{2}}} + \dfrac{1}{\left\{1+\dfrac{2r(R-\rho)}{(R-\rho)^2+r^2}.\cos.\varphi\right\}^{\frac{3}{2}}}\right\} \\ -\dfrac{R+\rho}{\left\{(R+\rho)^2+r^2\right\}^{\frac{3}{2}}}.\left\{\dfrac{1}{\left\{1-\dfrac{2r(R+\rho)}{(R+\rho)^2+r^2}.\cos.\varphi\right\}^{\frac{3}{2}}} + \dfrac{1}{\left\{1+\dfrac{2r(R+\rho)}{(R+\rho)^2+r^2}.\cos.\varphi\right\}^{\frac{3}{2}}}\right\} \end{array}\right\} = o \ldots\ldots (B)$$

From this equation the value of F in terms of M may be found for any values of φ, the distances R, r and ρ being known; and if we suppose M constant during the observations, the variations in the intensity of the force F may be obtained from the observed variations in the value of φ.

If the angle φ does not differ from a right angle by more than 10° or even 20°, by expanding the several fractions, no sensible error will arise by limiting the series to a few of the

first terms, and we shall in these cases thus obtain a much more convenient equation for computation. Since

$$\frac{1}{(1-a\cos.\phi)^{\frac{3}{2}}} = 1 + \frac{3}{2}\,.\,a\cos.\phi + \frac{3\cdot5}{2\,.\,4}a^2\cos.^2\phi + \frac{3\cdot5\cdot7}{2\,.\,4\,.\,6}a^3\cos.^3\phi$$
$$+ \frac{3\cdot5\cdot7\cdot9}{2\,.\,4\,.\,6\,.\,8}a^4\cos.^4\phi + \&c.$$

and

$$\frac{1}{(1+a\cos.\phi)^{\frac{3}{2}}} = 1 - \frac{3}{2}\,.\,a\cos.\phi + \frac{3\cdot5}{2\,.\,4}a^2\cos.^2\phi - \frac{3\cdot5\cdot7}{2\,.\,4\,.\,6}a^3\cos.^3\phi$$
$$+ \frac{3\cdot5\cdot7\cdot9}{2\,.\,4\,.\,6\,.\,8}a^4\cos.^4\phi + \&c.$$

$$\frac{1}{(1-a\cos.\phi)^{\frac{3}{2}}} + \frac{1}{(1+a\cos.\phi)^{\frac{3}{2}}} = 2 + \frac{3\cdot5}{4}a^2\cos.^2\phi + \frac{3\cdot5\cdot7\cdot9}{4\,.\,6\,.\,8}a^4\cos.^4\phi$$
$$+ \&c.$$

So that the equation (B) will become

$$M - F\,.\left\{\begin{array}{l} \dfrac{R-\rho}{\left\{(R-\rho)^2+r^2\right\}^{\frac{3}{2}}}\cdot\left\{2+3\,.\,5\,.\,\dfrac{r^2(R-\rho)^2}{\left\{(R-\rho)^2+r^2\right\}^2}\,.\cos.^2\phi\right\} \\ -\dfrac{R+\rho}{\left\{(R+\rho)^2+r^2\right\}^{\frac{3}{2}}}\cdot\left\{2+3\,.\,5\,.\,\dfrac{r^2(R+\rho)^2}{\left\{(R+\rho)^2+r^2\right\}^2}\,.\cos.^2\phi\right\}\end{array}\right\} = o \quad (C)$$

neglecting the terms which contain the fourth and higher powers of cos. ϕ,

Taking one of the cases which I investigated, and from which the others do not differ very considerably, the values of the co-efficients of cos. ϕ in the denominators of the fractions in the equation (B) are .25691 and .15951; so that the greatest of the terms neglected would be

$$\frac{3\cdot5\cdot7\cdot9}{4\,.\,6\,.\,8}\times(.25691)^4.\cos.^4\phi \text{ and } \frac{3\cdot5\cdot7\cdot9}{4\,.\,6\,.\,8}\times(.15951)^4\cos.^4\phi.$$

Now, supposing that ϕ is 70°, if these terms are employed in determining the value of F, it will be 218.7705 . M, and 218.8184 . M, if they are neglected; making a difference of .0479 M, or only affecting the fifth figure in this extreme

case. If, instead of expanding the fractions, we computed them in the form which they have in the equation (B), we could hardly be supposed to obtain the *absolute* values of F more nearly than this; although in either case the relative values would be obtained to a much greater degree of accuracy. In the observations which I made, the values of φ were seldom much less than 80°, and in such cases the error would be considerably less. In an instance where φ was 82° 37′, the value of F was 222.5630 M, employing the terms containing $\cos.^4 \varphi$, and 222.5640 M, neglecting them. Seeing then that no sensible error would arise from neglecting these terms, I have in all cases made use of the equation (C), for determining the values of F. I now proceed to the experiments which I made for this purpose.

On this occasion I made use of the same compass which I had already used in the greater part of the observations detailed in my former paper, and distinguished there as No. 1; the magnets were also the same that I had used with this compass. The length of the needle is very accurately 6 inches. In order to determine the distance between the points which I ought to consider as the poles of the needle, I fixed it at right angles to the meridian; and bringing another needle, freely suspended, near to it, I moved the centre of this needle along a line parallel to the axis of the first, and noted the points opposite to which the axis of the second was exactly in the magnetic meridian; these points I considered as the poles of the first needle. The distance between the points thus determined was 4.28 inches.

In my former paper I have stated the length of each of the magnets to be 12 inches; more accurately, the length of the

two joined together was 23.84 inches; so that the length of each might be taken to be very accurately 11.92 inches: they are .95 inch wide, and .375 inch thick. In all my observations the same magnet was always placed on the same side of the centre of the needle; so that in ascertaining the situations of their poles I distinguish one as the north, the other as the south magnet. The distances of the poles of the magnets from their ends, determined in the same manner as for the needle, were measured on each side, and a mean of the whole taken to obtain the distances between the poles; they were these:

	North Magnet.		South Magnet.	
	North Pole.	South Pole.	North Pole.	South Pole.
	0. 82 inch. 0. 81	0.86 inch. 0.86	0.84 inch. 0.84	0. 77 inch. 0. 76
Mean	0.815	0.86	0.84	0.765

Taking half the sum of these, 1.64 inches, from the length of each magnet, we have 10.28 inches for the distance between the poles.

A meridian line being drawn on a firm table, standing on a stone floor, the compass was accurately adjusted on it, so that the needle pointed to zero on the graduated circle. The magnets were fixed at the bottoms of earthen pans, secured in such a way to rectangular pieces of board that their positions could not be accidentally changed, and projecting from these boards were small pieces of brass, on each of which a line was drawn to indicate the position of the axis of the magnet; the horizontal distance of the edge of each of the

projections nearest to the needle from the corresponding end of the magnet within the pan, was exactly 3 inches; I could therefore, in any instance, determine very accurately the distance of the centre of the magnet from that of the needle. The pans were placed on the table, so that the indexes on the pieces of brass coincided with the meridian line. Water was now poured into the pans, and the temperature of the magnets was varied by varying the temperature of the water. The temperature of each magnet was ascertained by a thermometer placed in the water, with its bulb resting on that pole of the magnet which was nearest to the centre of the needle. In my first observations I however made use of only one thermometer, which was moved, during them, from one magnet to the other. In Plate I. Fig. 1, an apparatus of the same nature, which I subsequently made use of, is represented. This differs from that employed in these experiments only in having the boxes containing the magnets made to slide on a ruler, whose axis being in the magnetic meridian, and the axes of the magnets adjusted in the boxes also in the meridian, they can be made to approach or recede from the needle, in that line, which saves considerable trouble in the adjustments when observations are to be made at different distances. Fig. 2 and 3 represent the plan and elevation of another apparatus which I had constructed for Mr. Foster, and which he has taken with him on the North-western Expedition, to enable him to make observations on the daily variation, particularly with a view of ascertaining the times of maximum east and west, and also of zero, should any of the stations at which he may find himself be favourable to the employment of such an ap-

Fig. 1.

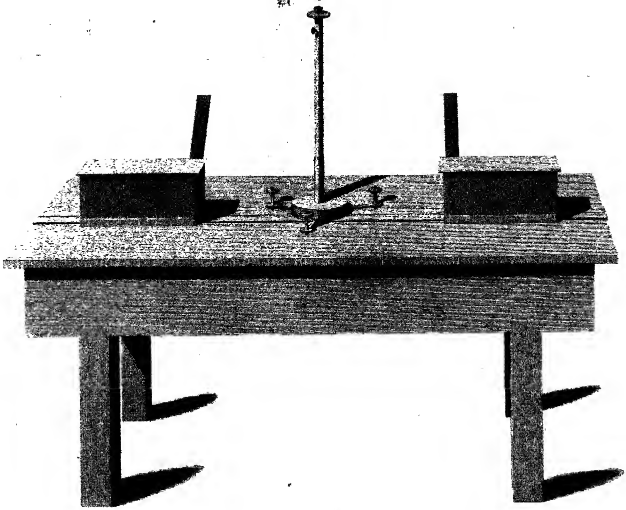


Fig. 2

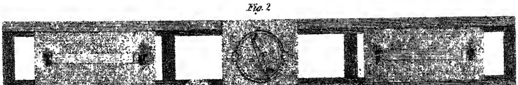
N  S

Fig. 3.

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paratus. N, S, (Fig. 2,) denote the ends of the instrument to be placed towards the magnetic north and south; *c*, the centre of the needle; *n* and *s*, its north and south *poles*; ν_N, σ_N, the north and south *poles* of the north magnet; ν_S, σ_S, the north and south *poles* of the south magnet: the magnets being fixed on boards which, sliding in grooves, may be made to approach or recede from the needle at pleasure.

In the observations which I first made on the effects of changes of temperature, the centres of the magnets were at the same distances from the centre of the needle as they were during the observations on the diurnal changes in the directions of the needle, which it was my object to reduce, and which will be given in the conclusion of this paper: this distance was 21.21 inches. We have therefore in this case $R = 21.21$ inches, and, from what I have before said, $r = 2.14$ inches, and $\varrho = 5.14$; if we substitute these values in the equation (C), it will become

$$M. - F. (.004690814 + .000829329 \cos. \varphi) = 0. \qquad (\alpha)$$

Observing then the value of φ at any particular temperature of the magnets, by means of this equation I could readily obtain the corresponding value of F in terms of M; and by varying the temperature of the magnets, I obtained the variation of the intensity of their forces, corresponding to such change of temperature. The observations contained in the following table were made thus: I first noted the time, which is set down in the first column, and then the temperature of the north magnet; after which I placed the thermometer on the pole of the south magnet: I next observed the westerly point, at which the needle was held in equilibrio

by the terrestrial forces and those of the magnets, slightly agitating the needle, that it might the more readily assume the true position; from this it was led, by means of a very small and weak magnet, held on the outside of the compass-box, towards the easterly point of equilibrium, which was observed in the same manner; and from this it was led in the same way towards the southerly point, which, however, was not observed with an intention of deducing any thing by means of the equation (α), which was not calculated for such a value of φ. After these observations of the points of equilibrium, the temperature of the south magnet being observed, the time set down in the seventh column, at which the observations concluded, was noted. The temperature of the water in the pans was now increased or diminished, according to circumstances, by the addition of other water, and the pans covered over, to prevent any rapid changes of temperature during the observations: after allowing a short time for the magnets to acquire the temperature of the water, the observations were repeated. To prevent any ambiguity, with regard to the time indicated being morning or evening, I have, except when otherwise expressed, adopted the astronomical division of the day, from noon to noon. The scale made use of for the temperature was in all cases that of Fahrenheit.

Table of the positions of the Points of Equilibrium, corresponding to different Temperatures of the Magnets retaining a Magnetic Needle in equilibrio. 6th June, 1823.

Time of commencing observation.	Temperature of N. Magnet.	Points of Equilibrium.			Temperature of S. Magnet.	Time of concluding observation.	Mean temperature of the Magnets.	$10^h 15^m$ { Barom. 30.12; Therm. attached 60.25
		West.	East.	South.				
h. m.	°	° ′	° ′	° ′	°	h. m.	°	
7 54	62.0	82 44	82 30	0 12W	62.1	8 01	62.05	
8 10	59.3	86 00	84 50	0 12	58.8	8 17	59.05	
8 31	79.0	74 40	74 56	0 12	76.3	8 35	77.65	
8 40	75.0	76 02	75 54	0 12	73.0	8 44	74.00	
8 52	71.0	77 30	77 24	0 10	70.3	8 56	70.65	
9 11	67.3	79 16	78 52	0 14	67.0	9 16	67.15	
9 34	63.8	80 42	80 50	0 18	63.8	9 40	63.80	
9 53	62.0	81 50	82 16	0 16	62.1	10 00	62.05	

Taking half the sum of the easterly and westerly arcs for the value of ϕ, and substituting them successively for ϕ in the equation (α), I obtain the values of $\frac{F}{M}$ corresponding to the respective mean temperatures of the magnets. These I have arranged in the following table; placing in the second column the differences of the successive temperatures, and in the fifth the corresponding differences in the values of $\frac{F}{M}$; these, divided by the numbers in the second column, will give the variation of the value of $\frac{F}{M}$, corresponding to a change in the temperature of the magnets of 1° on FAHRENHEIT's scale: these variations in the values of $\frac{F}{M}$ are contained in the last column of the table, and are denoted by $\Delta . \frac{F}{M}$.

Table of the Magnetic Intensities corresponding to different Temperatures of the Magnets.

Mean temperature of the Magnets.	Diff. of Temp. in successive observations.	Mean of the observed values of φ.	Magnetic Intensity or values of $\frac{F}{M}$.	Diff. of successive values of $\frac{F}{M}$.	Variation of $\frac{F}{M}$ for 1° Fah. or $\Delta \cdot \frac{F}{M}$.
62°.05		82° 37′	212.5620		
59.05	− 3.00	85 25	212.9423	+0.3803	0.1268
77.65	+18.60	74 48	210.6228	−2.3195	0.1247
74.00	− 3.65	75 58	210.9892	+0.3664	0.1004
70.65	− 3.35	77 27	211.4178	+0.4286	0.1279
67.15	− 3.50	79 04	211.8353	+0.4175	0.1193
63.80	− 3.35	80 46	212.2167	+0.3814	0.1138
62.05	− 1.75	82 03	212.4640	+0.2473	0.1413

The differences in the deduced values of the variation of $\frac{F}{M}$ for a change of temperature in the magnets of 1° in the last column, are not greater than we may suppose to have arisen from small inaccuracies in the observations, or slight changes in the terrestrial intensity during the time in which they were made; the latter indeed appear to have taken place, since, at the same temperature, the value of φ was 82° 37′ at the beginning of the observations, and 82° 03′ at their conclusion. The value 0.1247 deduced from the observations at the temperatures 59.05 and 77.65 I should consider as nearest the truth, since whatever may have been the errors, the divisor is here larger than in any other case; and, in taking a mean, this value should be taken with the mean of all the others: the contrary may be said of the value 0.1413, which should have only half the weight of any of the others. I therefore first take in this manner the mean of all the values excluding 0.1247, and then the mean of this

mean and 0.1247, and I thus get .1226 as the mean variation of the intensity of the magnets for a change in their temperature of 1°, between the temperatures 59.05 and 77.65, an increase of temperature always causing a decrease of intensity, and *vice versa.*

In the results in the last column of this table there are no marked indications of an increase in the values of $\Delta . \frac{F}{M}$ arising from an increase of the temperature at which the observations were made. Having afterwards, when I carried the observations to a greater extent in the scale of temperature, clearly ascertained that this was the case, I determined therefore not to take the mean of the values of $\Delta . \frac{F}{M}$, as I have here pointed out, but as I had made observations at every convenient opportunity, to take out from them, in the first place, all the values of $\Delta . \frac{F}{M}$, where the mean between the temperatures from which they were derived agreed nearly with the lowest temperature of the observations which it was my object to reduce; in the same manner, to take those which agreed most nearly with the mean temperature to which these observations were to be reduced; and likewise those agreeing with their highest temperature: taking then the mean of each of these, from these three means, I derived a value of $\Delta \frac{F}{M}$, from which I determined the variation of the angle ϕ, corresponding to any change of temperature. I have mentioned this here, that my reason for giving so many of the observations may be apparent. Observations, precisely similar to the preceding, were made on the 7th of June: they are contained in the following table.

Table of the positions of the Points of Equilibrium corresponding to different Temperatures of the Magnets retaining a Magnetic Needle in equilibrio, 7th June.

Time of commencing observation.	Temperature of N. Magnet.	Points of Equilibrium.			Temperature of S. Magnet.	Time of concluding observation.	Mean temperature of the Magnets.
		West.	East.	South.			
h. m.	°	° ′	° ′	° ′	°	h. m.	°
9 50	57.0	83 18	83 16	0 10W	57.0	9 53	57.00
10 04	66.3	78 16	78 04	0 16	67.7	10 07	67.00
10 15	71.0	76 22	76 10	0 14	70.7	10 18	70.85
10 27	75.0	74 36	74 42	0 12	75.0	10 31	75.00
10 44	60.0	81 18	80 26	0 20	61.0	10 47	60.50

$10^h\ 50^m$ Barom 30.12, Therm. att. 60.50

Previous to making these observations I had slightly changed the distances of the magnets from the centre of the needle: the distances of their nearest ends were now 15.26 inches, or that of their centres 21.22 inches from the needle's centre. Substituting this value of R in the equation (C) it becomes

$$M - F\,(.004683954 + .000827265 \cos.^2 \varphi) = o; \qquad (\alpha_1)$$

and from this I calculated the following table.

Table of the Magnetic Intensities corresponding to different Temperatures of the Magnets.

Mean temperatures of the Magnets.	Diff. of Temp. in successive observations.	Mean of the observed values of φ.	Magnetic Intensity or value of $\frac{F}{M}$.	Diff. of successive values of $\frac{F}{M}$.	Variation of $\frac{F}{M}$ for 1° Fah. or $\Delta \cdot \frac{F}{M}$.
°		° ′			
57.00		83 17	212.9803		
	+10.00			−1.0594	0.1059
67.00		78 10	211.9209		
	+ 3.85			−0.5302	0.1377
70.85		76 16	211.3907		
	+ 4.15			−0.5059	0.1219
75.00		74 39	210.8848		
	−14.50			+1.6641	0.1148
60.50		80 52	212.5489		

After these observations the magnets were wiped dry, and their poles of contrary names joined by bars of soft iron: to this circumstance I attribute the increase which, when next I used them, I found had taken place in their intensities. In the observations subsequent to these, I made use of two thermometers, one for each magnet, and observed the temperatures of both magnets at the beginning and at the conclusion of the observation.

Table of the positions of the Points of Equilibrium corresponding to different Temperatures of the Magnets retaining a Magnetic Needle in Equilibro. 13th of June.

Time of commencing observation.	Temperature of		Points of Equilibrium.			Temperature of		Time of concluding observation.	Mean Temperature of the Magnets.	$8^h\ 40^m$ { Barom. 29.93 / Therm. att. 65.66
	N. Mag.	S. Mag.	West.	East.	South.	N. Mag.	S. Mag.			
h. m.	°	°	° ′	° ′	° ′	°	°	h. m.		
7 12	63.0	62.6	80 28	80 28	0 18 E	63.0	62.6	7 14	62.80	
7 35	61.1	61.0	81 12	81 16	0 16	61.2	61.0	7 40	61.08	
7 55	71.1	71.1	75 36	75 44	0 18	71.0	71.0	8 00	71.05	
8 26	66.2	65.8	77 40	77 48	0 18	66.1	65.7	8 32	65.95	

I have just mentioned that, on making these observations, I found the intensities of the magnets increased: on this account I was under the necessity of increasing their distances from the needle. The distances of their nearest ends from the centre of the needle were in this case 15.45 inches, or of their centres 21.41 inches: this value of R being substituted in the equation (C) gives

$$M - F\,(.004553604 + .0007880523 \cos.^2 \phi) = 0. \qquad (\alpha_2).$$

As before, I calculate the following table from this equation.

Table of the Magnetic Intensities corresponding to different Temperatures of the Magnets.

Mean Temperature of the Magnets.	Diff. of Temp. in successive observations.	Mean of the observed values of φ.	Magnetic Intensity or value of $\frac{F}{M}$.	Diff. of successive values of $\frac{F}{M}$.	Variation of $\frac{F}{M}$ for 1° Fah. or $\Delta \cdot \frac{F}{M}$.
62°.80		80° 28′	218.5687		
61.08	−1.72	81 14	218.7269	+0.1582	0.0920
71.05	+9.97	75 40	217.3014	+1.4255	0.1430
65.95	−5.10	77 44	217.9040	+0.6026	0.1182

There is only one, the first, of the values of $\Delta \cdot \frac{F}{M}$, which differs much from those already obtained, but the difference of the temperatures in the observations from which it is derived is so small, that any errors would be rendered very sensible; and if the thermometers happened not to indicate the precise temperatures of the magnets at the times of observation, it would be quite sufficient to account for this discrepancy.

In his paper on the daily variation of the horizontal and dipping needles under a reduced directive power, Mr. BARLOW has described some anomalies which he observed between the daily changes in the direction of a needle when placed in the house and when in the open air, and also the steps which he took to discover their cause. He mentions, "that in certain positions of the needle towards the east and west, the daily motion, although it proceeded with the same determinate uniformity in both cases, yet it took place in different directions; passing in the one instance from the east, or west, towards the south, and in the other towards the north, at the

same corresponding hours of the day, the motion in both instances being equally distinct, regular, and progressive."* These anomalies, I also noticed, although, as I have mentionod in my former paper, I did not find the reversion, in the directions in the two cases, to take place with the same regularity and uniformity that Mr. Barlow observed it to have. In that paper I also stated my opinion, that these anomalies had arisen from the difference in the changes of temperature in the magnets when in doors and when in the open air, and that the observations in the two cases would be found to agree when they were freed from the influence of difference of temperature in the magnets.

As I had already made observations in doors, in which I noted the temperature of the magnets, it was now my intention to make corresponding observations in the open air, in order that by reducing the obervations to the same standard of temperature, their agreement or disagreement might be put beyond doubt. For this purpose the whole apparatus was placed in my garden, exposed to the sun and air, on a table having its legs driven firmly into the ground; and for several days I observed, at stated intervals, the positions of the points of equilibrium; when I had an opportunity I also made experiments, similar to the preceding, for the purpose of determining the value of $\Delta \frac{F}{M}$, to be applied to the correction of the observations in doors and in the open air.

On adjusting the magnets to the needle, I again found that

* In the Postscript to this paper, Mr. Barlow, to whom I had communicated my views with regard to the effects of temperature, refers to the experiments which I had made, for the explanation of these apparent anomalies.

their intensities had increased, owing, I consider, to the same circumstance as before, and I therefore increased their distances from the needle; but after making the first days observations, and comparing them with those made in doors, I found it necessary slightly to diminish these distances, in order that, at the same temperature of the magnets, the situations of the points of equilibrium might more nearly agree in the two cases. During the observations of the first day, the distance of the nearest ends of the magnets from the centre of the needle were 15.62 inches: so that the value of R is here 21.58 inches, and the equation C becomes,

$$M - F\,(.004441190 + .0007549085\ \cos.^2 \varphi) = o \qquad (\alpha_3)$$

The observations are contained in the following table.

Table of the positions of the Points of Equilibrium corresponding to different Temperatures of the Magnets retaining a Magnetic Needle in Equilibrio. 17th and 18th June.

Time of commencing observation.		Temperature of the Magnets. North.	Temperature of the Magnets. South.	Points of Equilibrium. West.	Points of Equilibrium. East.	Points of Equilibrium. South.	Temperature of the Magnets. North.	Temperature of the Magnets. South.	Time of concluding observation.	Mean Temperature of the Magnets.	Barom.	Therm. attached.
June	h. m	°	°	° ′	° ′	° ′	°	°	h. m.	°		
17	19 27	49.6	48.8	79 58	80 46	0 42 E	49.8	49.0	19 32	49.30		
	19 53	60.25	60.25	75 24	75 46	0 32 E	60.25	60.25	19 56	60.25		
	20 15	68.4	68.6	72 02	72 40	0 24 E	67.8	68.2	20 19	68.25		
	20 36	74.9	75.0	69 36	70 08	0 24 E	74.2	74.3	20 41	74.60		
	21 02	61.8	62.0	74 12	74 24	0 14 E	61.5	61.7	21 06	61.75		
	21 26	74.0	74.2	70 06	70 24	0 08 E	73.4	73.6	21 30	73.80	30.29	55.75
18	7 36	55.7	55.5	75 00	75 06	0 00	55.7	55.4	7 40	55.58		
	8 00	66.2	66.0	70 58	71 28	0 02 E	66.0	65.8	8 04	66.00		
	8 25	73.8	73.8	68 12	68 52	0 02 E	73.4	73.4	8 28	73.60		
	8 51	56.8	57.4	74 38	74 44	0 02 E	56.4	57.0	8 56	56.90	30.20	56.00

From these I calculate the following table by means of the equation (α_3).

Table of the Magnetic Intensities corresponding to different Temperatures of the Magnets.

Mean Temperature of the Magnets.	Diff. of Temp. in successive observations.	Mean of the observed values of φ.	Magnetic Intensity, or value of $\frac{F}{M}$.	Diff. of successive values of $\frac{F}{M}$.	Variation of $\frac{F}{M}$ for 1° Fah or $\Delta . \frac{F}{M}$.
°		° ′			
49.30	+10.95	80 22	224.0981	−1.2810	0.1179
60.25	+ 8.00	75 35	222.8171	−1.1125	0.1391
68.25	+ 6.35	72 21	221.7046	−0.9848	0.1551
74.60	−12.85	69 52	220.7198	+1.6769	0.1305
61.75	+12.05	74 18	222.3967	−1.5189	0.1260
73.80		70 15	220.8778		
55.58	+10.42	75 03	222.6462	−1.3807	0.1315
66.00	+ 7.60	71 13	221.2655	−1.1123	0.1461
73.60	−16.70	68 32	220.1532	−2.3613	0.1314
56.90		74 39	222.5145		

After making these obsevations, the distances of the magnets from the needle were slightly diminished, for the reasons I have already mentioned: their nearest ends were now 15.56 inches from the centre of the needle, or the value of R was 21.52 inches. By substituting this value in the equation C, it becomes

$$M - F . (.004480432 + .0007664093 \cos.^2 \varphi) = o. \qquad (\alpha_4)$$

The observations which I made with the magnets at this distance from the needle, and the results which I obtain from them, are contained in the two following tables.

Table of the positions of the Points of Equilibrium corresponding to different temperatures of the Magnets retaining a Magnetic Needle in equilibrio. 18th, 19th, 20th, 22nd June.

Time of commencing observation.		Temperature of the Magnets.		Points of Equilibrium.			Temperature of the Magnets.		Time of concluding observation.	Mean Temperature of the Magnets.	Barom.	Therm. attached.
		North.	South.	West.	East.	South.	North.	South.				
June	h. m.	°	°	° ′	° ′	°	°	°	h. m.	°		°
18	19 24	55.2	55.2	85 18	84 24	0 20 E	55.6	55.6	19 32	55.40	30.23	57.10
	19 47	74.0	74.2	75 16	74 24	0 26 E	73.5	73.5	19 51	73.80		
	20 08	55.5	55.3	85 36	84 32	0 14 E	55.5	55.3	20 14	55.40		
19	7 25	56.0	55.0	83 08	82 06	0 06 E	55.8	55.0	7 30	55.45		
	7 52	64.6	64.3	77 38	77 20	0 00	64.3	64.0	7 56	64.30	30.17	55.3
	8 16	74.4	74.2	73 28	72 58	0 00	73.5	73.5	8 21	73.90		
	8 40	64.7	65.0	77 44	77 06	0 12 W	63.7	63.7	8 47	64.28		
	9 01	55.6	55.8	82 34	81 30	0 12 W	55.3	55.5	9 04	55.55	30.17	55.7
	19 25	55.2	54.8	84 12	83 02	0 18 E	55.5	55.1	19 29	55.15	30.11	56.75
	19 50	67.3	66.5	77 26	76 44	0 22 E	66.8	66.4	19 55	66.75		
20	17 29	51.3	50.0	85 16	85 14	0 14 E	51.3	50.0	17 34	50.65	30.10	60.4
	17 50	56.8	55.0	81 46	81 28	0 26 E	56.6	54.8	17 55	55.80		
	18 14	51.7	51.1	84 44	84 38	0 16 E	51.6	51.1	18 19	51.35		
	18 40	57.6	57.0	80 42	80 46	0 20 E	57.4	56.8	18 46	57.20		
22	18 01	54.7	53.7	82 52	82 46	0 10 E	54.5	53.4	18 06	54.08	30.16	55.3
	18 22	52.0	51.0	85 10	85 14	0 10 E	51.6	50.8	18 28	51.35		
	18 48	56.3	56.0	82 08	81 40	0 14 E	55.8	55.7	18 53	55.94		

Table of the Magnetic Intensities corresponding to different Temperatures of the Magnets.

Mean Temperature of the Magnets.	Diff. of Temp. in successive observations.	Mean of the observed values of ϕ.	Magnetic intensity or value of $\frac{F}{M}$.	Diff. of successive values of $\frac{F}{M}$.	Variation of $\frac{F}{M}$ for 1° Fah. or $\Delta \cdot \frac{F}{M}$.
°		° ′			
55.40	+18.40	84 51	222.8859	−2.2756	0.1237
73.80	−18.40	74 50	220.6103	+2.3010	0.1251
55.40		85 04	222.9113		
55.45	+ 8.85	82 37	222.5640	−1.1496	0.1299
64.30	+ 9.60	77 29	221.4144	−1.3595	0.1415
73.90	− 9.62	73 13	220.0549	+1.3413	0.1394
64.28	− 9.73	77 25	221.3962	+1.0648	0.1220
55.55		82 02	222.4610		
55.15	+11.60	83 37	222.7217	−1.4204	0.1224
66.75		77 05	221.3013		
50.65	+ 5.15	85 15	222.9322	−0.5474	0.1063
55.80	− 4.45	81 37	222.3848	+0.4712	0.1059
51.35	+ 5.85	84 41	222.8660	−0.6580	0.1125
57.20		80 44	222.2080		
54.08	− 2.73	82 49	222.5974	+0.3289	0.1203
51.35	+ 4.59	85 12	222.9263	−0.4886	0.1064
55.94		81 54	222.4377		

After having made the preceding observations, and concluded those on the diurnal changes in the points of equilibrium, I proposed applying the balance of torsion, as another means of determining the variations in the magnetic intensities, arising from changes in the temperatures of the magnets; but in my first observations with this instrument, there were such discrepancies, arising from the over torsion of the wire in consequence of its want of elasticity, being silver and too fine for the weight of the needle, and likewise too short, that, although they pointed out very clearly the same general results which I afterwards obtained from unexceptionable observations, I would not make use of them for determining a mean value of $\Delta . \frac{F}{M}$ to be applied to the correction of the observations on the diurnal changes, for the variation in the temperature of the magnets. As a considerable time intervened before I had an opportunity of repeating these experiments, and in making them, I had, by increasing the temperature of the magnets beyond a certain point, permanently destroyed a portion of their intensities, I considered it better to obtain the mean value of $\Delta . \frac{F}{M}$, which was requisite, from the results of the experiments made more nearly at the same time as the observations which it was my object to reduce. I therefore determined this value from the preceding results.

The temperatures of the magnets in the observations on the daily changes of the points of equilibrium were, with a few exceptions above and below, comprised between 54° and 65°, and I determined the mean value of $\Delta \frac{F}{M}$ to be applied to the correction of these observations in this manner: from

the preceding results I first took those values of $\Delta \frac{F}{M}$ derived from observations where the mean of the temperatures was near to 65°, and from these obtained a mean value of $\Delta . \frac{F}{M}$ when the mean temperature of the magnets was nearly 65°: in like manner I obtained a mean value of $\Delta . \frac{F}{M}$ when the mean temperature of the magnets was nearly 60°, and also when nearly 54°: taking a mean of these three means, I obtained a value of $\Delta . \frac{F}{M}$, which could not be sensibly different from its true value in any of the observations which were to be corrected. These are collected in the following table.

Table of Results for obtaining the Mean value of $\Delta . \frac{F}{M}$.

Mean Temperature 65° nearly.					Mean Temperature 60° nearly.					Mean Temperature 54° nearly.				
Date.	Temperatures.		Mean.	Value of $\Delta \frac{F}{M}$	Date.	Temperatures.		Mean.	Value of $\Delta . \frac{F}{M}$.	Date.	Temperatures.		Mean.	Value of $\Delta . \frac{F}{M}$.
June 6	62.05°	67.15°	64.600°	.1233	June 6	59.05°	62.05°	60.550°	.1268	June 17	49.30°	60.25°	54.775°	.1179
13	61.08	71.05	66.065	.1430	17	49.30	68.25	58.775	.1263	20	50.65	55.80	53.225	.1063
17	60.25	68.25	64.250	.1391	18	55.58	66.00	60.790	.1315	20	51.35	55.80	53.575	.1059
17	61.75	73.80	67.775	.1260	19	55.45	64.30	59.875	.1299	20	51.35	57.20	54.275	.1125
18	56.90	73.60	65.250	.1314	19	55.55	64.28	59.915	.1220	22	51.35	54.08	52.715	.1203
18	55.40	73.80	64.600	.1237	19	55.15	66.75	60.950	.1224	22	51.35	55.94	53.645	.1064
18	55.40	73.80	64.600	.1251			Mean	60.143	.12648			Mean	53.701	.11155
19	55.45	73.90	64.675	.1360										
19	55.55	73.90	64.725	.1311										
		Mean	65.171	.13097	Mean value of $\Delta . \frac{F}{M}$, to be applied in correcting the observations on the Diurnal changes in the positions of the Points of Equilibrium									0.12300

To apply the value of $\Delta . \frac{F}{M}$, thus determined, to the correction of the observed directions of the needle, for the changes which took place in the temperature of the magnets, let $\Delta \varphi$ be the increment of the azimuth of the needle corres-

ponding to the increment Δ F of the intensity of the magnets. When therefore F becomes F $+ \Delta$ F, putting the equation (C) in the form

$$M - (P + Q \,.\, \cos.^2 \varphi) \,.\, F = o,$$

it becomes

$$M - \left\{P + Q \,.\, \cos.^2 (\varphi + \Delta\,\varphi)\right\}(F + \Delta\,F) = o,$$

whence

$$\cos.^2 (\varphi + \Delta\,\varphi) = \frac{1}{Q} \,.\, \left\{\frac{M}{F + \Delta\,F} - P\right\}. \qquad (E)$$

This formula, though sufficiently simple, is not in the most convenient form for calculating the values of $\Delta\,\varphi$, corresponding to different values of φ, owing to the common tables of logarithms giving only seven places of figures. The values of P and $\frac{M}{F + \Delta F}$ agree in the first two or three figures, so that there will remain only five figures towards the beginning of the table, and four figures towards the end of it, from which the values of $\varphi + \Delta\,\varphi$ are to be derived, consequently they cannot be calculated to the greatest accuracy. But if $\frac{M}{F + \Delta F}$ be expanded, the first figure of $\frac{M}{F + \Delta F} - P$ being in the 5th place of decimals, the first figure of $\frac{(\Delta F)^2}{F^3}$ will be in the 9th place, and the first figure of $\frac{(\Delta F)^3}{F^4}$ will be in the 13th place; and therefore we should obtain the value of $\frac{M}{F + \Delta F} - P$ true to the 11th place of decimals, or true to 7 places of figures when we neglect the term $\frac{(\Delta F)^3}{F^4}$. Now in the cases which I had to compute, the first two figures in the value of $\frac{(\Delta F)^2}{F^3}$ were the same for all the arcs, and conse-

quently by using these, the value of $\frac{M}{F + \Delta F} - P$ for the determination of $\cos.^2 (\varphi + \Delta \varphi)$ would be true to the 6th figure, which would give $\varphi + \Delta \varphi$ to the tenth of a second. Such a degree of accuracy may appear quite uncalled for by the nature of the observations, but from the manner which I adopted for correcting them, it was necessary to guard against any accumulation of error.

From what I have said, we have

$$\cos.^2 (\varphi + \Delta \varphi) = \frac{1}{Q} \left\{ \frac{M}{F} - M \cdot \left(\frac{\Delta F}{F^2} - \frac{(\Delta F)^2}{F^3} \right) - P \right\};$$

but
$$\frac{M}{F} = P + Q \cos.^2 \varphi,$$

and therefore,

$$\cos.^2 (\varphi + \Delta \varphi) = \frac{1}{Q} \left\{ Q \cos.^2 \varphi + \frac{M (\Delta F)^2}{F^3} - \frac{M . \Delta F}{F^2} \right\}. \qquad (G)$$

Having, as we have seen, determined by observation ΔF in terms of M, and $\frac{M}{F}$, being computed from the equation (C), for any angle φ, the value of $\Delta \varphi$ would be readily computed from this formula: that is, we could obtain from it the correction to be made in any observed angle, for a change of 1° in the temperature of the magnets, whether that temperature were increasing or decreasing, only observing that ΔF is minus for an increase of temperature, and plus for a decrease.

The method which I have adopted for reducing the observed values of φ to what they would have been, had the temperature of the magnets been constant, is this: the observed values of φ being comprised between 74° and 86°, I computed the values of $\Delta \varphi$, both plus and minus, at inter-

vals of 30 minutes, from 74° to 86°, by means of the formula (G): from these and their several orders of differences, I interpolated the values of $\Delta\,\varphi$ at intervals of 6′ minutes: forming these, with their differences, into tables, I obtained from them, by inspection, the value of $\Delta\,\varphi$ corresponding to any observed angle: adding the plus value of $\Delta\,\varphi$ to the observed angle, when the temperature of the magnets was above the mean temperature to which the observations were to be reduced, I obtained the value of φ at a temperature of the magnets one degree lower than that observed: proceeding in the same manner with this corrected value of φ, I obtained another value at a temperature one degree lower than the last, or two degrees below the observed temperature: with this I proceeded again in the same manner, and so on, until the observed value of φ was reduced to its value at the standard temperature of the magnets. If the observed temperature was below the mean temperature, I successively subtracted the different minus values of $\Delta\,\varphi$ to obtain the corrected value of φ. This will perhaps be better understood when I come to the observations and their corrections; but I thought it necessary to explain the use which I made of these tables previous to giving them.

In the observations which I made within doors on the daily variation in the positions of the points of equilibrium, the distances of the nearest ends of the magnets from the centre of the needle were 15.21 inches, or the distances of their centres from the centre of the needle 21.21 inches; so that, as we have before seen, the equation (C) here becomes

$$M - (.004690814 + .000829329 \cos.^2 \varphi).\,F = o; \quad (\alpha)$$

consequently the equation (G) becomes

$$\cos.^2(\phi+\Delta\phi)=\frac{1}{.000829329}\times\Big\{.000829329\cos.^2\phi+.0000000016$$
$$\mp.123\times(.004690814+.000829329\cos.^2\phi)^2\Big\}=o\,;\quad\ldots.(\gamma)$$

.0000000016, being the value of $\frac{M.(\Delta F)^2}{F^2}$ in all the values of ϕ for which I had to compute; and .123 the value of $\Delta.\frac{F}{M}$ already found: the upper sign to be used when $\Delta.\frac{F}{M}$ is plus, or when the observed temperature of the magnets is above the mean temperature to which the observations are to be reduced, and the lower sign, when $\Delta\cdot\frac{F}{M}$ is minus.

This formula is not so ill adapted for calculation as it may at first sight appear, since for each value of ϕ it is only necessary to refer to the tables eight times to obtain the values both of $\phi+\Delta\,\phi$ and $\phi-\Delta_1\,\phi$, or of ϕ_1 and $\phi_{\underline{1}}$.

The values of ϕ, in the observations in doors, being comprised between 77° and 86°, I calculated the two following tables as the basis of the tables by which these observations were to be corrected, for the difference between the observed temperature of the magnets and the standard temperature.

1. *Table of the* increments *in the Azimuths of the Points of Equilibrium corresponding to a* decrement *of* 1° *in the Temperature of the Magnets, with their several orders of differences, calculated at intervals of* 30′ *in the Azimuths from* 77° *to* 86° *: the distances of the centres of the Magnets from the centre of the Needle being* 21.21 *inches.*

φ	$\Delta\varphi$	$\Delta^2\varphi$	$\Delta^3\varphi$	$\Delta^4\varphi$	$\Delta^5\varphi$	$\Delta^6\varphi$	$\Delta^7\varphi$
77° 00′	26′.455	0′.988					
30	27.443	1.080	0′.092	0′.012			
78 00	28.523	1.184	0.104	0.016	0′.004		
30	29.707	1.304	0.120	0.020	0.004		
79 00	31.011	1.444	0.140	0.024	0.004		
30	32.455	1.608	0.164	0.027	0.003		
80 00	34.063	1.799	0.191	0.035	0.008		
30	35.862	2.025	0.226	0.047	0.012		
81 00	37.887	2.298	0.273	0.064	0.017		
30	40.185	2.635	0.337	0.080	0.016		
82 00	42.820	3.052	0.417	0.109	0.029	0′.013	
30	45.872	3.578	0.526	0.159	0.050	0.021	
83 00	49.450	4.263	0.685	0.234	0.075	0.025	0′.032
30	53.713	5.182	0.919	0.366	0.132	0.057	0.061
84 00	58.895	6.467	1.285	0.616	0.250	0.118	0.210
30	65.362	8.368	1.901	1.194	0.578	0.328	0.651
85 00	73.730	11.463	3.095	2.751	1.557	0.979	
30	85.193	17.309	5.846				
86 00	102.502						

2. *Table of the* decrements *in the Azimuths of the Points of Equilibrium corresponding to an* increment *of* 1° *in the Temperature of the Magnets, with their several orders of differences, calculated at intervals of* 30′ *in the Azimuths from* 77° *to* 86°: *the distances of the centres of the Magnets from the centre of the Needle being* 21.21 *inches.*

φ	$\Delta\varphi$	$\Delta^2\varphi$	$\Delta^3\varphi$	$\Delta^4\varphi$	$\Delta^5\varphi$
° ′	′	′	′	′	′
77 00	25.675	0.890	0.073	0.011	0.000
30	26.565	0.963	0.084	0.011	0.002
78 00	27.528	1.047	0.095	0.013	0.001
30	28.575	1.142	0.108	0.014	0.002
79 00	29.717	1.250	0.122	0.016	0.004
30	30.967	1.372	0.138	0.020	0.004
80 00	32.339	1.510	0.158	0.024	0.007
30	33.849	1.668	0.182	0.031	0.008
81 00	35.517	1.850	0.213	0.039	0.004
30	37.367	2.063	0.252	0.043	0.010
82 00	39.430	2.315	0.295	0.053	0.018
30	41.745	2.610	0.348	0.071	0.018
83 00	44.355	2.958	0.419	0.089	0.020
30	47.313	3.377	0.508	0.109	0.027
84 00	50.690	3.885	0.617	0.136	0.046
30	54.575	4.502	0.753	0.182	
85 00	59.077	5.255	0.935		
30	64.332	6.190			
86 00	70.522				

By means of the values of $\Delta\,\varphi$ and their several orders of differences, contained in these tables, interpolating in the usual manner, I calculated the following tables.

I. *Table of the* increments *in the Azimuths of the Points of Equilibrium corresponding to a* decrement *of* 1° *in the Temperature of the Magnets, calculated at intervals of* 6′ *in the Azimuths from* 77° *to* 86°; *the distances of the centres of the magnets from the centre of the needle being* 21.21 *inches: to be applied to the correction of the observed Azimuths, when the Observed Temperature of the Magnets is* above *the Mean Temperature to which the observations are to be reduced.*

φ	77°		78°		79°		80°		81°	
	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.
′00	′26.455	.191	′28.523	.228	′31.011	.277	′34.063	.343	′37.887	.436
06	.646	.194	.751	.232	.288	.283	.406	.351	38.323	.447
12	.840	.197	.983	.237	.571	288	.757	.360	.770	.459
18	27.037	.201	29.220	.241	.859	.295	35.117	.368	39.229	.471
24	.238	.205	.461	.246	32.154	.301	.485	.377	.700	.485
30	.443	.209	.707	.250	.455	.308	.862	.386	40.185	.498
36	.652	.212	.957	.255	.763	.315	36.248	.395	.683	.512
42	.864	.216	30.212	.261	33.078	.322	.643	.405	41.195	.527
48	28.080	.220	.473	.266	.400	.328	37.048	.414	.722	.541
54	.300	.223	.739	.272	.728	.335	.462	.425	42.263	.557
60	.523		31.011		34.063		.887		.820	

φ	82°		83°		84°		85°	
	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.
′00	′42.820	.574	′49.450	.791	′58.895	1.173	′73.730	1.985
06	43.394	.591	50.241	.820	60.068	1.228	75.715	2.120
12	43.985	.609	51.061	.851	61.296	1.288	77.835	2.272
18	44.594	.629	51.912	.883	62.584	1.354	80.107	2.445
24	45.223	.649	52.795	.918	63.938	1.424	82.552	2.641
30	45.872	.670	53.713	.955	65.362	1.499	85.193	2.865
36	46.542	.691	54.668	.994	66.861	1.579	88.058	.120
42	47.233	.714	55.662	1.034	68.440	1.665	91.178	3.411
48	47.947	.739	56.696	1.077	70.105	1.759	94.589	3.753
54	48.686	.764	57.773	1.122	71.864	1.866	98.342	4.160
60	49.450		58.895		73.730		102.502	

II. *Table of the* decrements *in the Azimuths of the Points of Equilibrium corresponding to an* increment *of* 1° *in the Temperature of the Magnets, calculated at intervals of* 6′ *in the Azimuths from* 77° *to* 86°; *the distances of the centres of the Magnets from the centre of the Needle being* 21.21 *inches: to be applied to the correction of the observed Azimuths when the observed Temperature of the Magnets is* below *the Mean Temperature to which the observations are to be reduced.*

φ	77°		78°		79°		80°		81°	
	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.
′	′		′		′		′		′	
00	25.675	.173	27.528	.203	29.717	.241	32.339	.290	35.517	.354
06	.848	.175	.731	.206	.958	.245	.629	.296	.871	.362
12	26.023	.178	.937	.209	30.203	.250	.925	.301	36.233	.370
18	.201	.181	28.146	.213	.453	.255	33.226	.308	.603	.378
24	.382	.183	.359	.216	.708	.259	.534	.315	.981	.386
30	.565	.187	.575	.220	.967	.264	.849	.320	37.367	.394
36	.752	.189	.795	.224	31.231	.269	34.169	.327	.761	.403
42	.941	.192	29.019	.228	.500	.274	.496	.333	38.164	.413
48	27.133	.196	.247	.233	.774	.280	.829	.340	.577	.422
54	.329	.199	.480	.237	32.054	.285	35.169	.348	.999	.431
60	.528		.717		.339		.517		39.430	

φ	82°		83°		84°		85°	
	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.
′	′		′		′		′	
00	39.430	.442	44.355	.562	50.690	.733	59.077	.985
06	.872	.452	.917	.576	51.423	.755	60.062	1.017
12	40.324	.462	45.493	.591	52.178	.776	61.079	1.049
18	.786	.474	46.084	.607	52.954	.799	62.128	1.084
24	41.260	.485	.691	.622	53.753	.822	63.212	1.120
30	.745	.497	47.313	.639	54.575	.847	64.332	1.157
36	42.242	.509	.952	.657	55.422	.873	65.489	1.195
42	.751	.522	48.609	.675	56.295	.899	66.684	1.236
48	43.273	.534	49.284	.693	57.194	.927	67.920	1.279
54	.807	.548	.977	.713	58.121	.956	69.199	1.323
60	44.355		50.690		59.077		70.522	

These tables are calculated for the distance 21.21 inches, that at which the centres of the magnets were from the centre of the needle during the observations which I made in-doors, and they would, without any great error, serve also for the correction of the observations in the open air, where the distances were 21.52 inches; but I would not, for the sake of avoiding the labour of computing fresh tables, which however was by no means inconsiderable, leave any doubt on the nature of the diurnal changes in the two cases.

We have already seen that, when $R = 21.52$, the equation (C) becomes,

$$M - F\,.\,(.004480432 + .0007664093 \cos.^2\phi) = 0, \quad (\alpha_4)$$

so that in this case the equation G becomes

$$\cos.^2(\phi + \Delta\phi) = \frac{1}{.0007664093}\Big\} \times .0007664093 \cos.^2\phi + .0000000014$$
$$\mp .123 \times (.004480432 + .0007664093 \cos.^2\phi)^2\Big\} = 0 \quad (\gamma_1)$$

where .0000000014 is the value of $\frac{M\,.\,(\Delta F)^2}{F^3}$ in all the values of ϕ between 74° and 86°, the values which I had in this case to compute.

From this formula I calculated the following tables, as in the preceding case, excepting that, as in the observations in the open air, the temperature of the magnets varied more considerably, I had, in correcting them, more frequently to repeat the additions and subtractions, and therefore from 82° to 86°, where the values of $\Delta\phi$ change rapidly, I calculated the values of $\Delta\phi$ at intervals of 15′ for the fundamental tables, and interpolated at intervals of 3′ for the tables to be applied to the correction of the observations.

3. *Table of the* increments *in the Azimuths of the Points of Equilibrium corresponding to a* decrement *of* 1° *in the Temperature of the Magnets, with their several orders of differences, calculated at intervals of* 30′ *in the Azimuths from* 74° *to* 82°, *and at intervals of* 15′ *in those from* 82° *to* 86°; *the distances of the centres of the Magnets from the centre of the Needle being* 21.52 *inches.*

φ	$\Delta\varphi$	$\Delta^2\varphi$	$\Delta^3\varphi$	$\Delta^4\varphi$	$\Delta^5\varphi$	$\Delta^6\varphi$	$\Delta^7\varphi$
° ′	′	′	′	′	′	′	′
74 00	21.653	0.607					
30	22.260	0.653	0.046				
75 00	22.913	0.704	0.051	0.005			
30	23.617	0.761	0.057	0.006			
76 00	24.378	0.824	0.063	0.005			
30	25.202	0.895	0.071	0.008	0.001		
77 00	26.097	0.975	0.080	0.009	0.002		
30	27.072	1.066	0.091	0.011	0.002		
78 00	28.138	1.169	0.103	0.012	0.003		
30	29.307	1.287	0.118	0.015	0.004		
79 00	30.594	1.424	0.137	0.019	0.005		
30	32.018	1.585	0.161	0.024	0.004	0.003	
80 00	33.603	1.774	0.189	0.028	0.007	0.003	
30	35.377	1.998	0.224	0 035	0.010	0.005	
81 00	37.375	2.267	0.269	0.045	0.015		
30	39.642	2.596	0.329	0.060			
82 00	42.238	1.446					
15	43.684	1.560	0.114				
30	45.244	1.689	0.129	0.015	0.003		
45	46.933	1.836	0.147	0.018	0.002		
83 00	48.769	2.003	0.167	0.020	0.005	0.001	
15	50.772	2.195	0.192	0.025	0.006	0.002	
30	52.967	2.418	0.223	0. 31	0.008	0.001	
45	55.385	2.680	0.262	0.039	0.009	0.007	
84 00	58.065	2.990	0.310	0.048	0.016	0.004	
15	61.055	3.364	0.374	0.064	0.020	0.010	
0	64.419	3.822	0.458	0.084	0.030	0.015	0.005
45	68.241	4.394	0.572	0.114	0.045	0. 33	0.018
85 00	72.635	5.125	0.731	0.159	0.078	0.061	0.028
15	77.760	6.093	0.968	0.237	0.139	0.129	0.068
30	83.853	7.437	1.344	0.376	0.268		
45	91.290	9.425	1.988	0.644			
86 00	100.715						

4. *Table of the* decrements *in the Azimuths of the Points of Equilibrium corresponding to* an increment *of* 1° *in the Temperature of the Magnets, with their several orders of differences, calculated at intervals of* 30′ *in the Azimuths from* 74° *to* 82°, *and at intervals of* 15′ *in those from* 82° *to* 86°; *the distances of the centres of the Magnets from the centre of the needle being* 21.52 *inches.*

φ	$\Delta\varphi$	$\Delta^2\varphi$	$\Delta^3\varphi$	$\Delta^4\varphi$
° ′	′	′	′	′
74 00	21.249	0.566		
30	21.815	0.606	0.040	
75 00	22.421	0.650	0.044	0.004
30	23.071	0.699	0.049	0.005
76 00	23.770	0.754	0.055	0 006
30	24.524	0.814	0.060	0.005
77 00	25.338	0.879	0.065	0.005
30	26.217	0.952	0.073	0.008
78 00	27.169	1.035	0.083	0.010
30	28.204	1.130	0.095	0.012
79 00	29.334	1.235	0.105	0.010
30	30.569	1.354	0.119	0.014
80 00	31.923	1.491	0.137	0.018
30	33.414	1.649	0.158	0.021
81 30	35.063	1.831	0.182	0.024
30	36.894	2.042	0.211	0.029
82 00	38.936	1.111		
15	40.047	1.177	0.066	
30	41.224	1.250	0.073	0.007
45	42.474	1.331	0.081	0.008
83 00	43.805	1.417	0.086	0.005
15	45.222	1.511	0.094	0.008
30	46.733	1.614	0.103	0.009
45	48.347	1.728	0.114	0.011
84 00	50.075	1.853	0.125	0.011
15	51.928	1.990	0.137	0.012
30	53.918	2.142	0.152	0.015
45	56.060	2.311	0.169	0.017
85 00	58.371	2.500	0.189	0.020
15	60.871	2.708	0.208	0.019
30	63.579	2.939	0.231	0.023
45	66.518	3.199	0.260	0.029
86 00	69.717			

From these tables, interpolating as before, I constructed the two following.

III. *Table of the* increments *in the Azimuths of the Points of Equilibrium corresponding to a* decrement *of* 1° *in the Temperature of the Magnets, calculated at intervals of* 6′ *in the Azimuths from from* 74 *to* 82°, *and of* 3′ *in those from* 82° *to* 86°; *the distances of the centres of the Magnets from the centre of the Needle being* 21.52 *inches: to be applied to the correction of the observed Azimuths when the Observed Temperature of the Magnets is* above *the Mean Temperature to which the observations are to be reduced.*

φ	74°		75°		76°		77°	
′	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.
00	21.653	.118	22.913	.137	24.378	.160	26.097	.188
06	.771	.120	23.050	.138	.538	.162	.285	.192
12	.891	.121	.188	.141	.700	.164	.477	.195
18	22.012	.123	.329	.143	.864	.168	.672	.198
24	.135	.125	.472	.145	25.032	.170	.870	.202
30	.260	.127	.617	.148	.202	.173	27.072	.206
36	.387	.129	.765	.149	.375	.176	.278	.209
42	.516	.130	.914	.152	.551	.179	.487	.213
48	.546	.133	24.066	.154	.730	.182	.700	.217
54	.679	.134	.221	.157	.912	.185	.917	.221
60	.913		.378		26.097		28.138	

φ	78°		79°		80°		81°	
′	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.
00	28.138	.225	30.594	.273	33.603	.339	37.375	.430
06	.363	.229	.867	.279	.942	.346	.805	.442
12	.592	.234	31.146	.285	34.388	.355	38.247	.453
18	.826	.238	.431	.290	.643	.363	.700	.465
24	29.064	.243	.721	.297	35.006	.371	39.165	.477
30	.307	.247	32.018	.303	.377	.380	.642	.491
36	.554	.253	.321	.310	.757	.390	40.133	.504
42	.807	.257	.631	.317	36.147	.399	.637	.518
48	30.064	.262	.948	.324	.546	.410	41.155	.534
54	.326	.268	33.272	.331	.956	.419	.689	.549
60	.594		.603		37.375		42.238	

Table III. continued.

φ	82°		83°		84°		85°	
′	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.
00	42.238	.281	48.769	.387	58.065	.572	72.635	.960
03	.519	.285	49.156	.393	.637	.584	73.595	.991
06	.804	.289	.549	.400	59.221	.597	74.586	1.024
09	43.093	.293	.949	.408	.818	.612	75.610	1.057
12	.386	.298	50.357	.415	60.430	.625	76.667	1.093
15	.684	.302	.772	.423	61.055	.641	77.760	1.132
18	.986	.307	51.195	.430	.696	.656	78.892	1.174
21	44.293	.312	.625	.439	62.352	.672	80.064	1.216
24	.605	.317	52.064	.447	63.024	.689	81.280	1.262
27	.922	.322	.511	.456	.713	.706	82.542	1.311
30	45.244	.327	.967	.464	64.419	.725	83.853	1.364
33	.571	.332	53.431	.474	65.144	.743	85.217	1.422
36	.903	.338	.905	.484	.887	.764	86.639	1.482
39	46.241	.343	54.389	.493	66.651	.784	88.121	1.549
42	.584	.349	.882	.503	67.435	.806	89.670	1.620
45	.933	.355	55.385	.514	68.241	.829	91.290	1.698
48	47.288	.361	.899	.524	69.070	.853	92.988	1.784
51	.649	.367	56.423	.536	.922	.878	94.772	1.876
54	48.016	.373	.959	.547	70.800	.904	96.648	1.978
57	.389	.380	57.506	.559	71.704	.929	98.626	2.089
60	.769		58.065		72.635		100.715	

IV. *Table of the* decrements *in the Azimuths of the Points of Equilibrium corresponding to an* increment *of* 1° *in the Temperature of the Magnets, calculated at intervals of* 6′ *in the Azimuths from* 74° *to* 82°, *and of* 3′ *in those from* 82° *to* 86°; *the distances of the centres of the Magnets from the centre of the Needle being* 21.52 *inches: to be applied to the correction of the observed Azimuths when the observed Temperature of the Magnets is* below *the Mean Temperature to which the observations are to be reduced.*

φ	74°		75°		76°		77°	
′	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.
00	21.249	.110	22.421	.126	23.770	.146	25.338	.170
06	.359	.112	.547	.128	.916	.149	.508	.173
12	.471	.113	.675	.130	24.065	.151	.681	.176
18	.584	.115	.805	.132	.216	.153	.857	.179
24	.699	.116	.937	.134	.369	.155	26.036	.181
30	.815	.118	23.071	.136	.524	.158	.217	.184
36	.933	.119	.207	.137	.682	.160	.401	.188
42	22.052	.122	.344	.140	.842	.163	.589	.190
48	.174	.123	.484	.142	25.005	.165	.779	.193
54	.297	.124	.626	.144	.170	.168	.972	.197
60	.421		.770		.338		27.169	

φ	78°		79°		80°		81°	
′	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.
00	27.169	.200	29.334	.238	31.923	.287	35.063	.351
06	.369	.203	.572	.243	32.210	.292	.414	.358
12	.572	.207	.815	.247	.502	.298	.772	.366
18	.779	.211	30.062	.251	.800	.304	36.138	.374
24	.990	.214	.313	.256	33.104	.310	.512	.382
30	28.204	.218	.569	.261	.414	.316	.894	.391
36	.422	.222	.830	.265	.730	.323	37.285	.399
42	.644	.226	31.095	.271	34.053	.330	.684	.408
48	.870	.230	.366	.276	.383	.337	38.092	.417
54	29.100	.234	.642	.281	.720	.343	.509	.427
60	.334		.923		35.063		.936	

Table IV. continued.

φ	82°		83°		84°		85°	
r	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.
00	′38.936	.217	′43.805	.276	′50.075	.360	′58.371	.484
03	39.153	.220	44.081	.280	.435	.366	.855	.492
06	.373	.222	.361	.283	.801	.370	59.347	.500
09	.595	.225	.644	.287	51.171	.376	.847	.508
12	.820	.227	.931	.291	.547	.381	60.355	.516
15	40.047	.230	45.222	.294	.928	.387	.871	.524
18	.277	.233	.516	.299	52.315	.392	61.395	.533
21	.510	.235	.815	.302	.707	.398	.928	.541
24	.745	.238	46.117	.306	53.105	.403	62.469	.551
27	.983	.241	.423	.310	.508	.410	63.020	.559
30	41.224	.244	.733	.314	.918	.416	.579	.569
33	.468	.247	47.047	.319	54.334	.422	64.148	.578
36	.715	.250	.366	.322	.756	.428	.726	.587
39	.965	.253	.688	.327	55.184	.435	65.313	.598
42	42.218	.256	48.016	.332	.619	.441	.911	.607
45	.474	.260	.347	.336	56.060	.448	66.518	.618
48	.734	.263	.683	.341	.508	.455	67.136	.629
51	.997	.266	49.024	.345	.963	.462	.765	.639
54	43.263	.269	.369	.351	57.425	.469	68.404	.651
57	.532	.273	.720	.355	.894	.477	69.055	.662
60	.805		50.075		58.371		69.717	

I now proceed to the observations for the correction of which these tables were calculated. My principal object in making these observations, was to ascertain how far they would enable me to determine the diurnal changes in the terrestrial magnetic intensity, and whether a series of such observations would not afford very accurate measures of such changes; and I have before stated that I made them both within-doors and in the open air, in order to ascertain whether I had, in my former paper, assigned the true cause of the apparent anomalies which were noticed by Mr. Barlow and myself in these different situations.

The first observations were made in-doors, in the same

room as those from the 20th to the 27th of April, described in my former paper. The compass was placed on an horizontal table, with its centre at the distance of 5 feet from the middle of the only window in the room, and which was nearly in the direction of the magnetic meridian from it. I mention this circumstance, not that I myself consider it of importance, but as a datum for those who may be disposed to attribute the diurnal changes in the direction of the needle to the influence of light. The only iron in the room is a large lock to the door and the weights to the window, which, when the observations were made, were always in the same position. The magnets were placed on the table with their axes, as nearly as I could adjust them, in the meridian, the north pole of each being, as I have before mentioned, towards the north, and the distances of their centres from the centre of the needle 21.21 inches.

The method which I at first adopted for determining the changes that took place in the temperature of the magnets, was by placing a thermometer with the bulb *near* the southern extremity of the north magnet. In this manner I continued to observe for five days: I then placed the bulb of the thermometer *on* the southern extremity of the north magnet; and continued the observations for five days longer. I consider that the changes in the thermometer would, in either case, very nearly indicate the changes in the temperature of the magnets, especially as those changes were very gradual, and did not exceed 10° during the whole time in which the observations were made.

In the present state of our knowledge respecting the causes of magnetical phænomena, it is difficult to say how far

atmospheric changes may influence the direction and intensity of the terrestrial magnetism; I consider, therefore, in order that all possible information should be derived from a series of such observations as I am about to describe, that they should be accompanied with very precise observations of all the atmospheric changes which take place, particularly those of an electric nature. It was not always in my power to note these with the requisite precision; and as the observations were not continued for a sufficient length of time to enable me to derive any thing from those which I made on the force and direction of the wind, the character and appearance of the clouds, &c. I omit them: I have however inserted the changes which I noticed in the state of the barometer.

Table of observations, made within doors, *on the Diurnal Changes in the positions of the Points of Equilibrium at which a Magnetic Needle was retained by the joint action of Terrestrial Magnetism and of two bar Magnets, having their axes horizontal and in the magnetic meridian, and their centres at the distance* 21.21 *inches from the centre of the Needle.*

Date and Time of Observation.		Temp. of the Magnets.	Points of Equilibrium. Westerly.	Easterly.	South.	Barometer.	Therm. attached.
	h. m.		° ′	° ′	° ′		°
22d May 1823. Morning.	6 00	59.75	82 06	80 24	0 04 W	29.75	59.66
	7 30	60.00	82 44	81 06	0 02 W	29.75	59.66
	8 55	60.75	82 20	81 06	0 06 W	29.74	60.00
	10 30	60.75	82 34	81 30	0 14 W	29.75	60.50
Afternoon.	0 00	61.00	82 04	80 00	0 44 W	29.75	60.50
	1 45	61.15	81 38	79 16	0 34 W	29.76	59.75
	3 05	60.75	81 20	80 20	0 10 W	29.76	60.00
	4 35	60.50	81 34	80 40	0 06 E	29.75	60.25
	6 10	60.00	81 20	80 46	0 04 E	29.80	57.75
	7 40	59.50	81 50	80 52	0 02 W	29.82	58.00
	9 30	58.75	82 04	80 32	0 10 W	29.86	58.50

Table of observations made within doors, &c.

Date and time of Observation.		Temperature of the Magnets.	Points of Equilibrium. Westerly.	Points of Equilibrium. Easterly.	Points of Equilibrium. South.	Barometer.	Therm. attached.
	h. m.		° ′	° ′	° ′		° ′
23d May, 1823. Morning.	6 30	58.50	83 14	82 22	0 14 E	29.82	59.00
	7 35	59.00	83 38	82 26	0 08 E	29.82	58.75
	9 10	59.00	84 38	83 36	0 18 E	29.81	58.00
	10 35	60.00	84 40	83 20	0 00	29.82	58.50
Afternoon	0 10	59.80	83 12	81 22	0 28 W	29.85	59.00
	1 35	59.75	82 38	80 20	0 34 W	29.85	59.20
	3 05	59.50	82 50	80 20	0 28 W	29.87	59.33
	4 35	59.25	82 42	80 04	0 18 W	29.89	59.50
	6 05	59.50	83 12	80 42	0 06 W	29.91	60.25
	7 27	59.00	82 40	81 00	0 00	29.94	59.00
	9 40	58.00	83 26	81 18	0 12 W	29.98	58.00
24th May. Morning.	6 00	57.00	84 12	82 26	0 02 E	29.98	57.66
	7 30	57.66	85 04	83 34	0 04 W	29.97	55.66
	9 05	57.75	86 46	84 14	0 00	29.97	55.80
	10 30	58.75	86 00	84 14	0 16 W	29.96	56.50
Afternoon.	0 05	60.00	84 26	82 02	0 28 W	29.96	59.25
	1 35	59.90	82 46	81 04	0 38 W	29.96	61.10
	3 00	60.80	83 56	81 32	0 34 W	29.96	61.20
	4 30	60.20	81 08	79 42	0 26 W	29.96	60.00
	6 00	60.20	82 10	80 38	0 06 W	29.95	60.00
	7 35	59.30	81 48	80 38	0 14 W	29.93	59.50
	9 35	58.75	82 34	80 42	0 16 W	29.91	57.75
25th May. Morning.	6 00	58.25	83 10	82 08	0 04 E	29.77	59.10
	7 35	59.33	84 12	82 10	0 04 E	29.74	60.10
	9 05	60.10	84 24	82 36	0 12 W	29.74	61.00
	10 45	61.00	82 46	80 38	0 26 W	29.74	62.50
Afternoon.	0 00	61.00	82 26	80 26	0 34 W	29.74	62.50
	1 40	61.10	82 06	80 00	0 28 W	29.74	62.40
	3 00	61.50	81 32	79 34	0 26 W	29.74	61.75
	4 30	No observation.					
	6 00	61.00	81 20	79 26	0 24 W	29.74	61.50
	7 30	60.50	81 36	80 04	0 18 W	29.74	60.25
	9 45	59.75	82 04	80 42	0 14 W	29.74	61.25
26th May. Morning.	6 00	59 00	83 26	81 32	0 06 W	29.75	59.75
	7 30	59 75	83 10	81 52	0 08 E	29.75	59.50
	9 00	59 75	84 36	83 20	0 16 E	20.74	59.25
	10 30	60 50	86 00	83 50	0 14 W	29.75	59.75
Afternoon.	0 40	61 10	83 12	80 42	0 46 W	29.76	61.75
	1 30	61 50	82 40	80 22	0 54 W	29.77	62.50
	3 00	61 50	82 00	79 52	0 36 W	29.80	61.75
	5 00	61 50	81 42	80 08	0 14 W	29.81	62.10
	5 55	61 50	81 10	79 38	0 14 W	29.83	62.00
	7 40	61.00	81 00	79 38	0 10 W	29.85	61.50
	9 30	60.00	81 34	79 50	0 16 W	29.89	60.00

Table of observations made within dooors, &c.

Date and time of Observation.		Temperature of the Magnets.	Points of Equilibrium.			Barometer.	Therm. attached.
			Westerly.	Easterly.	South.		
	h. m.	°	° ′	° ′	° ′	° ′	°
27th May, 1823. Morning.	6 00	59.00	83 00	81 52	0 04 E	29.98	59.25
	7 35	60.00	82 36	81 38	0 14 E	29.99	57.50
	9 05	60.90	83 26	82 18	0 06 E	30.00	59.80
	10 30	61.50	83 34	82 20	0 10 W	30.01	61.25
Afternoon.	0 00	62.50	82 36	80 58	0 40 W	30.02	63.80
	1 30	63.00	81 34	79 54	0 46 W	30.02	64.30
	3 00	62.60	81 32	79 44	0 34 W	30.02	64.40
	4 30	61.75	81 34	80 08	0 10 W	30.02	61.75
	6 00	61.00	81 40	80 38	0 04 W	30.02	60.25
	7 30	60.20	81 30	80 04	0 06 W	30.02	59.50
	9 40	60.00	81 58	80 20	0 08 W	30.03	59.50
	11 25	60.00	81 40	80 00	0 02 E	30.04	60.60
28th May. Morning.	6 07	59.75	82 30	81 42	0 14 E	30.07	61.00
	7 30	60.10	83 12	81 52	0 04 E	30.07	59.00
	9 00	60.50	83 28	81 52	0 00 W	30.09	62.00
	10 25	61.00	83 40	81 42	0 24 W	30.09	64.75
Afternoon.	0 00	61.20	83 34	81 22	0 44 W	30.09	66.00
	1 30	61.50	82 04	80 06	0 52 W	30.09	66.25
	3 00	62.30	80 48	79 04	0 46 W	30.09	66.90
	4 30	63.25	80 56	79 46	0 12 W	30.09	66.50
	6 00	63.00	80 44	79 38	0 08 W	30.09	65.50
	7 20	62.75	80 10	79 20	0 16 W	30.09	64.00
	9 30	61.25	81 34	79 58	0 12 W	30.10	58.80
	11 20	61.00	81 54	80 14	0 16 W	30.15	61.80
29th May. Morning.	6 20	60.00	82 46	81 18	0 06 W	30.18	60.60
	7 30	60.40	82 52	81 18	0 04 W	30.19	60.30
	9 00	61.00	82 58	81 24	0 04 W	30.19	55.00
	10 30	62.75	83 24	82 10	0 20 W	30.19	57.00
Afternoon.	0 10	63.00	82 46	80 48	0 22 W	30.19	59.25
	1 30	62.00	82 42	80 16	0 36 W	30.19	60.75
	3 00	62.30	81 52	79 38	0 32 W	30.19	62.20
	4 30	63.75	80 46	79 16	0 18 W	30.19	63.25
	6 10	63.30	80 38	78 56	0 20 W	30.19	63.00
	7 30	62.00	80 54	79 18	0 18 W	30.19	60.25
	9 45	60.75	81 24	79 36	0 10 W	30.19	58.50
	11 20	60.50	81 42	80 12	0 10 W	30.20	60.50
30th May. Morning.	6 05	60.30	82 10	80 50	0 02 E	30.22	60.50
	7 30	61.30	82 12	80 38	0 02 W	30.23	59.40
	9 00	62.00	82 30	81 40	0 12 E	30.23	62.75
	10 30	62.25	82 46	81 50	0 02 W	30.24	64.25
Afternoon.	0 10	64.00	82 04	80 22	0 40 W	30.24	65.50
	1 45	63.50	81 58	79 42	0 36 W	30.24	65.40
	3 00	63.75	80 52	78 44	0 28 W	30.24	65.75
	4 30	64.00	80 10	78 38	0 06 W	30.24	66.75
	5 55	63.50	79 50	78 16	0 18 W	30.24	66.00
	7 22	63.20	80 28	78 44	0 16 W	30.24	65.00
	9 40	62.25	80 52	79 04	0 16 W	30.25	63.00
	11 10	62.00	81 00	79 26	0 16 W	30.26	63.25

Table of observations made within doors, &c.

Date and time of observation.		Temperature of the Magnets.	Points of Equilibrium.			Barometer.	Therm. attached.
			Westerly.	Easterly.	South.		
	h. m.	°	° ′	° ′	° ′	° ′	°
31st May, 1823. Morning.	6 10	62.00	81 50	80 12	0 04 E	30.27	63.00
	7 30	62.50	81 56	80 26	0 08 E	30.28	62.00
	9 00	63.00	82 56	81 18	0 06 E	30.28	64.50
	10 30	63.40	82 32	80 40	0 16 W	30.27	65.75
Afternoon.	0 00	63.66	81 32	79 34	0 24 W	30.28	66.00
	1 30	63.75	81 04	79 02	0 32 W	30.28	66.33
	3 00	64.20	80 10	78 38	0 30 W	30.28	66.33
	4 30	65.00	80 10	78 36	0 20 W	30.28	67.75
	5 55	65.00	79 40	78 24	0 16 W	30.27	65.66
	7 30	64.66	79 44	78 20	0 16 W	30.27	63.50
	9 25	63.33	79 54	78 50	0 16 W	30.26	61.66
	11 30	63.00	80 28	79 00	0 16 W	30.26	64.00

In all the observations which I have made, I have considered the magnetic meridian to be the line of direction of a needle at the time when that direction is most stationary, that is at about seven o'clock in the evening; and in arranging the magnets for the foregoing and similar observations, I have not only always found much difficulty, but have seldom succeeded, in determining so accurately the axes of the magnets, and adjusting them so precisely in the meridian, that, at that time, the needle should be held in equilibrio exactly at south, and also at points towards the west and east equidistant from the north, which evidently ought to be the case with a perfect adjustment. Partly from this difficulty in adjusting the magnets, of which those who have attempted similar arrangements will be best aware, and partly from the changes which, even during the evening, take place in the direction and intensity of the terrestrial forces, the east and west points of equilibrium, in the foregoing observations, are not, during the evening, at equal

distances from the north, nor is the south point exactly at south.* In order to reduce the situations of these points to their distances from what ought to be considered as their meridian, I take the mean of the azimuths of the westerly point at the evening observations, which is 81° 27′, and also of the corresponding azimuths of the easterly points, 79° 57′; half their difference will be the mean error in the point which has been considered as zero of the compass with reference to these points: so that if 45′ be subtracted from each of the azimuths of the westerly point, and added to those of the easterly, these points will be reduced very nearly to what would have been their positions had all the adjustments been perfect. With regard to the southerly point of equilibrium, the mean of the evening observations gives its position 12′ W; this therefore should be subtracted from the westerly and added to the easterly, in order to reduce the observed deviations to those from the meridian. These reductions I have made in the following table, preparatory to the reduction to be made in consequence of the changes in the temperature of the magnets.

Table of the preceding observations reduced to their Mean Magnetic Meridian.

Time of Observation.	May 22.				23				24				25				26				Mean Positions of the South Point.
	Temperature of the Magnets.	Points of Equilibrium.			Temperature of the Magnets.	Points of Equilibrium.			Temperature of the Magnets.	Points of Equilibrium.			Temperature of the Magnets.	Points of Equilibrium.			Temperature of the Magnets.	Points of Equilibrium.			
		West.	East.	South.		West.	East.	South.		West.	East.	South.		West.	East.	South.		West.	East.	South.	
h. m.	°	° ′	° ′	° ′	°	° ′	° ′	° ′	°	° ′	° ′	° ′	°	° ′	° ′	° ′	°	° ′	° ′	° ′	
6 00	59.75	81 21	81 09	0 08 E	58.50	82 29	83 07	0 26 E	57.00	83 27	83 11	0 14 E	58.25	82.25	82 53	0 16 E	59.00	82 41	82 17	0 06 E	0 14.0 E
7 30	60.00	81 59	81 51	0 10 E	59.00	82 53	83 11	0 20 E	57.66	84 19	84 19	0 08 E	59.33	83.27	82 55	0 16 E	59.75	82 25	82 37	0 20 E	0 14.8 E
9 00	60.75	81 35	81 51	0 06 E	59.00	83 53	84 21	0 30 E	57.75	86 01	84 59	0 12 E	60.10	83.39	83 21	0 00	59.75	83 51	84 05	0 28 E	0 15.2 E
10 30	60.75	81 49	82 15	0 02 W	60.00	83 55	84 05	0 12 E	58.75	85 15	84 59	0 04 W	61.00	82.01	81 23	0 14 W	60.50	85 15	84 35	0 02 W	0 02.0 W
Noon.	61.00	81 19	80 45	0 32 W	59.80	82 27	82 07	0 16 W	60.00	83 41	82 47	0 16 W	61.00	81.41	81 11	0 22 W	61.10	82 27	81 27	0 34 W	0 24.8 W
1 30	61.15	80 53	80 01	0 22 W	59.75	81 53	81 05	0 22 W	59.90	82 01	81 49	0 26 W	61.10	81.21	80 45	0 16 W	61.50	81 55	81 07	0 42 W	0 25.6 W
3 00	60.75	80 35	81 05	0 02 E	59.50	82 05	81 05	0 16 W	60.80	83 11	82 17	0 22 W	61.50	80.47	80 19	0 14 W	61.50	81 15	81 37	0 24 W	0 14.8 W
4 30	60.50	80 49	81 25	0 18 E	59.25	81 57	80 49	0 06 W	60.20	80 23	80 27	0 14 W	No observation.				61.50	80 57	80 53	0 02 W	0 01.0 W
6 00	60.00	80 35	81 31	0 16 E	59.50	82 27	81 27	0 06 E	60.20	81 25	81 23	0 06 E	61.00	80 35	80 11	0 12 W	61.50	80 25	80 23	0 02 W	0 02.8 E
7 30	59.50	81 05	81 37	0 10 E	59.00	81 55	81 45	0 12 E	59.30	81 03	81 23	0 02 W	60.50	80 51	80 49	0 06 W	61.00	80 15	80 23	0 02 E	0 03.2 E
9 30	58.75	81 19	81 17	0 02 E	58.00	82 41	82 03	0 00	58.75	81 51	81 27	0 04 W	59.75	81 19	81 27	0 02 W	60.00	80 49	80 35	0 04 W	0 01.6 W
	May 27.				28				29				30				31				
6 00	59.00	82 15	82 37	0 16 E	59.75	81 45	82 27	0 26 E	60.00	82 01	82 03	0 06 E	60.30	81 25	81 35	0 14 E	62.00	81 05	80 57	0 16 E	0 15.6 E
7 30	60.00	81 51	82 23	0 26 E	60.10	82 27	82 37	0 16 E	60.40	82 06	82 03	0 08 E	61.30	81 27	81 23	0 10 E	62.50	81 11	81 11	0 20 E	0 16.0 E
9 00	60.90	82 41	83 03	0 18 E	60.50	82 43	82 37	0 12 E	61.00	82 13	82 09	0 08 E	62.00	81 45	82 25	0 24 E	63.00	82 13	82 03	0 18 E	0 16.0 E
10 30	61.50	82 49	83 05	0 02 E	61.00	82 55	82 27	0 12 W	62.75	82 39	82 55	0 08 W	62.25	82 01	82 35	0 10 E	63.40	81 47	81 25	0 04 W	0 02.4 W
Noon.	62.50	81 51	81 43	0 28 W	61.20	82 49	82 07	0 32 W	63.00	82 01	81 33	0 10 W	64.00	81 19	81 07	0 28 W	63.66	80 47	80 19	0 12 W	0 22.0 W
1 30	63.00	80 49	80 39	0 34 W	61.50	81 19	80 51	0 40 W	62.00	81 57	81 01	0 24 W	63.50	81 13	80 27	0 24 W	63.75	80 19	79 47	0 20 W	0 28.4 W
3 00	62.60	80 47	80 29	0 22 W	63.30	80 03	79 49	0 34 W	62.30	81 07	80 23	0 20 W	63.75	80 07	79 29	0 16 W	64.20	79 25	79 23	0 18 W	0 22.0 W
4 30	61.75	80 49	80 53	0 04 W	63.25	80 11	80 31	0 00	63.75	80 01	80 01	0 06 W	64.00	79 25	79 23	0 06 E	65.00	79 25	79 21	0 08 W	0 02.4 W
6 00	61.00	80 55	81 23	0 08 E	63.00	79 59	80 23	0 04 E	63.30	79 53	79 41	0 08 W	63.50	79 05	79 01	0 06 W	65.00	78 55	79 09	0 04 W	0 01.4 W
7 30	60.20	80 45	80 49	0 06 E	62.75	79 25	80 05	0 01 W	62.00	80 09	80 03	0 06 W	63.20	79 43	79 29	0 04 W	64.66	78 59	79 05	0 04 W	0 02.4 W
9 30	60.00	81 13	81 05	0 04 E	61.25	80 49	80 43	0 00	60.75	80 39	80 21	0 02 E	62.25	80 07	79 49	0 04 W	63.33	79 09	79 35	0 04 W	0 00.4 W
11 20	60.00	80 55	80 45	0 14 E	61.00	81 09	80 59	0 04 W	60.50	80 57	80 57	0 02 E	62.00	80 15	80 11	0 04 W	63.00	79 43	79 45	0 04 W	0 00.8 W

To reduce these observed positions of the points of equilibrium to their true positions, that is, those which they would have had if the temperature of the magnets had been the same at each of the observations, it is necessary to apply a correction by means of Tables I. and II.; and that the nature of this reduction may be evident, I shall give an instance of the process at length of applying the tables to the correction of the observations, when the temperature at which they were made was *below* the standard temperature, and also when it was *above* that temperature. As the observations were made with the magnets at temperatures varying nearly equally *above* and *below* 60°, I consider that, the standard temperature to which to reduce them. The two following are instances of this reduction.

1st. Observed temperature *below* the standard temperature.

24th May, $6^h\ 00^m$ A.M.	Westerly.	Easterly.	
Points of Equilibrium	83° 27	83 11	at temp. 57°
Correction for 1° Temp. Table II.	— 47.002	— 45.397	
Points of Equilibrium -	82 39.998	82 25.603	at temp. 58°
Correction for 1° Temp. -	— 42.581	— 41.380	
Points of Equilibrium -	81 57.417	81 44.223	at temp. 59°
Correction for 1° Temp. -	— 39.244	— 38.316	
Reduced Poiuts of Equilibrium	81 18.173	81 05.907	at standard temp. 60°.

2nd. Observed temperature *above* the standard temperature.

29th May, Noon.	Westerly.	Easterly.	
Points of Equilibrium -	82° 00′	81° 34′	at temp. 63°
Correction for 1° Temp. Table I.	+ 42.820	+ 40.517	
Points of Equilibrium -	82 42.820	82 14.517	at temp. 62°
Correction for 1° Temp. -	+ 47.329	+ 44.241	
Points of Equilibrium -	83 30.149	82 58.758	at temp. 61°
Correction for 1° Temp. -	+ 53.713	+ 49.296	
Reduced Points of Equilibrium	84 23.862	83 48.054	at standard temp. 60°.

By processes similar to these, making use of Table I. or II. according as the observed temperature of the magnets is above or below the standard temperature 60°, the observed positions of the points of equilibrium are reduced to what would have been their positions had the temperature of the magnets been 60° at each observation.

A. *Table of the positions of the Points of Equilibrium at which a Magnetic Needle was retained at different hours during the day, by the joint action of two bar Magnets and of Terrestrial Magnetism, reduced to their true positions at the Standard Temperature* (60°) *of the Magnets.* Note. The observations were made within doors.

Time of Observation.	May 22.			23.			24.			25.			26.			Mean true positions of the Points of Equilibrium.		
	Difference of Temperature corrected for	Points of Equilibrium. West.	East.	Difference of Temperature corrected for	Points of Equilibrium. West.	East.	Difference of Temperature corrected for	Points of Equilibrium. West.	East.	Difference of Temperature corrected for	Points of Equilibrium. West.	East.	Difference of Temperature corrected for	Points of Equilibrium. West.	East.	Westerly.	Easterly.	South, as before.
h. m.	°	° ′	° ′	°	° ′	° ′	°	° ′	° ′	°	° ′	° ′	°	° ′	° ′	° ′	° ′	° ′
6 00	+0.25	81 12	81 00	+1.50	81 28	82 01	+3.00	81 18	81 06	+1.75	81 15	81 39	+1.00	81 58	81 36	81 26.2	81 28.4	0 14.0 E
7 30	0.00	81 59	81 51	+1.00	82 09	82 27	+2.33	82 25	82 25	+0.67	82 56	82 26	+0.25	82 15	82 26	82 20.8	82 19.8	0 14.8 E
9 00	—0.75	82 05	82 22	+1.00	83 03	83 28	+2.25	83 41	82 58	—0.10	83 45	83 26	+0.25	83 39	83 52	83 14.6	83 13.2	0 15.2 E
10 30	—0.75	82 20	82 48	0.00	83 55	84 05	+1.25	84 01	83 47	—1.00	82 44	82 03	—0.50	85 54	85 08	83 46.8	83 34.2	0 02.0 W
Noon.	—1.00	81 58	81 22	+0.20	82 19	81 59	0.00	83 41	82 47	—1.00	82 22	81 50	—1.10	83 18	82 12	82 43.6	82 02.0	0 24.0 W
1 30	—1.15	81 36	80 40	+0.25	81 43	80 56	+0.10	81 57	81 45	—1.10	82 05	81 26	—1.50	83 01	82 06	82 04.4	81 22.6	0 25.6 W
3 00	—0.75	81 02	81 34	+0.50	81 45	80 47	—0.80	*83 52	82 53	—1.50	81 44	81 13	—1.50	82 15	81 33	81 41.5	81 16.8	0 14.8 W
4 30	—0.50	81 07	81 45	+0.75	81 28	80 23	—0.20	80 30	80 34	No observation.			—1.50	81 55	81 50	81 15.0	81 08.0	0 01.0 W
6 00	0.00	80 35	81 31	+0.50	82 06	81 08	—0.20	81 33	81 31	—1.00	81 11	80 46	—1.50	81 19	81 17	81 20.8	81 14.4	0 02.8 E
7 30	+0.50	80 47	81 18	+1.00	81 16	81 07	+0.70	80 38	80 57	—0.50	81 10	81 08	—1.00	80 50	80 58	80 56.2	81 05.4	0 03.2 E
9 30	+1.25	80 33	80 32	+2.00	81 19	80 46	+1.25	81 03	80 41	+0.25	81 10	81 18	0.00	80 49	80 35	80 58.8	80 46.2	0 01.6 W
	May 27.			28.			29.			30.			31.					
6 00	+1.00	81 35	81 55	+0.25	81 35	82 17	0.00	82 01	82 03	—0.30	81 37	81 47	—2.00	82 24	82 15	81 50.4	82 03.4	0 15.6 E
7 30	0.00	81 51	82 23	—0.10	82 31	82 41	—0.40	82 24	82 20	—1.30	82 20	82 16	—2.50	82 54	82 54	82 24.0	82 30.8	0 16.0 E
9 00	—0.90	83 23	83 48	—0.50	83 07	83 00	—1.00	82 57	82 53	—2.00	83 12	84 05	—3.00	84 42	84 29	83 28.2	83 39.0	0 16.0 E
10 30	+1.50	84 04	84 24	—1.00	83 44	83 13	—2.75	85 06	85 31	—2.25	83 44	84 29	—3.40	84 25	83 58	84 13.4	84 19.0	0 02.4 W
Noon.	+2.50	83 45	83 35	—1.20	83 48	83 00	—3.00	84 25	83 47	—4.00	84 21	84 04	—3.67	83 17	82 41	83 55.2	83 25.4	0 22.0 W
1 30	—3.00	82 49	82 37	—1.50	82 19	81 48	—2.00	83 26	82 20	—3.50	83 45	82 44	—3.75	82 44	82 03	83 00.6	82 18.4	0 28.4 W
3 00	—2.60	82 29	82 07	—2.30	81 25	81 09	—2.30	82 40	81 48	—3.75	82 29	81 41	—4.20	81 53	81 51	82 11.2	81 43.2	0 22.0 W
4 30	—1.75	81 56	82 01	—3.25	82 13	82 38	—3.75	82 21	82 21	—4.00	81 45	81 43	—5.00	82 27	82 23	82 08.4	82 13.2	0 02.4 W
6 00	—1.00	81 32	82 02	—3.00	81 47	82 17	—3.30	81 52	81 38	—3.50	81 02	80 57	—5.00	81 46	82 04	81 35.8	81 47.6	0 01.4 W
7 30	—0.20	80 52	80 56	—2.75	80 58	81 45	—2.00	81 20	81 13	—3.20	81 36	81 20	—4.67	81 38	81 47	81 16.8	81 24.2	0 02.4 W
9 30	0.00	81 13	81 05	—1.25	81 36	81 30	—0.75	81 06	80 47	—2.25	81 28	81 07	—3.33	81 01	81 32	81 16.8	81 12.2	0 00.4 W
11 20	0.00	80 55	80 45	—1.00	81 43	81 33	—0.50	81 16	81 16	—2.00	81 27	81 23	—3.00	81 28	81 31	81 21.8	81 17.6	0 00.8 E

* In taking the mean, I reject this observation as evidently irregular.

To obtain from these corrected observations the diurnal variation of the terrestrial magnetic intensity, I take half the sum of the mean easterly and westerly arcs at different hours during the day as the mean azimuths of the points of equilibrium at those hours, and substituting these azimuths successively for φ in the equation (α),

$$M - F(.004690814 + .000829329 \cos.^2 \varphi) = o,$$

I obtain the values of M in terms of F at those hours: dividing each of these values by the minimum value of M, which in every case appears to happen at about $10^h\ 30^m$ in the morning, I obtain the relative terrestrial magnetic intensities at the times of observation. These results are contained in the following table.

B. *Table of the mean Terrestrial Magnetic Intensities at different hours during the day, deduced from the preceding observations.* Note. The observations were made within doors.

Time of Observation.	Mean of the Observations of May 22, 23, 24, 25, 26.		Mean of the Observations of May 27, 28, 29, 30, 31.		Mean of the two sets.
	Azimuth of the Points of Equilibrium.	Terrestrial Magnetic Intensity.	Azimuth of the Points of Equilibrium.	Terrestrial Magnetic Intensity.	Terrestrial Magneric Intensity.
h. m.	° ′		° ′		
6 00	81 27.3	1.00175	81 56.9	1.00170	1.00173
7 30	82 19.9	1.00100	82 27.4	1.00128	1.00114
9 00	83 13.9	1.00031	83 33.6	1.00046	1.00039
10 30	83 40.5	1.00000	84 16.2	1.00000	1.00000
Noon.	82 22.8	1.00096	83 40.3	1.00038	1.00067
1 30	81 43.5	1.00151	82 39.5	1.00112	1.00132
3 00	81 29.1	1.00173	81 57.2	1.00170	1.00172
4 30	81 11.5	1.00199	82 10.8	1.00151	1.00175
6 00	81 17.7	1.00190	81 41.7	1.00192	1.00191
7 30	81 00.9	1.00216	81 20.5	1.00224	1.00220
9 30	80 52.6	1.00229	81 14.5	1.00233	1.00231
11 20			81 19.7	1.00225	1.00225

From the mean obtained here, it appears that the terrestrial magnetic intensity was the least between 10 and 11 o'clock in the morning, the time, nearly, when the sun was on the magnetic meridian; that it increased from this time until between 9 and 10'clock in the evening; after which it decreased, and continued decreasing during the morning until the time of the minimum.

Having by this reduction of the observations made within doors, determined the nature of the changes in the direction of the needle in that situation, independent of the changes which took place in the temperature of the magnets, and thence deduced the diurnal changes in the intensity of the terrestrial forces acting upon the needle, I shall now detail similar observations which I made in the open air, for the purpose of comparing with them, when these had also been cleared of the effects due to changes in the temperature of the magnets, in order to determine how far there was any thing anomalous in the directions of the needle when in doors and when in the open air. I have already mentioned that, for the purpose of making these observations, the apparatus was placed on a table fixed firmly in my garden, the magnets being placed in earthen pans containing water. The observations were made in the same manner as those in doors, excepting that, as the magnets were here liable to greater changes of temperature, their temperatures were noticed at the beginning, and also at at the conclusion of each of the observations: they are contained in the following table, where the time set down is that at which the observation commenced, the time occupied in making the whole of each being from four to six minutes.

Table of observations, made in the open air, *on the Diurnal Changes in the positions of the Points of Equilibrium at which a Magnetic Needle was retained by the joint action of Terrestrial Magnetism, and of two bar Magnets, having their axes horizontal and in the Magnetic Meridian, and their centres at the distance* 21.52 *inches from the centre of the needle.*

Date and Time of Observation.		Temperature of the Magnets.		Points of Equilibrium.			Temperature of the Magnets.		Mean Temperature of the Magnets.	Barom.	Therm. attached.
		North.	South.	West.	East.	South.	North.	South.			
	h. m.	°	°	° ′	°	°	°	°	°		°
19th June, 1823. Morning.	6 05	55.2	54.0	85 50	85 02	0 20 E	54.8	54.0	54.50	30.23	58.2
	7 24	55.2	55.2	85 18	84 24	0 20 E	55.6	55.6	55.40	30.23	57.1
	8 53	57.3	56.7	85 32	83 38	0 06 E	57.4	57.0	57.10		
	10 30			No observation.							
Afternoon.	0 05	61.8	60.0	82 06	80 18	0 34 W	61.8	60.0	60.90	30.20	58.2
	1 26	62.5	61.0	81 02	78 46	0 44 W	62.5	61.0	61.75	30.19	59.8
	2 56	63.8	62.0	79 14	77 58	0 28 W	63.8	62.0	62.90	30.19	60.6
	4 26	63.0	61.0	80 00	78 58	0 20 W	62.8	61.0	61.95	30.18	59.5
	6 10	59.0	58.0	81 36	80 20	0 00	58.8	57.8	58.40	30.18	56.8
	7 25	56.0	55.0	83 08	82 06	0 06 E	55.8	55.0	55.45		
	9 00	55 6	55.8	82 34	81 30	0 12 W	55.3	55.5	55.55	30.17	55.7
20th June. Morning.	6 12	55.7	55.5	82 50	82 04	0 12 E	55.5	55.3	55.50		
	7 25	55.2	54.8	84 12	83 02	0 18 E	55.5	55.1	55.15	30.11	56.75
	9 00	66.25	64.0	77 52	78 08	0 18 E	66.25	64.0	65.13	30.11	60.25
	10 27	68.0	66.0	77 18	76 40	0 20 W	68.0	66.0	67.00	30.11	63.0
Afternoon,	0 12	71.0	68.8	77 04	74 42	0 26 W	71.0	68.8	69.90	30.09	64.3
	1 26	70.7	69.0	75 24	74 20	0 38 W	70.7	69.0	69.85	30.09	64.6
	3 00	70.0	68.4	75 20	74 02	0 32 W	69.8	68.3	69.14	30.08	65.1
	4 30	69.0	67.5	75 16	74 28	0 16 W	69.0	67.5	68.25	30.07	65.8
	6 00	67.6	66.3	75 42	74 54	0 02 W	67.6	66.3	66.95	30.07	64.8
	7 30	65.8	64.2	76 18	75 38	0 02 W	65.8	64.2	65.00	30.08	64.9
	9 00	61.5	60.1	79 14	77 38	0 10 W	61.4	60.0	60.73	30.09	60.0
21st June. Morning.	5 50	56.8	55.0	81 46	81 28	0 26 E	56.6	54.8	55.80	30.10	60.4
	7 26	56.3	55.7	81 46	81 20	0 22 E	56.1	55.7	55.95	30.13	57.5
	9 00	55.4	54.8	82 48	82 54	0 08 E	55.4	54.8	55.10	30.15	57.8
	10 26	56.5	56.0	82 48	82 36	0 06 W	56.7	56.0	56.30	30.15	57.7
Afternoon.	0 00	58.6	58.0	81 50	80 40	0 24 W	58.8	58.0	58.35	30.16	58.5
	1 30	60.0	58.8	80 08	78 52	0 26 W	60.0	58.8	59.40	30.16	58.6
	3 00	60.5	58.7	79 54	78 40	0 24 W	60.5	58.7	59.60	30.17	58.5
	4 30	59.3	57.6	79 10	78 32	0 04 W	59.3	57.6	58.45	30.17	57.4
	6 00	57.5	56.0	79 38	79 04	0 06 E	57.4	55.9	56.70	30.17	57.0
	7 30	55.5	54.5	81 00	80 32	0 04 E	55.5	54.5	55.00	30.18	55.7
	9 00	53.9	53.1	82 18	81 38	0 00	53.8	53.0	53.65	30.20	55.4

Observations made in the open air, &c.

Date and Time of Observation.		Temperature of the Magnets.		Points of Equilibrium.			Temperature of the Magnets.		Mean Temperature of the Magnets.	Barom.	Therm. attached.
		North.	South.	West.	East.	South.	North.	South.			
	h. m.	°	°	° ′	° ′	° ′	°	°	°		°
22d June, 1823. Morning.	6 00			No observation.							
	7 44	55.1	54.5	83 44	83 02	0 14 E	55.0	54.4	54.75		
	9 07	65.5	64.0	78 36	78 04	0 08 E	65.3	64.0	64.70	30.21	55.0
	10 30	60.0	58.4	82 34	81 18	0 06W	59.9	58.3	59.15	30.21	54.5
	11 58	57.5	55.6	83 52	82 24	0 26W	57.4	55.5	56.50	30.22	55.8
Afternoon.	1 29	56.3	54.8	83 18	81 54	0 48W	56.5	54.8	55.60	30.22	56.7
	2 58	57.0	55.8	81 32	80 34	0 40W	57.0	55.8	56.40	30.21	57.4
	4 21	57.0	56.0	81 52	80 44	0 18W	57.0	56.0	56.50	30.20	56.7
	5 57	54.8	54.2	82 38	8 48	0 00	54.8	54.2	54.50	30.20	56.2
	7 28	53.5	52.4	83 44	8 08	0 06W	53.5	52.2	52.90	30.19	56.1
	8 53	52.0	51.0	84 54	8 52	0 00	52.0	51.0	51.50	30.19	56.4
23d June. Morning.	6 01	54.7	53.7	82 52	82 46	0 10E	54.5	53.4	54.08	30.16	55.3
	7 28	55.0	54.5	83 52	82 58	0 20E	55.2	54.5	54.80	30.15	54.8
	8 55	56.25	55.4	82 52	82 46	0 02W	56.25	55.4	55.83	30.14	55.5
	10 25	56.5	54.75	83 10	82 50	0 14W	56.5	54.75	55.63	30.14	55.0
Afternoon.	0 14	57.9	55.9	83 28	81 00	0 40W	58.2	56.2	57.05	30.10	56.2
	1 29	58.5	56.0	81 34	79 54	0 40W	58.5	56.0	57.25	30.11	55.4
	2 59	57.7	56.5	80 52	79 20	0 28W	57.7	56.5	57.10	30.11	56.1
	4 30			No observation.							
	6 00			No observation.							
	7 42	52.3	51.5	83 08	82 34	0 04W	52.3	51.5	51.90	30.10	56.7
	9 00	50.8	50.6	84 22	83 44	0 00	50.8	50.4	50.65	30.10	52.8

The mean of the azimuths of the westerly point at 7^h 30^m in the evening is 81° 28′, and of the easterly at the same time 80° 48′; so that to reduce the situations of the westerly and easterly points to their distances from what ought to be considered as their meridian, 20′ must be subtracted from each of the azimuths of the westerly point, and added to each of those of the easterly, similarly to what was done with the observations made in doors. The mean of the observations gives the position of the south point at the same hour 0′.4 W., or so nearly in the meridian, that the observations of this point require no reduction. The observed

azimuths, so reduced to their mean meridian, are to be corrected for the difference between the standard temperature and that of the magnets. By means of tables III. and IV, repeating the processes described for the reduction of the observations made in doors to the standard temperature 60°, I reduce these observed positions of the points of equilibrium to what would have been their positions had the temperature of the magnets been 60° at each of the observations. These reductions are successively effected in the two following tables.

Table of the preceding Observations reduced to their Mean Magnetic Meridian.

	June 19				20				21.				22.				23.				
Time of Observation.	Mean Temperature of the Magnets.	Points of Equilibrium. West.	East.	South.	Mean Temperature of the Magnets.	Points of Equilibrium. West.	East.	South.	Mean Temperature of the Magnets.	Points of Equilibrium. West.	East.	South.	Mean temperature of the Magnets.	Points of Equilibrium. West.	East.	South.	Mean Temperature of the Magnets.	Points of Equilibrium. West.	East.	South.	Mean Positions of the South Point.
h. m.	°	° ′	° ′	° ′	°	° ′	° ′	° ′	°	° ′	° ′	° ′	°	° ′	° ′	° ′	°	° ′	° ′	° ′	° ′
6 00	54.50	85 30	85 22	0 20 E	55.50	82 30	82 24	0 12 E	55.80	81 26	81 48	0 26 E		No observation.			54.08	82 32	83 06	0 10 E	0 19.0 E
7 30	55.40	84 58	84 44	0 20 E	55.15	83 52	83 22	0 18 E	55.95	81 26	81 40	0 22 E	54.75	83 24	83 22	0 14 E	54.80	83 22	83 18	0 20 E	0 18.0 E
9 00	57.10	85 12	83 58	0 06 E	65.13	77 32	78 28	0 18 E	55.10	82 28	83 14	0 08 E	64.70	78 16	78 24	0 08 E	55.83	82 32	83 06	0 02 W	0 11.3 E
10 30		No observation.			67.00	76 58	77 00	0 20 W	56.30	82 28	82 56	0 06 W	59.15	82 14	81 38	0 06 W	55.63	82 50	83 10	0 14 W	0 10.7 W
Noon.	60.90	81 46	80 38	0 34 W	69.90	76 44	75 02	0 26 W	58.35	81 30	81 00	0 24 W	56.50	83 32	82 44	0 26 W	57.05	83 08	81 20	0 40 W	0 25.3 W
1 30	61.75	80 42	79 06	0 44 W	69.85	75 04	74 40	0 38 W	59.40	79 48	79 12	0 26 W	55.60	82 58	82 14	0 48 W	57.25	81 14	80 14	0 40 W	0 37.3 W
3 00	62.90	78 54	78 18	0 28 W	69.14	75 00	74 22	0 32 W	59.60	79 34	79 00	0 24 W	56.40	81 12	80 54	0 40 W	57.10	80 32	79 40	0 28 W	0 32.0 W
4 30	61.95	79 40	79 18	0 20 W	68.25	74 56	74 48	0 16 W	58.45	78 50	78 52	0 04 W	56.50	81 32	81 04	0 18 W		No observation.			0 12.7 W
6 00	58.40	81 16	80 40	0 00	66.95	75 22	75 14	0 02 W	56.70	79 18	79 24	0 06 E	54.50	82 18	82 08	0 00		No observation.			0 01.3 W
7 30	55.45	82 48	82 26	0 06 E	65.00	75 58	75 58	0 02 W	55.00	80 40	80 52	0 04 E	52.90	83 24	83 28	0 06 W	51.90	82 48	82 54	0 04 W	0 01.3 W
9 00	55.80	82 14	81 50	0 12 W	60.75	78 54	77 58	0 10 W	53.65	81 58	81 58	0 00	51.50	84 34	84 12	0 00	50.65	84 02	84 04	0 00	0 03.3 W

C. *Table of the positions of the Points of Equilibrium at which a Magnetic Needle was retained at different hours during the day, by the joint action of two bar Magnets and of Terrestrial Magnetism, reduced to their true positions at the Standard Temperature (60°) of the Magnets.* Note. The observations were made in the open air.

	June 19.			20.			21.			22.			23.			Mean true Positions of the Points of Equilibrium.		
Time of Observation.	Difference of Temperature corrected for	Points of Equilibrium. West.	East.	Difference of Temperature corrected for	Points of Equilibrium. West.	East.	Difference of Temperature corrected for	Points of Equilibrium. West.	East.	Difference of Temperature corrected for	Points of Equilibrium. West.	East.	Difference of Temperature corrected for	Points of Equilibrium. West.	East.	Westerly.	Easterly.	South as before.
h. m.	°	° ′	° ′	°	° ′	° ′	°	° ′	° ′	°	° ′	° ′	°	° ′	° ′	° ′	° ′	° ′
6 00	+5.50	81 05	81 02	+4.50	79 48	79 44	+4.20	79 05	79 23	No observation.			+5.92	79 03	79 20	79 26.5	79 33.5	0 19.0 E
7 30	+4.60	81 22	81 13	+4.85	80 31	80 11	+4.05	79 10	79 21	+5.25	79.59	79 58	+5.20	80 06	79 56	79 53.3	79 50.0	0 18.0 E
9 00	+2.90	82 39	81 47	−5.13	80 03	81 14	+4.90	79 31	80 04	−4.70	80 43	80 53	+4.17	79 57	80 22	80 05.7	80 43.7	0 11.3 E
10 30	No observation.			−7.00	80 22	80 24	+3.70	80 10	80 31	+0.85	81 40	81 06	+4.37	80 04	80 19	80 44.0	80 40.3	0 10.7 W
Noon.	−0.90	82 23	81 10	−9.90	81 48	79 20	+1.65	80 31	80 03	+3.50	81 05	80 29	+2.95	81 07	79 38	81 08.0	79 57.3	0 25.3 W
1 30	−1.75	81 47	80 01	−9.85	79 21	78 49	+0.60	79 29	78 54	+4.40	80 09	79 36	+2.75	79 40	78 40	79 39.7	79 06.3	0 37.3 W
3 00	−2.90	80 26	79 45	−9.14	78 55	78 05	+0.40	79 22	78 48	+3.60	79 12	78 57	+2.80	79 02	78 16	79 09.7	78 36.7	0 32.0 W
4 30	−1.95	80 45	80 21	−8.25	78 23	78 13	+1.55	78 06	78 08	+3.50	79 31	79 08	No observation.			78 40.0	78 29.7	0 12.7 W
6 00	+1.60	80 20	79 47	−6.95	78 19	78 09	+3.30	77 43	77 48	+5.50	79 05	78 58	No observation.			78 22.3	78 18.3	0 01.3 W
7 30	+4.55	79 57	79 41	−5.00	78 07	78 07	+5.00	78 05	78 15	+7.10	79 01	79 04	+8.10	78 10	78 14	78 24.3	78 28.7	0 01.3 W
9 00	+4.20	79 43	79 24	−0.75	79 17	78 19	+6.35	78 26	78 26	+8.50	78 59	78 47	+9.35	78 18	78 19	78 54.0	78 30.7	0 03.3 W

The character of the diurnal changes in the positions of the points of equilibrium is very nearly the same for each day, but, in taking the mean, I can only make use of the observations of the 20th, 21st, 22d, since on the 19th no observation could be made at $10^h\ 30^m$, and the azimuths are all greater on this day than on any of the subsequent, and two observations were unavoidably omitted on the 23d.

Comparing the results with those obtained from the observations made in doors, we find them agree as nearly as could possibly be expected. From table A it appears, that when the observations were made in doors, the westerly point receded from the north until half past 10 o'clock in the morning, and approached the north during the remainder of the day until about 9 in the evening; and from table C, that when they were made in the open air, the westerly point receded from the north until about half past eleven in the morning, and approached it until six or seven in the evening, after which it again gradually receded. This is not a greater variation in the times of the maxima than we find on different days, either in the in-door observations, or in those in the open air. The easterly point appears to have receded from the north until about 10 o'clock in the morning, when the observations were made in doors and likewise when they were made in the open air; and to have approached it until between nine and ten in the evening in the former case, and until six in the latter.

Taking, as before, half the sum of the mean easterly and westerly arcs at different hours during the day as the mean azimuths of the points of equilibrium at those hours, and substituting these for ϕ in the equation (α_4),

$$M - F\,(.00448032 + .0007664093 \cos.^2 \phi) = o,$$

I obtain the values of M in terms of F, at those hours; and dividing each of these values by the minimum value of M, I find, as before, the relative terrestrial magnetic intensities at the times of observation.

D. *Table of the mean Terrestrial Magnetic Intensities at different hours during the day, deduced from the preceding observations.* Note. The observations were made in the open air.

Time of Observation.	Mean of the Observations of June 20, 21, 22.	
	Azimuth of the Point of Equilibrium.	Terrestrial Magnetic Intensity.
h. m.		
6 00	79 30.0	1.00112
7 30	79 51.7	1.00061
9 00	80 24.7	1.00028
10 30	80 42.2	1.00000
Noon.	80 32.7	1.00015
1 30	79 23.0	1.00134
3 00	78 53.2	1.00188
4 30	78 34.8	1.00223
6 00	78 20.3	1.00251
7 30	78 26.5	1.00239
9 00	78 42.3	1.00209

From these it appears, that the minimum intensity happened nearly at the time the sun passed the magnetic meridian, and rather later than in May, which was also the case with the time of the sun's passage over the meridian:* the

* The diurnal variation, both in the direction of the needle and in the magnetic intensity, appears to have a reference to the position of the sun with regard to the magnetic meridian; it is therefore probable, that the sun is the principal cause of both

intensity increased until about six o'clock in the afternoon, after which time it appears to have decreased during the

these phœnomena. The circumstance of the situation of the magnetic pole in what appears to be, independent of elevation, the coldest region of the globe, supported as it is by the fact of a diminution of temperature causing an increase of magnetic intensity, would lead us to infer, that the effect produced by the sun is principally to be attributed to the heat developed by it; but should any periodical effects, corresponding to the time of the sun's rotation about its axis, be observable in the diurnal variation, we must suppose that the sun, like the earth, is endued with magnetism, and look for a cause of this magnetism, common to all the planets. Being engaged more than two years ago in making some experiments on the effects produced on the needle by unpolarized iron, I discovered that a peculiar polarity was imparted to the iron by simply making it revolve about an axis; and this naturally suggested the question to me, whether the magnetism of the earth, and consequently, that of the other planets and the sun, might not be owing to their rotation? From the effects which I have observed to be produced on iron by its rotation, it appears probable, if the magnetism of these bodies be not caused by their rotation, that at least the effects will be modified by, and, to a certain extent, dependent on such rotation. Since first observing the fact, that simple rotation will cause a peculiar polarity, if I may be allowed the expression, in iron, I have made a great variety of experiments on the subject, which have enabled me to trace the laws according to which this polarity in the iron affects a magnetic needle, independently of the effect produced by the mass. It would lead me to too great a length here to state the several effects that are produced by the rotation of iron, or the laws which govern them; but I will briefly mention one. Let us imagine a plane to pass through the centre of an horizontal needle, at right angles to the meridian, and making an angle with the horizon equal to the dip; then, if the plane of a circular plate of iron coincide with this plane, and the plate be fixed on an axis passing through its centre at right angles to its plane, so that it can be made to revolve in its own plane, the direction of the needle will be different, according as the several points of the plate are brought into any particular position by making it revolve in one direction or the opposite, excepting in four positions of the centre of the plate. If the centre of the plate be successively placed to the east or west of the centre of the needle in the same horizontal line, and over the needle in the plane of its meridian, then the deviation of the needle due to the rotation of the plate will be in contrary directions in the two cases, the plate revolving in the same direction in both. These and other peculiar effects arise

evening, and to have been decreasing from an early hour in the morning.

The general agreement of these intensities with those deduced from the observations made in doors, is as near as could be expected, considering that an interval of twenty days had elapsed between the two sets of observations. From this, and the agreement in the manner in which the westerly and easterly points of equilibrium approach and recede from the north in the two cases, which I have before pointed out, we may conclude, that there is nothing anomalous in the action which takes place on the needle under the different circumstances of its being placed in doors or in the open air; and that the apparent anomaly in the directions of the needle in the two cases, which was observed by Mr. BARLOW and myself, arose from the cause which I have assigned for it in my former paper; namely, the difference in the changes of temperature in the magnets when in doors and when in the open air.

The diurnal changes in the terrestrial magnetic intensity have been determined by Professor HANSTEEN, by means of the vibrations of a needle delicately suspended. From these observations it appears, that in general the time of minimum intensity was between ten and eleven o'clock in the morning; that the maximum happened between four and seven

entirely from the rotation of the iron, and are not produced by any friction on the axis. As the effects are not very considerable, to render them conspicuous it is necessary to make use of a plate eighteen inches in diameter, and to have its centre within sixteen inches of that of the needle. If the needle is under the influence of magnets, as in the foregoing observations, the effects produced by the rotation of the plate are considerable.

for the month of May 1820, and about seven o'clock in the evening for the month of June. The intensity which, in these observations, is taken as unity, is that deduced from an observation made during an aurora borealis; but for the purpose of comparison, I have, for the months of May and June, taken the intensity deduced from his observations at $10^h\ 30^m$ in the morning as unity, reduced the intensities, which he gives for other times in the day, to this standard, and placed them in the following table, with the corresponding intensities deduced from my own observations.

Intensity deduced from HANSTEEN'S Observations in 1820.			Intensity deduced from the preceding Observations in 1823.		
Time.	May.	June.	Time.	May.	June.
h. m.			h. m.		
8 00 A. M.	1.00034	1.00010	7 30 A. M.	1.00114	1.00061
10 30	1.00000	1.00000	10 30	1.00000	1.00000
4 00 P. M.	1.00299	1.00251	4 30 P. M.	1.00175	1.00223
7 00	1.00294	1.00302	7 30	1.00220	1.00239
10 30	1.00191	1.00267	9 30	1.00231	1.00209

The principal difference to be observed in the nature of the changes of intensity during the day, in the two cases, is, that from my observations, the intensity appears to decrease more rapidly in the morning, and increase more slowly in the afternoon, than it does from those of Professor HANSTEEN; but the general character of these changes is as nearly the same as we can expect from methods so different, at different times, and at places where both the variation and dip of the needle are different. My object however was, to point out

what might be deduced from a series of such observations as I have detailed, rather than to compare the results deduced from them with those obtained by others, for which purpose it would have been necessary to have continued them for a greater length of time.

We have seen that with the magnets I made use of, their intensity being nearly 218 M, at the temperature 60°, a change in their temperature of 1° would cause a change of intensity of 0.123 M; or taking the intensity of the magnets 1, for each degree of increase in temperature we should have a decrease of intensity of 0.000564. Now if the same, or nearly the same, take place with all magnets, it is evidently necessary, in all cases where the terrestrial magnetic intensity is to be deduced from the vibrations of a needle, that great care should be taken to make the observations at the same temperature; or, the precise effect of change of temperature having been previously ascertained, to correct the observations according to the difference of the temperatures at which they were made. I am not aware that any one has yet attempted to make such a correction; but it is manifest from the experiments I have described, that it is indispensible, in order to deduce correct results from the times of vibration of a needle in different parts of the earth, where the temperatures at which the observations are made are almost necessarily different, that these temperatures should be registered, and the times of vibration reduced to a standard of temperature. It appears to me, that the effects will be the most sensible in large and powerful needles; and consequently, in making use of such, the reduction for a variation of temperature will

be most necessary. There would be no difficulty in this reduction, if we could give in terms of the intensity of any magnet the increment or decrement of intensity corresponding to a certain decrement or increment of temperature at all temperatures. To determine this accurately, would however require a great variety of experiments to be made with magnets of very different intensities; but as I have not made these, I must content myself for the present with pointing out some of the facts which I have ascertained from more extended experiments than those I have already given, reserving the detail of these experiments for another opportunity, should they be deemed of sufficient interest.

These experiments were made with a balance of torsion, the needle being suspended by a brass wire $\frac{1}{450}$ inch in diameter: by them I ascertained the following facts:

1. Commencing with a temperature — 3° FAHRENHEIT, up to a temperature 127°, as the temperature of the magnets increased, their intensity decreased. Owing to the almost total absence of snow during the winter, I was unable to reduce lower the temperature of the large magnets which I made use of; but from an experiment I made at the Royal Institution, in conjunction with Mr. FARADAY, in which a small magnet, enveloped in lint well moistened with sulphuret of carbon, was placed on the edges of a basin containing sulphuric acid, under the receiver of an air pump, I found that the intensity of the magnet increased to the lowest point to which the temperature was reduced, and that the intensity decreased on the admission of air into the receiver, and consequent increase of temperature in the magnet. This

is in direct contradiction to the notion which has been entertained of destroying the magnetism of the needle by the application of intense cold.

2. With a certain increment of temperature, the decrement of intensity is not constant at all temperatures, but increases as the temperature increases.

3. From a temperature of about 80° the intensity decreases very rapidly as the temperature increases: so that, if up to this temperature, the differences of the decrements are nearly constant, to ascertain which requires a precision in the experiments that perhaps their nature does not admit of, beyond this temperature, the differences of the decrements also increase.

4. Beyond the temperature of 100°, a portion of the power of the magnet is permanently destroyed.

5. On a change of temperature, the most considerable portion of the effect, on the intensity of the magnet, is produced instantaneously; showing that the magnetic power resides on or very near the surface. This is more particularly observable when the temperature of the magnet is increased, little change of intensity taking place after the first effect is produced; on the contrary, when the temperature of the magnet is diminished, although nearly the whole effect is produced instantly, yet the magnet appears to continue to gain a small power for some time.

6. The effects produced on unpolarized iron by changes of temperature are directly the reverse of those produced on a magnet; an increase of temperature causing an increase in the magnetic power of the iron, the limits between which I observed being 50° and 100°. That the effect on iron of an

increase of temperature should be the reverse of that produced on a magnet, is, I think, a strong argument against the hypothesis, that the action of iron upon the needle arises from the *polarity* which is communicated to it from the earth.

It may be objected to the method which I have adopted for determining the diurnal changes in the terrestrial magnetic intensity, that, after the observations have been made, they require a correction for temperature, which can only be determined by experiments previously made on the magnets and needle employed. The same objection may, however, be made against the method of determining the intensity by the vibrations of a needle. As such a correction has not in the latter case been hitherto applied, the results which have been obtained relative either to the diurnal changes of intensity, or the intensities in different parts of the earth, by means of observations on the vibrations of a needle, will be so far incorrect as the needle may happen to have been affected by differences in the temperature. The method I have described, however, possesses advantages over the other: a very considerable one is, that whatever effects are produced may easily be observed with considerable precision, the time required for each observation being not more than five minutes; another is, that, the magnets being immersed in water, as far as regards them, we may command the temperature at which the observations are to be made, and thus limit the correction for temperature to a very small quantity; and it possesses another decided advantage, that whatever are the effects produced on the needle by atmospheric changes, they are, by means of it, rendered immediately visible, and can be observed as they occur.

It was my intention to commence a series of such observations at the beginning of the present year, and to continue them for as long a period as I was able; but circumstances prevented my commencing at the time I proposed, and ill health has since put it out of my power to engage in such continued observations as would be required: but I trust the task will be undertaken by others who feel interested in investigating the phœnomena connected with terrestrial magnetism.

II. *The Croonian Lecture. On the existence of Nerves in the Placenta.* *By Sir* Everard Home, *Bart. V. P. R. S.*

Read November 18, 1824.

In the Lecture which I gave last year, I attempted to trace the structure of the human brain to as great a degree of minuteness as is consistent with accuracy, by observing its appearance in the field of the microscope. This I should not have ventured to do under any other circumstances, than being assisted by the eye of Mr. Bauer in examining the appearances, and in having correct representations of them under his hand, to lay before the Society.

Without these peculiar advantages, I should have been afraid of being led into error, either by the fallacies to which microscopical observations are liable in themselves, or those which so frequently occur when the same eye is not employed both in ascertaining the appearances, and in directing the pencil by which they are delineated.

As Mr. Bauer continues to indulge me with the same advantages, I shall employ them in the present Lecture in prosecuting my enquiry respecting the nerves; for as no anatomist before me has had the assistance of such an able coadjutor, it may never happen again; and I should feel myself undeserving of it, were I not to employ it in extending our researches in minute anatomy.

This I have now been enabled to do in no common degree,

by discovering nerves in both the fœtal and maternal portions of the placenta. This discovery, I am proud to say, was not the result of accident, but of a regularly arranged plan for that purpose.

In examining, at my desire, the structure of the horns of the fallow deer during their growth, while covered with velvet, Mr. BAUER found them abundantly supplied with nerves. The circumstance of nerves being met with, where sensibility is not only unnecessary, but even where the parts are unfitted for their office till the nerves are removed; which takes place as soon as the horns are full grown; makes it appear that nerves answer some other purposes in the animal œconomy, besides regulating the actions of the arteries; an office which, many years since, I not only considered them to perform, but illustrated my opinion by the effects of irritation on the parvagum and great sympathetic nerve on the carotid artery. Since that time, by considering the incubation of the chick, I have been led to believe that the arteries are indebted to the nerves for their formation; and so strong was the conviction on my mind of this being the case, that even the circumstance of the placenta, whose blood vessels are very numerous, having been suspected to have no nerves, did not induce me to abandon it; since until it is proved that the placenta is devoid of nerves, there is no argument against me. This was a point, of all others, that no one could so well determine as Mr. BAUER. I therefore most earnestly requested him to employ his microscopical observations on this subject, and supplied him with the placenta of a seal, in which the arteries and veins had been injected; and as in that animal the umbilical vessels are not

twisted, the nerves will be more exposed, and the parts having been in spirit, will have lost that transparency belonging to them in a recent state, by which they are less readily distinguished from blood vessels.

In this specimen, Mr. Bauer has shown that nerves are not only conspicuous surrounding the umbilical arteries, but has demonstrated them in the portion belonging to the uterus. For the appearance they put on, I must refer to the annexed drawing.

I may here remark, that in no communication which I have made to the Society, assisted by Mr. Bauer's labours, has any appearance been mentioned or represented, that I have not myself distinctly seen; for although I am not equal to the nice adjustment of the microscope, which indeed appears peculiar to Mr. Bauer, yet, when adjusted by him, the appearances before they were described had been rendered visible to me.

In looking at objects so much magnified beyond what they appear to the naked eye, it will not be unnatural for many of the Members to ask, how I am sure that these are really nerves, and not the secondary order of blood vessels, too small to carry red blood, and therefore, when their contents have been coagulated, appear to be chords? My answer to this question is, to recommend an inspection of the drawings; in which it will be seen, that these are not continuations of other branches, but form a trellis-work upon the arterial trunks, in a manner totally different from any thing met with either in the ramifications of arteries or veins; and when they are dried upon glass, they reflect the light with a degree of splendour like the human hairs when these

are quite white. When the nerves are very minutely examined, each fibre appears to consist of a row of small globules connected with one another.

At the time the nerves in the placenta were discovered, Sir STAMFORD RAFFLES (whose misfortune in having lost the most valuable collection in Natural History ever made in the East Indies by the ship taking fire, every one must feel for) brought me from Sumatra the pregnant uterus of the tapir of that country; and as in that animal the umbilical chord is connected with the chorion (there being no placenta), I examined the transparent portion of the chorion along which the branches of the funis pass, before they arrive at the spongy part, and there the nerves are so conspicuous, that Mr. BAUER's representation of them of the natural size is annexed.

The principal object of the present Lecture is to establish the fact of nerves existing in the placenta; and in these animals in which there is no placenta, in the floculent chorion, which is substituted for it; and it is a curious fact, that they should be largest in the latter.

This discovery places the placentular circulation in a new point of view, since, from the known influence of the nerves on the blood vessels, it is reasonable to believe that, during life, there are branches of communication between those of the uterus and fœtus, although too minute to be explored in the dead body. The erection of the penis cannot be produced after death by injecting the arteries, although when the nerves are excited the smaller branches give a ready passage to the blood. Having traced nerves from the fœtus to the maternal portion of the placenta, it will add to the value of this com-

munication, to give some general account of the course of the nerves which supply the uterus of the mother, more especially as these are little known in the different classes of animals, even to those who are well versed in comparative anatomy.

That some very important office is performed by the uterine nerves is evident from their number, the different sources from which they originate, and the various ganglia by which the filaments are connected with one another; and that such a complex system of nerves is required for the well doing of the fœtus in utero cannot be doubted, since they become enlarged during pregnancy. Mr. CÆSAR HAWKINS has very kindly made the dissections necessary for this purpose, and I shall give in his own words the account he has drawn up of the distribution of the nerves connected with the organs of generation of the female in the human species, in the quadruped, the bird, and the frog.

"The nerves of the human uterus are supplied from six different plexuses. The spermatic plexus within the abdomen, the great hypogastric plexus between the common iliac arteries, and four within the pelvis, two of which are situated on each side of the uterus. All of these have the peculiar appearance of the sympathetic nerves, and they are intimately connected with all the other nerves of the viscera.

"The uterine nerves in the dog, cat, rabbit, and guinea pig, so nearly resemble those of the human uterus, that a minute description of them is unnecessary. The spermatic plexus is formed by branches of the renal plexus and two nearest lumbar ganglia of the sympathetic nerve; it supplies the horns of the uterus, the ovaria, and apex of the urinary bladder.

"The common hypogastric plexus, after having supplied the body of the uterus, gives off a large nerve of considerable length, which dips down into the pelvis, and unites with numerous branches of the third sacral nerve, and smaller branches from the second and fourth; a remarkable plexus is thus formed which contains several distinct ganglia. It distributes nerves to the body of the uterus, the vagina, bladder, and rectum, the integuments of the upper part of the pubes, and the muscles of the inferior outlet of the pelvis. A few branches pass down to communicate with the fourth sacral nerve, where it gives origin to the pudic nerve. These nerves arise from the plexus in such a way as to resemble the ramifications of the venæ vorticosæ in the choroid membrane of the eye.

"The difference therefore between the nerves of the human uterus, and those belonging to the uterus in the quadruped, consists in the formation of only one lateral hypogastric plexus, and consequently in the existence of only four nervous centres in the latter. There appear also to be more ganglia in the plexiform distribution of the sympathetic nerve. In the seal, several large ganglia are found in the broad ligaments of the uterus.

"The nerves belonging to the female organs of birds are distributed as follows:

"The sympathetic nerve is found close to the origin of the spinal nerves, protected by the double heads of the ribs between which it runs. The spinal nerves that correspond to the lumbar and sacral nerves in quadrupeds emerge near each other, and as the sympathetic nerve communicates with each of them, and forms a ganglion immediately after their

appearance from the vertebral foramina, there is an almost uninterrupted ganglion of considerable length; from which numerous filiments go off to supply the oviducts. Others run upwards and are distributed on the ovaria.

"Near the termination of the oviduct in the cloaca a plexus is formed, nearly similar to the lateral hypogastric plexus in quadrupeds, which is distributed in a corresponding manner to the oviduct and cloaca. There is also a similar pudental nerve.

"Fewer ganglia are formed near the aorta than in quadrupeds, and scarcely any branches are sent from the common hypogastric plexus to the oviduct.

"In the frog, as there is no proper sympathetic nerve, the abdominal viscera are supplied directly from the spinal nerves. These soon after they emerge from the vertebral canal, become slightly enlarged: this does not deserve to be called a ganglion. From each of the spinal nerves in the lower part of the back and loins, a small nerve is given off, which takes a direction towards the centre of the bodies of the vertebræ, where they unite with each other, and with the corresponding nerves of the opposite side. By this union a flat nervous web is formed, which stretches across the aorta and extends downwards into the pelvis: this is analogous to the splanchnic plexus in hot blooded animals.

"From the upper surface of this plexus many branches run upwards towards the intestines and kidneys, but the greater number are distributed on the ovaria.

"The lumbar nerves on each side give off several branches, which pass at once into the oviduct. The last lumbar nerves pass down upon the surface of the psoæ muscles, and near

the pubes give off a branch, which takes a circuitous course towards the lateral portion of the bladder, and the extremity of the oviduct. The continuation of the aortic, or abdominal plexus, in union with some branches of the sacral nerves, forms on each side of the pelvis a kind of plexus; which distributes branches to the cloaca and lower portion of the oviducts.

"The nerves corresponding with those which have been described in the frog, run almost entirely in straight lines, instead of having the intricate reticulated texture of the visceral nerves in hot blooded animals. The ganglia are indistinct, and the fibres that compose them resemble those of the muscular, more than visceral nerves."

At the time I was appointed to give this Lecture, I had completed an investigation, in which was traced to its origin the formation of the brain and spinal marrow in the ovum of the frog, and intended, upon this occasion, to have laid my observations, illustrated by a series of drawings made by Mr. Bauer, before the Society, but having been so fortunate as to discover the nerves of the placenta, I did not hesitate in giving this discovery the preference, and taking the earliest possible opportunity of communicating it to the Society.

Now that it is known by the discovery of the nerves in the placenta that the brain of the child, as well as every part of its body, is connected by the medium of nerves with the brain of the mother, we are led to understand the degree of dependence in which the fœtus is kept during the whole time of utero gestation.

The small pox being in some instances communicated from the mother to the child, which has until now been consi-

dered as an extraordinary fact* and not to be accounted for, is readily explained, since absorption depends fully as much upon nervous influence as the action of the arteries: a child in utero having an ague, is in itself almost a proof of the placenta having nerves.†

A child being born without a brain is not to be marvelled at, the nerves of the child being connected with the brain of the mother.

The immediate division of the navel-string at the moment of the birth, in some particular instances having hazarded the life of the child, hinted at by Dr. DENMAN, shows the accuracy of his observations.

* There are several cases in the Philosophical Transactions of children having the small-pox in utero, and one that was read before the Society, but not published.

Two of these were in England; in both of them the child took the infection on or about the 14th day.

One was in Jamaica; and the infection was taken by the child on the 8th day. This difference appears to deserve being recorded.

† Dr. PATRICK RUSSEL states a case of ague occurring in a child in utero, in Aleppo.

In June, 1767, a healthy young woman, in the seventh month of her third pregnancy, was attacked by a tertian fever, and the fœtus in utero appeared to suffer a paroxysm distinct from the mother.

The fits in the mother returned regularly about noon, and terminated by a profuse sweat in less than ten hours.

About 8 in the morning of the odd days, the woman felt the child tremble with great violence; she also felt a weight and coldness in the womb; the coldness went off in less than 15 minutes, and was succeeded for more than an hour by a glowing heat; the child was at intervals restless, as she had felt in her other children during pregnancy, but the trembling she never before experienced.

After the sixth paroxysm the bark effected a cure.

Dr. RUSSEL, while at Aleppo, met with a few similar instances, but had attributed them to the effect of imagination, which in this woman he could not do, as she was remarkable for her cheerful disposition, and good sense. *Trans. of a Society for the Improvement of Medical and Chirurgical Knowledge*, vol. iii. p, 96.

Till this discovery was made, we had no mode of estimating the influence that could be produced upon the child by the affections of the mind, or the body of the mother; and therefore the instances that have occurred, were considered as idle stories or accidental occurrences, for which no satisfactory reason could be assigned; and this upon no better ground than that they do not always take place under similar circumstances, which nothing connected with nerves ever does.

That they do sometimes occur no one can be so hardy as to deny, and when they do, we cannot now be at a loss for a mode of accounting for their doing so.

With the following well authenticated instances of this kind that have come under my own observation, I shall close this Lecture.

The mare that in the first instance had a foal by a quagga, and afterwards three in succession by a Persian horse, all of which were marked like the quagga, can in no other way be explained than through the influence of the mother upon its young. In proof of the fact, paintings from life of all the individual animals have a place in the Royal College of Surgeons in London. There is also an account of the occurrence stated in the Philosophical Transactions, in which all the particulars are detailed communicated by the Earl of Morton, to whom in the first instance the mare belonged.

A lady while pregnant with her tenth child, the former nine being perfectly formed, was robbed in the dusk of the evening near Woolwich by an artillery man who had a hare lip the fright occasioned by the alarm was so great: that she did not recover from it for several days; when brought to bed the child was found to have a hare-lip. I was called

upon to perform the operation. As the mother was very irritable and nervous, and the child unusually fractious, I used every means of soothing the child previous to the operation; but after it was performed, the child never ceased from crying for three days, and died at the end of that period.

A lady of an unusually nervous habit, who had several healthy children perfectly well formed, during a state of pregnancy, was opening the hall door of her house in the country, when a Newfoundland dog rushed upon her, and jumped up in play, putting his two fore paws upon her sides: the alarm and surprise was very great, but she soon recovered herself: when the child was born there was a claret mark upon the two parts of the belly that corresponded with the places on which the dog's paws had been placed. These I afterwards removed, and the parts readily recovered.

An Italian woman, twenty years of age, when by her reckoning three months and three weeks gone with her third child, was travelling in a caravan with the baggage of the Duke of Wellington's army, in the middle of the night, in a violent storm, while she was fast asleep, a small monkey with a long chain upon the roof of the caravan took refuge in it, crept under her loins, and fell asleep; she awoke, feeling uneasy from the pressure of the monkey, and put her hand down to scratch the part, but came upon the monkey's head, by which it awoke and bit her fingers, and in its alarm got fast hold of her loins. The woman went into fits, and was some minutes before she recovered herself: it was expected she would miscarry, but she went her full time. When the child was born it measured between seven and eight inches in height, and weighed one pound. This was in France. The child

was reared with great difficulty; it was carried afterwards to Ireland, and there was afflicted with a hacking cough; it was brought to England, in expectation of the cough being relieved, but died soon after, just before it completed its ninth year. I saw it before its death; at that time it was much emaciated, and measured exactly twenty-two inches. After death it was found that the fontinelle of the head had closed; no fat was any where met with but at the bottom of the orbits. The uterus was as small as in a fœtus between three and four months, not being at all developed, whereas in a new-born child it has acquired a considerable size; it was closely attached to the posterior surface of the urinary bladder, apparently by inflammation: the bladder was distended with urine, and the size of a turkey's egg. As the child had never passed its water freely from the time of its birth, this affection of the bladder must have taken place at the same time with the injury produced by the monkey's gripe upon the loins of the mother.

I examined an abortion, which was considered by Mr. Clarke, Teacher in Midwifery, to be about three months and a half after impregnation; and on comparing the ovaria with those of the dwarf, they were nearly of the same size, but not quite so long; the difference however was scarcely observable.

The child when I saw it could walk alone, but not more confident in itself, or firm on its legs, than an healthy infant at sixteen months. Its sight was very quick, particularly in seeing bright objects; was delighted with every thing showy, much pleased with ornaments in its own dress, could speak in a very low tone and shrill voice, and had some taste for

music, but had few English words; appeared very sensible of kindness, and remembered perfectly those from whom it had received attention.

The mother has had a fifth child in Ireland, which, like her former children, was of the common size when born, and has nothing particular in its appearance.*

EXPLANATION OF THE PLATES.

Plate II.

Fig. 1. A small portion of the placenta of the seal, exposing the chorion by which it is covered, through which are seen the arterial and venal branches injected with wax, magnified four diameters.

The folds of the chorion contain the branches of the nerves.

The cut edge of this portion exposes the structure of the placenta, which is more distinctly seen in Fig. 4.

Fig. 2. The same portion as Fig. 1. exposing the uterine surface, which appears to be a tissue of arteries, veins, and nerves, enveloped in a soft spongy coagulable lymph; magnified in the same degree as Fig. 1.

Fig. 3. A small portion of Fig. 2. magnified ten diameters, to show the tissue, and the parts of which it is composed

* Since this Lecture was read, two cases have been stated to me, in which I have the most implicit confidence. In one, a Lady in early pregnancy was frightened by a sailor with one arm, and her child was born under the same deformity. In another, this occurred late in pregnancy. No effect was produced on that child; but in her next child, although every alarm in her mind had subsided, the deformity was found to have taken place.

more distinctly; the broken off portions of nerves are shown projecting beyond the edges of the outline of the figure.

Fig. 4. A transverse section of the placenta magnified ten diameters, showing its structure: the nerves are so readily distinguished by their course from the blood vessels, as to require no explanation.

Fig. 5 and 6. A single floculus of the tissue separated, and magnified ten diameters, exposing the terminal branch of the umbilical artery (and its accompanying nerves), where it ends in pencilli of infinitely small ramifications.

A. The pencilli.

B. Surface of the chorion.

Fig. 7. The mode in which the arteries of the umbilical chord begin to ramify on the chorion, and dip down into the substances of the placenta, magnified two diameters; the nervous filaments are distinctly seen.

Plate III.

Fig. 1. A portion of the uterine surface of the chorion of the tapir; natural size.

Fig. 2. A very small portion of the same, magnified fifty diameters.

Fig. 3. Lateral view of a section of the same; magnified fifty diameters.

Fig. 4. A very small portion of the fœtal surface; magnified fifty diameters.

POSTSCRIPT.

Mr. BAUER and myself at the same time, unknown to each other, having detected nerves in the human navel-string and placenta, I beg to communicate this discovery to the Royal Society, as a valuable addition to the Croonian Lecture, to which I wish it to be annexed as a Postscript.

Mr. BAUER's delineation of these nerves makes it unnecessary to give any verbal description of them. They were found in a specimen belonging to Mr. BROOKS, which had been successfully injected, and preserved in spirit for forty years, which I borrowed for that purpose.

PLATE IV.

Fig. 1. A portion of the umbilical cord of the human placenta, in its natural form; magnified four diameters.

Fig. 2. The same portion unravelled, to show the situation of the nerves within it; magnified four diameters.

Fig. 3. A portion of the external membrane of the umbilical cord, with part of the cellular substance, and a nerve adhering to it; magnified four diameters.

Fig. 4. The same nerve shown separately; magnified ten diameters.

Fig. 5. A portion of the amnion, with a double branch of a nerve passing across it; magnified four diameters.

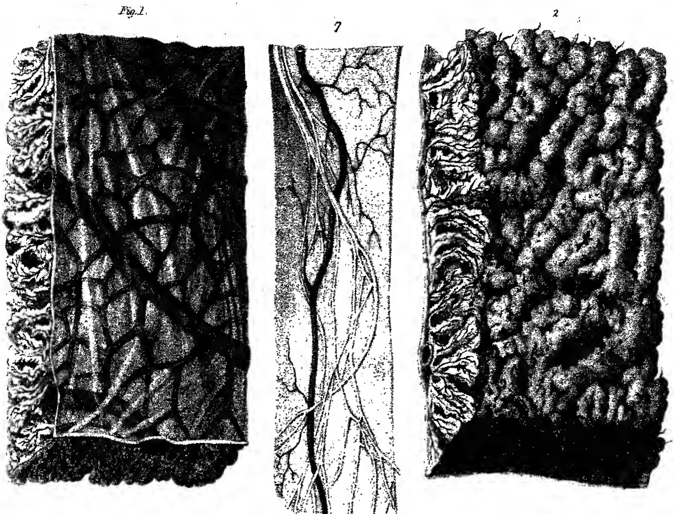
Fig. 1.
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Fig. 1.

2

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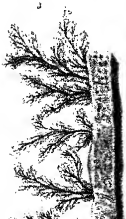

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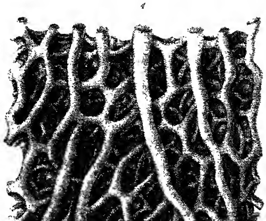

Fig 1

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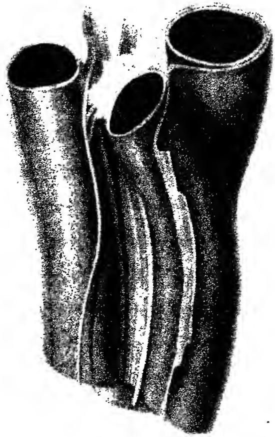

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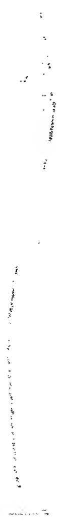

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III. *Observations on the changes the Ovum of the Frog undergoes during the formation of the Tadpole. By Sir* Everard Home, *Bart. V. P. R. S.*

Read November 25, 1824.

In the year 1822, I laid before the Society a series of observations on the progress of the formation of the chick in the egg of the pullet, illustrated by drawings from the pencil of Mr. Bauer, showing that in the ova of hot-blooded animals the first parts formed are the brain and spinal marrow. I have now brought forward a similar series on the progress of organization in the ova of cold-blooded animals, illustrated in the same manner by microscopical drawings made by the same hand.

By comparing together the first rudiments of organization in the ova of these very distinct classes of animals, I shall be able to prove that, in both, the same general principle is employed in the formation of the embryo.

This enquiry has its interest considerably encreased, by the ova not being composed of similar parts.

The ova of the frog, which have been selected for this investigation, are found to have no yelk. If we examine these ova in the ovaria in which they are formed, we find them to consist of small vesicles of a dark colour; when they enter the oviducts they enlarge in size, and acquire a gelatinous covering, which increases in quantity in their course along those tubes; but the ova can neither be said to have acquired

their full size, nor to have received their proportion of jelly, till they arrive at a cavity close to the termination of each oviduct, formed by a very considerable enlargement of those tubes, corresponding, in many respects to the cloaca in which the pullet's egg is retained till the shell becomes hard.

These large bags, in which the oviducts of the frog terminate, when distended with ova, put on an appearance so like the enlarged horns of the uterus of the quadruped when they are filled with young ones, that they have by some anatomists been called a double uterus. This, however, is an improper appellation.

When the ova are deposited in these reservoirs, they become completely formed, and in a state to be impregnated by the male influence, which is applied to them in the act of their expulsion. As they are pressed upon each other, by being confined in a small space, the gelatinous covering takes on an hexagonal figure, in the centre of which is the ovum.

The ova, when examined by a magnifying glass in a strong light, exhibit an appearance so similar to the molecule in the pullet's egg, as to be readily mistaken for it; but a more attentive inspection shows, that it is only a white portion in the ovum, seen through the covering of the vesicle. When the vesicle is punctured by the point of a needle, the contents are so fluid as readily to run out, leaving the strong transparent membranous bag lined with a fluid nigrum pigmentum, empty.

Immediately after impregnation there is no change in the appearance of the jelly, nor of the vesicle contained in it, in

this respect corresponding exactly with what happens to the pullet's egg. The first change that is produced towards the formation of an embryo is, the contents of the vesicle expand, its form changes from that of a sphere to an oval, and when cut through its contents are no longer fluid. In the act of coagulation, the central portion becomes of a lighter colour than that which surrounds it, swells out in the middle, and there is a distinct line by which the two portions are separated from one another: the central part, in its future changes, is converted into brain and spinal marrow; and after these organs have acquired a defined outline, the heart and other viscera are seen forming in the darker substance.

This does not exactly correspond with what takes place in the pullet's egg, that of the frog having no yelk. In the pullet's egg, the part within the inner circle of the molecule, when impregnated by the male, undergoes the necessary changes to form the brain and spinal marrow; the part within the outer circle forms the blood and its vessels; the supplies out of which the other organs are to be produced, are afterwards derived from the yelk.

The membrane that forms the vesicle which is destined to contain the embryo when it has become a tadpole, has a power of enlargement as the embryo increases in size, and then performs the office both of the shell and of the membrane that lines it in the pullet's egg, at the same time serving as a defence to protect it, and allow of the blood being aerated.

The nigrum pigmentum lining the vescicle can only answer some secondary purpose, since it is not met with in the aquatic salamander, whose mode of breeding very closely

resembles that of the frog. Upon reflecting that the frog's spawn is exposed to the scorching effect of the sun, and in places where there is no shelter, this nigrum pigmentum may be given to the eggs as a defence for the young during its growth, which cannot be required in those of the aquatic salamander, since they are separately inclosed within the twisted leaves of water plants, and screened from the full force of the sun's rays. The plant whose leaves the aquatic salamander most generally selects to lay its eggs upon is the Polygonum persicare.

Explanation of the Plates.

Plate V.

Fig. 1. A female frog laid open, just ready to shed her spawn; natural size.

Fig. 2. The ovaria and oviducts; natural size.

Fig. 3. Ova of different sizes taken from the ovarium; magnified five diameters.

Fig. 4. Ova from the upper part of the oviduct; magnified five diameters.

Fig. 5. Ova from the dilated portion of oviduct; natural size. A B. Two ova that had been a few minutes in water, to show the expansion of jelly; magnified five diameters.

Fig. 6. An ovum from which the jelly is removed; magnified ten diameters.

Fig. 7. The same ovum opened, to show that the contents are fluid; when they are allowed to coagulate and dried upon glass, ramifications are formed as in coagulating blood; magnified ten diameters.

Fig. 8. Some ova after being immersed in water for fourteen days; natural size.

Fig. 9. One of them magnified five diameter.

Fig. 10. The same ovum magnified ten diameters.

Fig. 11. The same ovum opened; its contents still fluid, in which oil is found; magnified ten diameters.

PLATE VI.

Fig. 1. Ova six hours after being spawned on water; natural size. A. one of these ova; magnified five diameters. B. The same ovum magnified ten diameters. C. Longitudinal section of the same ovum; its contents in a half coagulated state, and putting on an organized structure; magnified ten diameters.

The rest of the ova spawned at the same time were kept in water, to watch the progress of the formation of the tadpole.

One of these was not impregnated, consequently remained unchanged, while the others became gradually more and more organized. This abortive ovum is placed at the top of each cluster with a mark *

Fig. 2. The cluster of ova 12 hours after being spawned, diminished by that examined in fig. 1.; of the natural size. A. One magnified five diameters. B. The same magnified ten diameters. C. The same, forming a longitudinal section; magnified ten diameters.

Fig. 3. Twenty-four hours after being spawned.

Fig. 4, Thirty-six hours after being spawned. At this period the ovum has its form considerably changed, and the head and tail of the tadpole are distinctly seen.

Fig. 5. Three days after being spawned. At this period muscular motion is for the first time perceptible. The letter D shows the ovum laid open on one side; magnified ten times, as in letter C.

Fig. 6. Four days after being spawned. The letters correspond to these in Fig. 5.

Plate VII.

Fig. 1. Five days after being spawned. At this period the ova become separated, and the tadpoles begin to leave the ovum. A. Shows a tadpole in the act of extricating itself; magnified five diameters.

B. A back view of it; magnified eight diameters.

C. A belly view.

D. A side view.

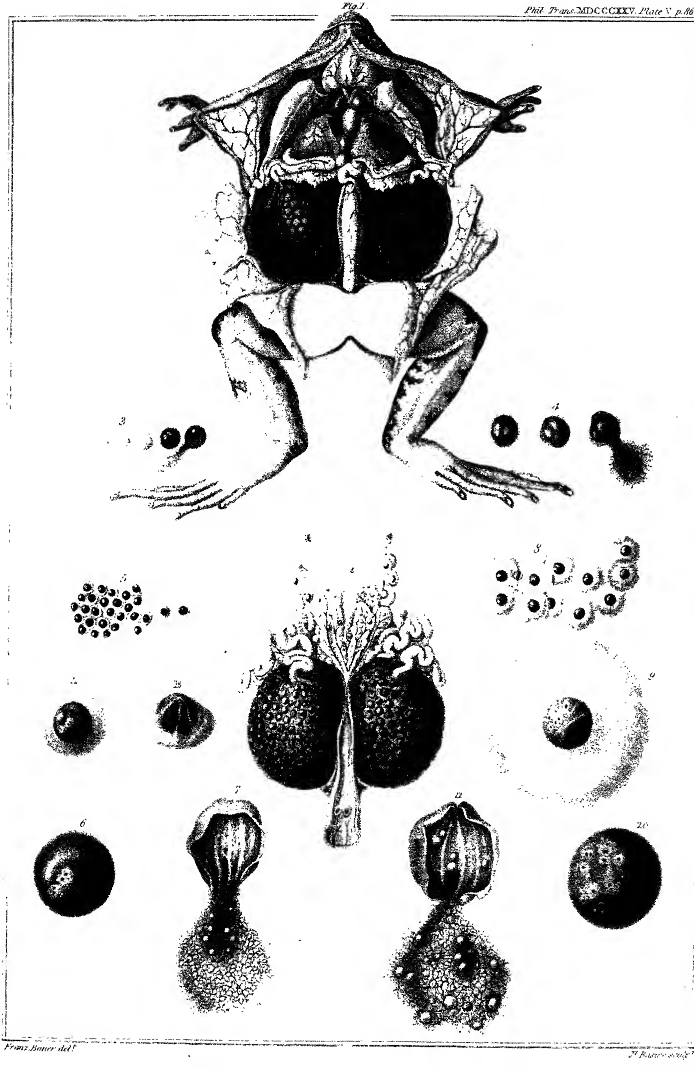
E. A longitudinal section; all the views magnified eight diameters.

Fig. 2. Six days after being spawned, four different views; magnified eight diameters, as in Fig. 1.

Fig. 3. Eight days after being spawned; all the views magnified eight diameters.

Fig. 4. Twelve days after being spawned; four views of the tadpole; magnified eight diameters.

The animal in twelve days had become so far advanced in its growth to make further progress in the investigation unnecessary, after the splendid figures that are before the public upon that subject in different publications.



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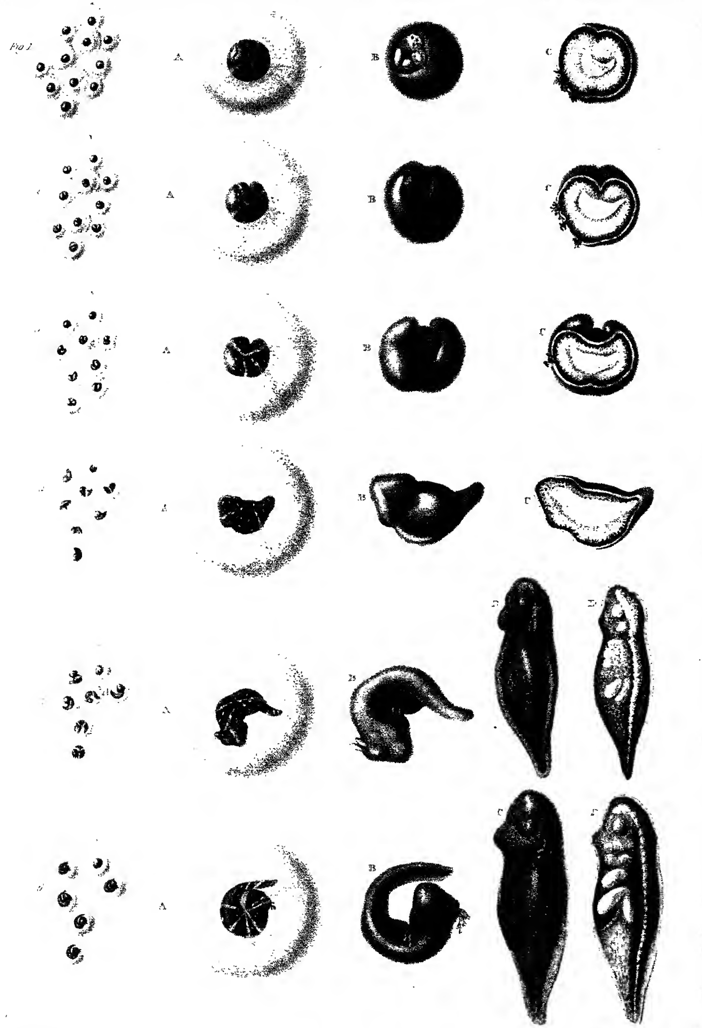

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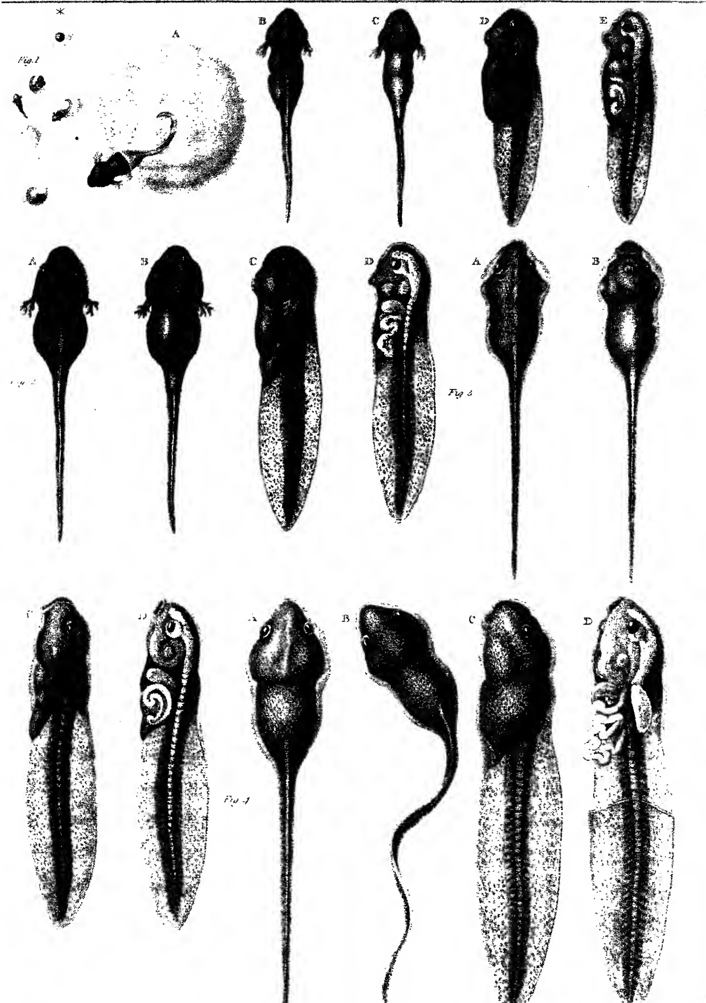

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IV. *A general Method of Calculating the Angles made by any Planes of Crystals, and the Laws according to which they are formed.* By the Rev. W. WHEWELL, *F. R. S. Fellow of Trinity College, Cambridge.*

Read November 25, 1824.

1. IT has been usual to calculate the angles of crystals and their laws of decrement from one another, by methods which were different as the figure was differently related to its nucleus; which were consequently incapable of any general expression or investigation, and which had no connexion with the notation by which the planes of the crystals were sometimes expressed. And the notation which has hitherto been employed, besides being merely a mode of registering the laws of decrement, without leading to any consequences, is in itself very inelegant and imperfect. The different modes of decrement are expressed by means of different arbitrary symbols; and these are combined in a manner which in some cases, as for instance in that of intermediary decrements, is quite devoid both of simplicity and of uniformity, and indeed, it may be added, of precision. The object of the present paper is to propose a system which seems exempt from these inconveniences, and adapted to reduce the mathematical portion of crystallography to a small number of simple formulæ of universal application. According to the method here explained, each plane of a

crystal is represented by a symbol indicative of the laws from which it results; the symbol, by varying the indices only, may be made to represent any law whatever: and by means of these indices, and of the primary angles of the substance, we obtain a general formula, expressing the dihedral angle contained between any *one plane* resulting from crystalline laws, and any *other.* In the same manner we can find the angle contained between any *two edges* of the derived crystal. Conversely, knowing the plane or dihedral angles of any crystal, and its primary form, we can by a direct and general process deduce the laws of decrement according to which it is constituted. The same formula are capable of being applied to the investigation of a great variety of properties of crystals of various kinds, as will be shown in the sequel. We shall begin with the consideration of the rhomboid, and the figures deduced from it; and we shall afterwards proceed to other primary forms.

§ 1. *The Rhomboid.*

2. Let there be a rhomboid, A *a*, Fig. 1. divided into a number of small equal rhomboids by planes parallel to its faces. Let any one of the points of division of each of its three upper edges be taken, as P, Q, R; and let a plane pass through these three points P, Q, R. Let the small rhomboids which are above this plane be removed, so as to leave a uniform assemblage of cavities. Then, the remaining surface P Q R, being composed of the trihedral angles of small rhomboids, if we suppose the small rhomboids to become smaller than the least distinguishable magnitude, the surface P Q R will appear a plane. And if we suppose these rhomboids to represent the

primary form of a crystalline body, P Q R will be a secondary surface deduced from a certain arrangement of these primary elements.

Let the three upper edges of the rhomboid, A x, A y, A z, be considered as three axes of co-ordinates; and let the corresponding co-ordinates be x, y, z. We can then express the plane P Q R by means of these co-ordinates. If, for instance, we consider an edge of the small rhomboid as unity, and if AP, AQ, AR contain respectively 9, 6, and 3 of these edges, the equation to the plane P, Q, R, will be

$$\frac{x}{9} + \frac{y}{6} + \frac{z}{3} = 1:$$

and if the numbers of small rhomboids in AP, AQ, AR be respectively h, k, l, the equation to the plane will be

$$\frac{x}{h} + \frac{y}{k} + \frac{z}{l} = 1.$$

If h, k, l be multiplied by any common quantity m, so that the equation becomes

$$\frac{x}{mh} + \frac{y}{mk} + \frac{z}{ml} = 1, \text{ or } \frac{x}{h} + \frac{y}{k} + \frac{z}{l} = m,$$

it is clear that the plane P Q R will continue parallel to its former position, and may be considered as deduced from the same law as before. Hence it appears, that in the equation $\frac{x}{h} + \frac{y}{k} + \frac{z}{l} = m$, the quantity m does not serve to determine the position or law of formation of the plane, and may be any whatever. If we make $m = 0$, the plane P Q R, still continuing parallel to its former position, will pass through the point A; and as we have to consider only the *angles* made by planes and their intersections, we may in such calculations suppose all our planes to pass through this point A.

Since therefore the direction of the plane P Q R is completely determined by the three quantities h, k, l, we may represent it by writing those three quantities thus $\left(\frac{1}{h}; \frac{1}{k}; \frac{1}{l}\right)$;* or, if the equation be $px + qy + rz = m$, we may represent the plane by the symbol $(p; q; r)$.

3. According to the law of symmetry which prevails in the production of crystalline forms, if one edge or face of the primary solid be modified in any manner, the other homologous edges and faces will be similarly modified. Hence, if one plane exist, other corresponding planes must also exist, and these we may call *co-existent planes* to the first.

Thus if we have a plane P Q R, Fig. 2, and if we take AP′ = AQ, and AQ′ = AP, we must also have a plane P′Q′R: for the edges Az, Ay being perfectly similarly situated, if one of them be affected in any manner, the other must be similarly affected. Hence, if we have a plane $(p; q; r)$, we must have one $(q; p; r)$. The same is also true of z; and by considering this in the same manner, it will be seen that the plane $(p; q; r)$ has the following co-existent planes

$$(q; p; r) \ (r; q; p) \ (p; r; q) \ (q; r; p) \ (r; p; q).$$

That is, there are all the permutations that can be made by altering the arrangement of the three quantities p, q, r; that the one which stands first in order being always the coefficient of x, the second that of y, and the third that of z.

These six planes may be represented by a single symbol

* We might represent the plane by $(h; k; l)$, which shows more immediately the law of its formation; but in all our subsequent calculations we have to use the reciprocals, and hence our formulæ are simplified by using the symbol $(p; q; r)$ where p, q, r are the coefficients of the equation.

(p, q, r); it being understood, that when quantities are only separated by *commas*, they are to be taken in all the ways in which they can be permuted. In the same manner $(p, q; r)$ may represent the two planes $(p, q; r)$ $(q, p; r)$, the permutations not extending to r, which is separated by a *semicolon*. In the case of the rhomboid, however, the permutations always include all the three quantities, in consequence of the similarity of its three edges.

4. We have hitherto considered only the planes produced by cutting off the upper angle; but we may represent in the same manner the plane produced by truncating any other angle. It may be observed that the angles x, y, z, fig. 3, which are separated from the *superior angle* A by an edge, are called *lateral angles*. The angles x', y', z', which are separated from A by a diagonal, are called *inferior angles*.

Let $p\,q\,r$, fig. 3, be a plane produced by a truncation at the lateral angles: xp, xq, xr being h, k, l respectively. Produce rA beyond A, and take $AP = xp$, $AQ = xq$, $AR = xr$; then the plane P Q R will be parallel to $p\,q\,r$, and may be taken instead of it. Now it is manifest that the equation to this plane is

$$-\frac{x}{h} + \frac{y}{k} + \frac{z}{l} = 1;$$

and therefore its symbol is $\left(-\frac{1}{h}; \frac{1}{k}; \frac{1}{l}\right)$. Or if $p = \frac{1}{h}$, $q = \frac{1}{k}$, $r = \frac{1}{l}$, the equation is $-p\,r + qy + rz = m$, and the symbol $(-p; q; r)$. Hence a plane which cuts off the lateral solid angles is distinguished by having one negative index.

In the same manner let $p\,q\,r$, fig. 4, cut off an inferior angle x', so that $x'p = h$, $x'q = k$, $x'r = l$: and taking

$AP = x'p$, $AQ = x'q$, $AR = x'r$, the plane PQR will be parallel to $p\,q\,r$, and its equation will be

$$\frac{x}{h} - \frac{y}{k} - \frac{z}{l} = 1 \text{ ; or } px - qy - rz = 1 \text{ :}$$

and its symbol $\left(\frac{1}{h}; -\frac{1}{k}; -\frac{1}{l}\right)$, or $(p; -q; -r;)$. Hence a plane which cuts off the inferior solid angles is distinguished by having two negative indices.

It may be observed, that in both these cases the *coexistent* planes are given by taking the permutations of p, q, r; and may be represented as before by $(-p, q, r)$ and $(p, -q; -r)$. There will in each case be six; two for each angle.

5. If one of the quantities AP, AQ, AR, or h, k, l, in any of these cases become infinite, we shall have a truncation of an *edge* of the rhomboid. Thus if AP, in fig. 2, become infinite, we have a plane cutting off the terminal edge $A\,x$, fig. 5. And since h is infinite, if $q = \frac{1}{k}$, $r = \frac{1}{l}$, the equation of this plane is $qy + rz = 1$; and its symbol $(0; q; r)$.

In the same manner, making $x'r$ infinite in fig. 4, we have, for a plane truncating the lateral edge $x'y$, an equation $px - qy = 1$, and a symbol $(p; -q; 0)$.

The terminal edges of $A\,x$, $A\,y$, $A\,z$, are not similarly affected with the lateral edges xy', $y'z$, zx', $x'y$, yz', $z'x$.

6. Instead of supposing the secondary faces to be produced by removing a part of the rhomboid $A\,a$, we may conceive, with HAUY, that this larger figure is composed by adding successive layers of the small component rhomboids to a rhomboidal nucleus; and that the secondary faces are produced by supposing the magnitude of these layers to decrease according to any law. And it will be easy to show

what symbols, according to the notation here proposed, correspond to the different laws in the old system. Thus

A decrement on the superior angle is expressed by (p, q, q), which corresponds to HAUY's symbol $\overset{A}{\frac{p}{q}}$.

On a lateral angle by $(-p, q, q)$ corresponding to $E^{\frac{q}{p}}$;

On an inferior angle by $(p, -q, -q)$ corresponding to $e^{\frac{q}{p}}$;

On a terminal edge by $(0, q, r)$ corresponding to $B^{\frac{r}{q}}$;

On a lateral edge by $(p, -q, 0)$ corresponding to $G^{\frac{q}{p}}$.

An intermediary decrement thus (p, q, r), corresponding to $\left(A^p B^q C\frac{q}{r}\right)$ and $(p, -q, -r)$ corresponding to $\left(O^p D^q F\frac{q}{r}\right)$.

The symbols of the faces of the primary form are $(p, 0, 0)$.

7. There is in fact, however, no necessity to suppose the secondary forms to be produced either by truncation of a primary one, or by addition to it. If we suppose that the small rhomboids, of which A*a* was assumed to be made up, are continued through all the space round the point A, we may conceive a plane to pass among these, parallel to PQR. And this plane will be represented by $(p; q; r)$ independently of any consideration of the rhomboid A*a* or the point A; for if we take *any point*, and from it draw lines to the plane, parallel to the three edges Ax, Ay, Az, these three lines will be as $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$. And any other plane may similarly pass among the small rhomboids, and be represented by $(p'; q'; r')$. And if we obtain any solid figure contained by such planes, we may, by supposing those of the small

rhomboids which lie without this plane to be removed, have a proper representation of a secondary crystalline form constituted by the aggregation of primary ones.

Before we proceed to the calculations founded on this mode of viewing the subject, we may observe, that by increasing or diminishing the three indices p, q, r in any ratio, the plane represented by them is not altered. Thus $(p; q; r)$ $(np; nq; nr)$ $\left(\frac{p}{r}; \frac{q}{r}; 1\right)$, &c. are the same plane. Hence $(p; q; q)$ is the same as $\left(\frac{p}{q}; 1; 1\right)$ $(p; p; 0)$ as $(1; 1; 0)$: and the primary faces are $(1, 0, 0)$.

8. PROP. To find the dihedral angle contained between two planes $(p; q; r)$ $(p'; q'; r')$, the dihedral angle at the terminal edges of the primary rhomboid being α.

If there be three co-ordinates any how situated so that the dihedral angle at the axis x between the planes xy and xz is α; the dihedral angle at the axis y, β; and at the axis z, γ: and if d be the cosine of the angle which a line perpendicular to the plane yz makes with x; e the cosine of the angle which a line perpendicular to xz makes with y; f the cosine of the angle which a line perpendicular to xy makes with z: and if θ be the angle of two planes whose equations are $Ax + By + Cz = m$, $A'x + B'y + C'z = m'$: we shall have (see Transactions of the Cambridge Philosophical Society, Vol. II. P. I. p. 200)

$$-\cos.\theta = \frac{\left\{\begin{array}{c}\frac{AA'}{d^2} + \frac{BB'}{e^2} + \frac{CC'}{f^2} \\ -\frac{A'B + AB'}{de}\cos.\gamma - \frac{A'C + AC'}{df}\cos.\beta - \frac{B'C + BC'}{ef}\cos.\alpha\end{array}\right\}}{\sqrt{\left\{\begin{array}{c}\left(\frac{A^2}{d^2} + \frac{B^2}{e^2} + \frac{C^2}{f^2} - \frac{2AB}{de}\cos.\gamma - \frac{2AC}{df}\cos.\beta - \frac{2BC}{ef}\cos.\alpha\right) \times \\ \times\left(\frac{A'^2}{d^2} + \frac{B'^2}{e^2} + \frac{C'^2}{f^2} - \frac{2A'B'}{de}\cos.\gamma - \frac{2A'C'}{df}\cos.\beta - \frac{2B'C'}{ef}\cos.\alpha.\right)\end{array}\right\}}}$$

In the case of the rhomboid, since the dihedral angles are equal, α, β, γ are equal; and hence also d, e, f are equal. Hence

$$-\cos.\theta = \frac{AA'+BB'+CC'-(A'B+AB'+A'C+AC'+B'C+BC')\cos.\alpha}{\sqrt{\left\{(A^2+B^2+C^2-2(AB+AC+BC)\cos.\alpha)(A'^2+B'^2+C'^2-2(A'B'+A'C'+B'C')\cos.\alpha)\right\}}}$$

And if we put $p, q, r, p', q'; r'$ for A, B, C, A′, B′, C′, we shall have the angle.

If we have to find the angle of two planes resulting from the same law, $(p'; q'; r')$ will be a permutation of $(p; q; r)$; and the denominator of $-\cos.\theta$ will be

$$p^2+q^2+r^2-2(pq+pr+qr)\cos.\alpha.$$

We shall take examples of the use of these formulæ.

Ex. 1. To find the angle made by two planes of carbonate of lime resulting from the law* $(4, -5, -5)$. (*Chaux Carbonatée Cuboide* of HAUY).

The primary form of carbonate of lime is a rhomboid in which the angle α is $105^\circ\ 5'$, and therefore $\cos.\alpha = -.2602$.

Two of the secondary planes will be $(4; -5; -5)$ and $(-5; 4; -5)$, and if θ be the angle contained by these

$$-\cos.\theta = \frac{-15-51\cos.\alpha}{66+30\cos.\alpha};\ \text{or}\ \cos.\theta = \frac{5-17\times.2602}{22+10\times.2602} = .0297$$

$$\therefore \theta = 88^\circ.18.$$

A variety of other rhomboids may be produced in this and other substances by other laws. In all cases, if two of the indices of the symbol be equal, as (p, q, q), there will only

* That this law is what HAUY calls a decrement on the inferior angles of 4 in breadth to 5 in height, and is in his notation represented by the symbol $e\frac{4}{5}$.

The angles obtained in the text differ slightly from these given by HAUY in consequence of his having assumed the angle of the primary rhomboid of carbonate of lime, $= 104^\circ.28'.40''$, for the convenience of using the cosine $= -\frac{1}{4}$.

be three coexistent planes; and if each of these planes be repeated, we shall have three pairs of parallel planes containing a rhomboid.

If the three indices in the symbol (p, q, r) be all different, we shall have six planes, and repeating each of these, we shall have a dodecahedron consisting of two six-sided pyramids. To this case belongs the following example:

Ex. 2. To find the angle of planes in carbonate of lime, resulting from the law $(1, -2, 0)$. (Decrement on the lateral edges by two rows in breadth. Symbol D^2. *Chaux Carbonatée Metastatique.* HAUY.)

Two adjacent* planes are $(1; -2; 0)$ $(1; 0; -2)$, and preserving the same notation as before

$$-\cos.\theta = \frac{1}{5 + 4\cos.\alpha} = -.2525, \theta = 104^\circ\ 38'.$$

By other laws we should find other dodecahedrons and their angles. But in many cases we have *two* laws, producing two sets of faces, and it may be required to find the angle between those of one set and of the other.

Ex. 3. To find the angles of planes $(2, -1, -1)$ and $(1, 0, 0)$. (Decrement by two rows in breadth on an inferior angle, combined with the primitive faces. Symbol e^2 P. *Chaux Carbonatée Imitable.* HAUY).

Adjacent faces* are $(2; -1; -1)$ and $1; 0; 0)$: and

$$-\cos.\theta = \frac{2 + 2\cos.\alpha}{\sqrt{(6 + 6\cos.\alpha)}} = 2\sqrt{\frac{1 + \cos.\alpha}{6}} = .7022; \theta = 134^\circ\ 37'.$$

9. We proceed now to the inverse problem; having given the angles of the secondary crystal to find the law of its planes. And we shall first suppose the secondary form to

* It will be shown afterwards how we may determine of co-existent planes which are adjacent.

be a rhomboid; in which case, as has already been observed, two of the indices in the symbol are equal.

Prop. Knowing the dihedral angles of the secondary rhomboid, to find the symbol of its planes,

Let (p, q, q) be the symbol of the planes, θ the angle of $(p; q; q)$ and $(q; p; q)$.

$$\therefore -\cos.\,\theta = \frac{2pq + q^2 - (p^2 + 2pq + 3q^2)\cos.\,\alpha}{p^2 + 2q^2 - 2(2pq + q^2)\cos.\,\alpha}$$

Here cos. θ being known, we have a quadratic equation to determine q in terms of p, which as the proportion $q : p$ only is wanted, is sufficient.

The equation will be

$$p^2(\cos.\,\theta - \cos.\,\alpha) + 2pq(1 - \cos.\,\alpha - 2\cos.\,\alpha\cos.\,\theta) + q^2(1 - 3\cos.\,\alpha + 2\cos.\,\theta - 2\cos.\,\alpha\cos.\,\theta) = 0$$

There will be for each value of θ two values of $\frac{q}{p}$, and therefore two laws according to which the same secondary form may be produced. It is to be noticed however, that the direction of the primitive faces, and consequently of the cleavage will be different in the two cases.

10. Prop. It is required to find according to what law we shall have a rhomboid similar to the primary one.

Here $\theta = \alpha$: therefore the first sum of the above equation vanishes, and the remaining part will be verified either by $q = 0$, or by

$$2p(1 - \cos.\,\alpha - 2\cos.^2\alpha) + q(1 - \cos.\,\alpha - 2\cos.^2\alpha) = 0, \text{ or } q = -2p.$$

Therefore $(1, 0, 0)$ and $(1, -2, -2)$ each give $\theta = \alpha$. The first indicates the primary face, and the form is the primary form. The other indicates a decrement by 2 in height on the inferior angle, which it appears gives a rhomboid identical with the primary rhomboid.

11 PROP. Knowing the lateral angles made, at the terminal edges, by the planes of any bipyramidal dodecahedron to find the symbols.

If we have planes (p, q, r) they will generally form a bipyramidal dodecahedron, and the six angles at the edges of each pyramid will be alternately greater and less. If p, q, r be the order of magnitude of the indices, p being the greatest, the order of the faces will be that represented in fig. (see hereafter the section on the arrangement of faces). Hence faces occur in the order $(p; q; r)$ $(q; p; r)$ $(r; p; q)$ &c.: and if θ be the angle of the two first, and θ' of the next, we shall have

$$-\cos.\theta = \frac{2pq + r^2 - (p^2 + q^2 + 2pr + 2qr)\cos.\alpha}{p^2 + q^2 + r^2 - 2(pq + pr + qr)\cos.\alpha}:$$

$$-\cos.\theta' = \frac{2qr + p^2 - (q^2 + r^2 + 2pq + 2pr)\cos.\alpha}{p^2 + q^2 + r^2 - 2(pq + pr + qr)\cos.\alpha}.$$

from which equations we have to determine q and r in terms of p.

To eliminate in these equations would lead to expressions of four dimensions, and it will generally be simpler to find q and r by trial. If we assume for p any number, as 12; q and r, which generally bear to it very simple ratios, will in most cases be whole numbers, and may be found by a few trials. And if the ratios of q and r to p involve quantities which are not divisors of 12, still the trials made on this supposition will indicate *nearly* the values of q and r; and by trying other values for p, we may obtain them accurately.

If two of the indices, as q, r be negative; the order of the faces will be $(p; -r; -q)(-r; p; -q)(-q; p; -r)$, &c. and the rest of the process will be the same as before.

12. PROP. Knowing the angles made by any plane with two primary planes, to find its symbol.

Let $(p; q; r)$ be the plane, and $(0, 1, 0)(0, 0, 1)$ the two primary planes; θ and θ' the given angles

$$\therefore \cos.\ \theta = \frac{q - (p + r) \cos.\ \alpha}{\sqrt{\left\{p^2 + q^2 + r^2 - 2(pq + pr + qr) \cos.\ \alpha\right\}}}$$

$$\cos.\ \theta' = \frac{r - (p + q) \cos.\ \alpha}{\sqrt{\left\{p^2 + q^2 + r^2 - 2(pq + pr + qr) \cos.\ \alpha\right\}}}$$

whence q and r must be found in terms of p, as in last proposition.

Or we may find them directly thus. Since one of the three p, q, r is indeterminate, assume $p^2 + q^2 + r^2 - 2(pq + pr + qr)$ $\cos.\ \alpha = 1$.

$\therefore \cos.\ \theta = q - r \cos.\ \alpha - p \cos.\ \alpha$; $\cos.\ \theta' = r - q \cos.\ \alpha - p \cos.\ \alpha$. Eliminating, we have

$$q \sin.^2 \alpha = \cos.\ \theta + \cos.\ \alpha \cos.\ \theta' + p \cos.\ \alpha (1 + \cos.\ \alpha);$$

$$r \sin.^2 \alpha = \cos.\ \theta' + \cos.\ \alpha \cos.\ \theta + p \cos.\ \alpha (1 + \cos.\ \alpha).$$

If we substitute these values in the assumed equation multiplied by $\sin.^4 \alpha$, viz.

$$\left\{p^2 + q^2 + r^2 - 2(pq + pr + qr) \cos.\ \alpha\right\} \sin.^4 \alpha = \sin.^4 \alpha$$

we shall have a quadratic equation in p; and hence p, q, r are found.

13. Prop. To find what laws will give prisms parallel to the axis of the primary rhomboid.

For this purpose the planes must be parallel to the axis; and the equation of a plane must be consistent with the equations of the axis, which are

$$y = x,\ z = x.$$

Let $(p; q; r)$ be the plane; $\therefore p\,x + q\,y + r\,z = 0$ is the equation to it, supposing it to pass through the origin; and since $y = x, z = x$; we have $px + qx + rx = 0 \therefore p = -(q + r)$.

If $r = q, p = -2q$; the planes are $(-2, 1, 1)$ and the

secondary rhomboid becomes a regular hexagonal prism. (Example. *Chaux Carbonatée Prismatique.* HAUY.)

In other cases the secondary form is an irregular hexagonal prism, the angles being equal, three and three alternately.

14. PROP. To find the symbol of a plane which truncates any edge of a given form.

Let two faces $(p; q; r)$ $(p'; q'; r')$ meet, and let $(P; Q; R,)$ be a plane which truncates the edge formed by their intersection: the plane must be parallel to this intersection; and the equations to the intersection must be consistent with the equation $Px + Qy + Rz = 0$. Now for the intersection we have $px + qy + rz = 0$, $p'x + q'y + r'z = 0$: whence

$$(pq' - p'q)\,x = (qr' - q'r)\,z,\quad (p'r - pr')\,x = (qr' - q'r)\,y.$$

Multiply $Px + Qy + Rz = 0$ by $(qr' - q'r)$ and substitute, and we have

$$P(qr' - q'r) + Q(p'r - pr') + R(pq' - p'q) = 0.$$

And if P, Q, R fulfil this condition, (P; Q; R) will be a plane truncating the edge as required.

15. PROP. To find the symbol of a plane which truncates an edge of any secondary rhomboid.

This is a particular case of last Prop. when instead of $(p; q; r)$ $(p'; q'; r')$, the planes are $(p; q; q)$ $(q; p; q)$. Hence the equation of condition becomes

$$P(q^2 - pq) + Q(q^2 - pq) + R(p^2 - q^2) = 0$$

$$\text{or}\quad Pq + Qq - R(p + q) = 0$$

Hence if $R = q$, $P + Q = p + q$, and with this condition, $(P; Q; q)$ is the plane required.

Ex. Required the planes which truncate the edges of the rhomboid produced by the law $(3, -1 -1)$.

Here $p + q = 2$; $\therefore$ the values which may be given to

P, Q are any number whose sum is 2. Thus (1, 1, — 1) (2, 0, — 1) are truncating faces.

(This rhomboid truncated by these two planes occurs in Haüy's *Chaux Carbonatée Progressive.* Fig. 41.)

The plane thus determined will always be parallel to the intersection of the two planes; but in order that it may truncate the edge, it must meet both of them on the really existing part of each plane. This condition is easily introduced in each particular case.

16. In order to express, by means of the symbols already introduced, any crystal whatever, we may write down the symbols of the faces by which it is bounded; indicating by the punctuation the permutations which are allowed. It will be convenient also to mark the number of the faces which arise from these permutations. In the rhomboid, when all the three indices are different, this number will be *six*. When two are alike, it it will be *three*. Thus (6) (p, q, r) may indicate that the crystal has six faces arising from the law expressed by (p, q, r) and (3) (p, p, r) may represent a crystal with three faces arising from the law (p, p, r); which is what would, according to Haüy, be called a decrement on an angle at the summit.

It often happens that faces in a crystal are repeated; that is, that there are faces parallel to one another, one of which may be considered as a repetition of the other. In that case we may distinguish them by placing a 2 before them as a multiplier. Thus 2 (3) (p, p, r) indicates a rhomboid produced by repeating each of the three faces represented by (p, p, r). This is in fact the mode in which a rhomboid is always produced. In the same manner 2 (6) $(p; q, r)$ is the

symbol of a dodecahedron, which results from repeating each of the six planes (p, q, r).

§. 2. *The Quadrangular Prism.*

17. The quadrangular prism may be right or oblique, and its base may be a square, a rectangle, a rhombus, or a parallelogram. But in all cases we may take one of its angles, and make that the origin of co-ordinates; and taking two of our co-ordinates along two edges of the base, and the third along the length of the prism, we shall be able to express the secondary planes in the same manner as in the case of the rhomboid. There will however be some additional considerations to introduce, since the edges of the prism may be of different magnitudes; and its angles not being symmetrical like those of a rhomboid, we shall no longer have the same coexistent planes which we had in the former case.

In order to introduce the first consideration, let x and y, fig. 6, be the co-ordinates in the direction of the edges of the base, and z in that of the length of the prism. Let the space bounded by the co-ordinate planes be filled with small similar prisms, and let their edges in the directions x, y, z be a, b, c respectively. Let a secondary plane PQR be formed, by taking away h prisms along the edge x, k along y, and l along z; then the lengths of AP, AQ, AR will be ha, kb, lc respectively; and the equation to the plane will be

$$\frac{x}{ha} + \frac{y}{kb} + \frac{z}{lc} = 1.$$

If we call $\frac{1}{ha}$, A; $\frac{1}{kb}$, B; $\frac{1}{lc}$, C; we shall have the angle between any two planes by the formula, Art. 8; putting for α, β, γ and for d, e, f, their values. But if we make $\frac{1}{h}, -p,$

$\frac{1}{k} = q$, $\frac{1}{l} = r$, $(p ; q ; r)$ may still be taken for the symbol of the plane. In this case $\frac{p}{a}$, $\frac{q}{b}$, $\frac{r}{c}$, are the co-efficients of the equation to the plane, and are to be used for A, B, C in calculating the angles which the planes make with each other.

We shall use the following terms; a *rhombic prism* is one whose base is a rhombus: an *oblique rhombic prism*, fig, 8, is one in which the sides are not at right angles to the base, the angles of the sides, as BA z, CA z being equal. A *doubly oblique prism*, fig. 7, is one in which the angles of the sides at the base BA z, CA z are unequal. Prisms are called *square* or *rectangular* when their bases are so: and when the base is a parallelogram with unequal sides, and angles not right angles, the prism is called *oblique-angled*. Besides these we have a prism which we may call the *oblique rectangular prism*,* fig. 9, in which besides the two rectangular ends we have two sides, as $c\,z$ and the opposite one, also rectangles.

1. *The doubly-oblique Prism*, fig. 7.

18. In this, since the angles are all different, no one of the solid angles (A, B, C, D) is similar to another. Hence if a plane be formed on one of the angles, there is no plane necessarily formed on another angle; consequently a plane as $(p ; q ; r)$ or $(p ; -q ; -r)$ does not necessarily imply any *co-existent* plane, and the symbol is to be written with the mark (;) between the indices, to show that no permutations are allowed.

Let the edges of the subtractive prisms in last article be a,

* We might consider B z as the base of prism, by which means it would be a right oblique angled prism. But the method adopted in the text seems to be more natural and simple.

in the direction AB, b in the direction AC, c in the direction Az. Then putting $\frac{p}{a}$, $\frac{q}{b}$, $\frac{r}{c}$ for A, B, C in the formula, Art. 8, we shall have the angles made by secondary planes.

Conversely, knowing the angles made by secondary planes we may determine A, B, C, as before, and when we have found in crystals the same substance, various values of A, B, C, we have

$$\frac{q}{p} = \frac{Bb}{Aa}, \; \frac{r}{p} = \frac{Cc}{Aa};$$

and a, b, c are to be assumed so that $q:p$ and $r:p$ may be numerical ratios as simple as possible.

2. *The oblique rhombic Prism*, fig. 8.

19. In this case the angles zAB, zAC, and the sides AB, AC are equal; and consequently the two faces zAB, zAC are symmetrical; and whatever secondary plane is formed with reference to one, we must have a co-existent plane corresponding to the other. Hence, if we have a plane $(p; q; r)$ we must have a plane $(q; p; r)$ and we may express both these by the symbol $(p, q; r)$ the (,) indicating that the co-ordinates x and y may be exhanged, z remaining the same. And this is true whether p, q, r be positive or negative.

Here having found p, q, and r we have ha, ka, lc, because a and b are equal, and their values are to be determined as before.

3. *The oblique rectangular Prism*, fig. 9.

20. Here the solid angles A and C are similar in all respects, A being contained by two right angles BAC, CAz and the angle BAz, and C by the angles DCA, ACo, oCD equal to them. Hence whatever plane be formed on A, we must have a coexistent plane on C, agreeing with it, except

that the ordinate in AC is in the opposite direction: that is $(p;q;r)$ $(p;-q;r)$ are co-existent planes. These may be included in the formula $(p;\pm q;r)$.

4. *The right oblique-angled Prism*, fig. 10.

21. It is obvious that the opposite angles A and D of the base of this prism are similar in all respects; and with any secondary plane formed on one of them, we must have a co-existent similar plane on the other. That is, we must have a second plane, when x and y are negative, as they were positive in the first. Hence $(p;q;r)$ $(-p;-q;r)$ are co-existent planes; and we may express them thus $(\pm p;\pm q,r)$ it being understood in such symbols that the upper signs are taken together, and the lower together.

5. *The right rhombic Prism*, fig. 10.

22. Here, the opposite angles A, D are similar, and also the adjacent sides. Hence with a plane $(p;q;r)$ we have co-existent planes $(-p;-q;r)(q;p;r)(-q;-p;r)$. These may be included in the symbol $(\pm p,\pm q;r)$ the upper signs being taken together as before, and p, q being permutable as is indicated by the comma.

6. *The right rectangular Prism*, fig. 11.

23. Here the four angles A, B, C, D are similar. Hence $(p;q;r)$ has co-existent planes

$$(-p;q;r)\ (p;-q;r)\ (-p;-q;r)$$

These may be included in the formula

$$\left(\begin{matrix}+\\-\\+\\-\end{matrix}p;\ \begin{matrix}+\\+\\-\\-\end{matrix}q;\ r\right)$$

the signs being taken in horizontal pairs.

7. *The right square Prism.*

24. In this case, besides the co-existent planes which we have in the last figure, we shall have those which arise from considering that the sides AB, AC are symmetrical, that is p and q are permutable. Here the symbol is $\left(\begin{smallmatrix}+\\+\\-\end{smallmatrix} p, \begin{smallmatrix}\pm\\+\end{smallmatrix} q; r\right)$ this will give eight secondary faces.

8. *The Cube.*

25. This differs from the last in having the edge in the direction z similar to those in x and y. Hence p, q, r may be permuted and the symbol is $\left(\begin{smallmatrix}+\\+\\=\end{smallmatrix} p, \begin{smallmatrix}+\\+\\-\end{smallmatrix} q, r\right)$ which gives 24 secondary faces.*

There is no necessity to vary the sign of r, for the plane $(p; q; -r)$ is the same as $(-p; -q; r)$.

§ 3. *The regular Tetrahedron and Octahedron.*

26. In this and other cases where the figure is bounded by more than three planes we shall make three of the primary faces co-ordinate planes, and the remaining primary faces will be expressed by different symbols. Also the co-existent planes will be differently represented accordingly as they are on one angle or another, and we shall in each case have to determine the different forms which will thus occur.

Let Axyz, fig. 12, be a regular tetrahedron, and let Ax, Ay, Az be three co-ordinates.

* In some cases however, we have only half the number of faces which the law of symmetry would give. Thus in the case of the pentagonal dodecahedron derived from the cube, the law is (2, 1, 0); but the faces which occur are (2; 1; 0) (1; 0; 2) (0; 2; 1) which by the changes of sign become 12. The other 12 which arise from the symbols (1; 2; 0) (2; 0; 1) (0; 1; 2) are excluded.

Let a plane $p\,q\,r$ be formed on the angle A; then, since all the angles are symmetrical, we must have a coexistent plane at any other angle, as x.

Let $Ap = h$, $Aq = k$, $Ar = l$; and let $xP = h$, $xQ = k$, $xR = l$; it is required to find the equation to the plane PQR.

Draw x M and y K parallel to PQ and we have, if $Ax = a$,

$xK = xy \cdot \frac{xP}{xQ} = a\frac{h}{k}$; $\therefore AK = a\left(1 - \frac{h}{k}\right)$.

Also $AM = Ay \cdot \frac{Ax}{AK} = \frac{}{1 - \frac{h}{k}}$

Similarly if xN be parallel to PR, we shall find $AN = \frac{a}{1 - \frac{h}{l}}$

Hence the equation of the plane N x M is

$$\frac{x}{a} + \left(1 - \frac{h}{k}\right)\frac{y}{a} + \left(1 - \frac{h}{l}\right)\frac{z}{a} = 1;$$

or $$\frac{x}{h} + \left(\frac{1}{h} - \frac{1}{k}\right)y + \left(\frac{1}{h} - \frac{1}{l}\right)z = \frac{a}{h}.$$

And the symbol of this plane will be

$$\left(\frac{1}{h}; \frac{1}{h} - \frac{1}{k}; \frac{1}{h} - \frac{1}{l}\right).$$

And the plane PQR is parallel to N x M, and will have the same symbol.

If $\frac{1}{h} = p$, $\frac{1}{k} = q$, $\frac{1}{l} = r$, the symbol of the plane PQR will be $(p; p - q; p - r)$.

In the same way we shall have at the angles y and z, planes

$$(q - p; q; q - r) \text{ and } (p - r; q - r; r).$$

But the edges Ax, Ay, Az are also similar, and therefore p, q, r may be permuted in any manner. Hence we have these co-existent planes

(p, q, r), $(p, p - q, p - r)$, $(q - p, q, q - r)$, $(r - p, r - q, r)$.

It being understood that in each parenthesis the indices which are separated by commas may undergo any permutation.

The first symbol (p, q, r) gives 6 planes, and the three others also 6 each, making in all 24.

If the primary form be known to be a regular tetrahedron, it is evident that the first symbol (p, q, r) must be understood as implying also the rest. But in order to express all the planes we may include them in one symbol thus

$$\{(p, q, r)(p, p-q, p-r) \text{ \&c.}\}$$

the &c. implying the coexistent planes.

27. Prop. To determine the symbol of the planes which truncate the *edges* of a tetrahedron.

The plane truncating the edge x is $(0; q, r)$: and hence by last article the general symbol includes the planes

$$(0, q, r), (q, q, q-r), (r, r-q, r)$$

which gives 12 planes. We omit $(0, -q, -r)$, which is identical with $(0, q, r)$.

If $q = r$ the planes are expressed by $(0, q, q)$, which gives 3 planes; but in order to truncate the six edges, each is used twice, and the symbol is $2(3)(0, q, q)$.

The *regular octahedron* is bounded by the same 4 planes as the tetrahedron, each being used twice; and its symbol is $2(4)\{(1, 0, 0)(1, 1, 1)\}$.

Its edges are also parallel to the edges of the tetrahedron, each being used twice. And any plane which can be deduced from the octahedron, may with equal simplicity be deduced from the tetrahedron.

28. Prop. In the regular tetrahedron to find the angle contained by planes $(0, 1, 1)$.

The plane angles of the tetrahedron are 60°; and hence, to find its dihedral angles, we have to find the angle of an equilateral spherical triangle whose sides are 60°. If α be this angle, we have

$$\therefore \cos. \alpha = \text{cotan. } 60 \text{ . tan. } 30 = \tan.^2 30 = \frac{1}{3}.$$

Let θ be the angle of the planes (0, 1, 1) (1, 0, 1), and we have by the formula

$$-\cos. \theta = \frac{1 - 3 \cos. \alpha}{2 - 2 \cos. \alpha} = 0 \text{ because } \cos. = \frac{1}{3}.$$

Hence the angle of the planes is a right angle. And in the same manner the angles made by the other planes will be right angles. The figure will be a *cube* bounded by the 3 planes (0, 1, 1) twice repeated.

Irregular Tetrahedrons and Octahedrons.

29. If we have an octahedron composed of two right quadrilateral pyramids, similar and equal, set base to base, we shall call this a *right octahedron;* and it will be termed *square*, *rectangular*, or *rhombic*, when the base is so. The tetrahedron, from which the right rectangular octahedron is derived, may be called the *direct symmetrical tetrahedron;* and that from which the right rhombic octahedron is derived, may be called the *inverse symmetrical tetrahedron*, on account of properties which will be explained immediately. Also, all the planes which can be derived from the octahedrons, may be derived more simply from the corresponding tetrahedrons; and we shall find the coexistent planes, and the angles made by the faces, in the same manner as in the previous cases.

§ 4. *Direct symmetrical Tetrahedron and rectangular Octahedron.*

30. Let A $x\,y\,z$, fig. 13, be a tetrahedron, and let all its edges be bisected, and the bisections joined by lines drawn in the faces. We shall thus have an octahedron DEFGHK. If we consider EFHK as the common base of the two pyramids of which the octahedron is composed, when EFHK is a rectangle, the octahedron is called rectangular; and when EFHK is a square, the octahedron is called square.

Let EFHK be a rectangle, the octahedron being a right one. Then all the faces of the octahedron will be isosceles triangles, of which DEF, DHK, GFE, GHK will be equal to each other, and the other four also equal to each other. Also, it is easily seen that the triangle A $y\,z$ has its sides double of those of EFG, and is similar to it; and similarly $x\,y\,z$ has its sides double of KHG. Therefore the two triangles A $y\,z$, $x\,y\,z$ are both isosceles, ($y\,z$ being the base,) and are equal in every respect; and similarly y A x and z A x are isosceles triangles equal in every respect.

Hence the solid angles at y and z are equal in every respect, and also those at A and x. And a plane passing through A x and through the middle of $y\,z$ would divide the tetrahedron symmetrically into two equal portions. Hence we have called this the direct symmetrical tetrahedron.

We may suppose the solid angle A to be filled with parallelepipeds, the planes of which are parallel to the planes A $x\,y$, A $x\,z$, A $y\,z$, in the same manner as the solid angle A, fig. 1. And by removing these parallelepipeds according to any law, as in fig. 1, we obtain a secondary plane, of which the symbol and the equation may be known from the law.

31. But since the solid angles at A and at x are symmetrical, for every plane at A we shall have a co-existent plane at x,* of which we shall find the equation.

We may as before suppose A x, A y, A z, to be co-ordinates, and with any plane $p\,q\,r$ at A we shall have a co-existent plane PQR at x, such that xP, xQ, xR are equal to Ap, Aq, Ar respectively.

Prop. The symbol of $p\,q\,r$ being $(p\,;\,q\,;\,r)$ to find the symbol of PQR.

Let the small component parallelepipeds have the edge in direction $\mathrm{A}\,x = a$, and the edges in directions Ay, Az each $= c$ (these being equal). Also, let $\mathrm{A}x = n\,a$, $\mathrm{A}y = \mathrm{A}z = n\,c$.† And let the plane $p\,q\,r$ be obtained by taking away h molecules in the direction Ax, k in the direction Ay, and l in the direction Az. Therefore $\mathrm{A}p = h\,a$, $\mathrm{A}q = k\,c$, $\mathrm{A}r = l\,c$: and the equation to the plane $p\,q\,r$ is

$$\frac{x}{h\,a} + \frac{y}{k\,c} + \frac{z}{l\,c} = 1;$$

* The parallelepipeds of which the solid is supposed to be made up at x, are not in the same position with those of which it is supposed to be made up at A. Those at x are bounded by planes parallel to Axy, Axz, yxz, as those at A are by the planes which meet at A. If the crystal be divisible according to all the planes of a tetrahedron or octahedron, there are four different kinds of parallelepiped of which it may be conceived to be composed, corresponding to the four angles A, x, y, z. And we may take any one of these kinds with equal propriety. In fact, the mode of conceiving secondary planes to be formed by removing parallelepipeds, is an assumption to be considered right only so far as it exhibits the dependence of secondary planes upon the simplicity of the ratios $p : q : r$.

† If we suppose Axyz to be made up of parallelepipeds, Ax, Ay, and Az having *equal numbers* of them, planes parallel to xyz will pass through all their angles. And if instead of parallelepipeds, we suppose that we have only points in space where the angles of the parallelepipeds would be, the planes which are determined by any adjacent three points will be the four planes, Axy, Axz, Ayz, xyz.

$$\text{or if}\quad p=\frac{1}{h},\ q=\frac{1}{k},\ r=\frac{1}{l},$$

$$p\frac{x}{a}+q\frac{y}{c}+r\frac{z}{c}=1,$$

the symbol of which is $(p\,;\,q\,;\,r)$.

Draw $y\,O$, $x\,M$ parallel to PQ, meeting $A\,x$ and $A\,y$. Then

$$x\,O=\frac{xy\,.\,xP}{xQ}=\frac{nc\,.\,ha}{kc}=\frac{nha}{k}$$

$$\therefore AO=A\,x-x\,O=n\,a\left(1-\frac{h}{k}\right)$$

$$AM=\frac{Ax\,.\,Ay}{AO}=\frac{na\,.\,nc}{na\left(1-\frac{h}{k}\right)}=\frac{nkc}{k-h}.$$

Similarly if $x\,N$ be parallel to PR, $AN=\frac{nlc}{l-h}$.

Hence the equation to the plane $x\,M\,N$ is

$$\frac{x}{na}+\left(1-\frac{h}{k}\right)\frac{y}{nc}+\left(1-\frac{h}{l}\right)\frac{z}{nc}=1$$

$$\text{or}\ p\frac{x}{a}+(p-q)\frac{y}{c}+(p-r)\frac{z}{c}=pn$$

and the equations to planes $p\,q\,r$ and PQR are

$$p\frac{x}{a}+q\frac{y}{c}+q\frac{z}{c}=1\,;$$

$$p\frac{x}{a}+(p-q)\frac{y}{c}+(p-r)\frac{z}{c}=m$$

and their symbols are $(p;\,q;\,r)$, $(p;\,p-q;\,p-r)$.

Also the edges $A\,y$, $A\,z$ are symmetrical; and hence we have two other co-existent planes $(p;\,r;\,q)\,(p-r;\,p-q)$.

These are included in the formula $\left\{(p;\,q,r)\,(p;\,p-q,p-r)\right\}$

The solid angles at y and z are also symmetrical; and a plane being supposed to be formed at y as before, we must have a co-existent plane at z. Let $p'\,q'\,r'$ be a plane cutting off the angle y, and b being the edge of a molecule in the direction $y\,z$, let $y\,p'$, $y\,q'$, $y\,r'=hb,\,kc,\,lc$ respectively, and let $z\,P'$, $z\,Q'$, $z\,R'=y\,p'$, $y\,q'$, $y\,r'$ respectively. Then $p'\,q'\,r'$

and P′ Q′ R′ will be co-existent planes; and the condition of their co-existence is included in the preceding symbol.

The quantities a, b, c are as $n\,a$, $n\,b$, $n\,c$, that is as $A\,x$, $y\,z$ and $A\,y$. Or, referring to the octahedron in fig. 13, they are as FH, FE, and FD.

The square Octahedron.

32. When EFHK, fig. 13, is a square, $A\,x$, $y\,z$ will be equal, and the solid angles at y and z will be symmetrical to those at A and x, and will be similarly affected. Hence for a plane at A there will be co-existent planes at y and z.

Prop. To find the symbols of co-existent planes in this case,

If we take $z\,P'$, $z\,Q'$, $z\,R'$, $= y\,p'$, $y\,q'$, $y\,r'$, $= A\,p$, $A\,q$, $A\,r$ respectively, we shall, as in last article, find the equation of the planes $p'\,q'\,r'$, P′ Q′ R′ to be

$$\left(1-\frac{k}{l}\right)\frac{x}{n\,a}+\frac{y}{n\,c}+\left(1-\frac{k}{h}\right)\frac{z}{n\,c}=m$$

$$\left(1-\frac{k}{l}\right)\frac{x}{n\,a}+\left(1-\frac{k}{h}\right)\frac{y}{n\,c}+\frac{z}{n\,c}=m'$$

and since $p=\frac{1}{h}$, $q=\frac{1}{k}$, $r=\frac{1}{l}$, these are equivalent to

$$(q-r)\frac{x}{a}+q\,\frac{y}{c}+(q-p)\frac{z}{c}=\frac{m\,n}{k}$$

$$(q-r)\frac{x}{a}+(q-p)\frac{y}{c}+q\,\frac{z}{c}=\frac{m'\,n}{k}$$

Hence with a plane $(p\,;\,q\,;\,r)$ we have co-existent planes

$(q-r\,;\,q\,;\,q-p)$ and $(q-r\,;\,q-p\,;\,q)$.

But we have also a co-existent plane $(p\,;\,r\,;\,q)$ and therefore also $(r-q\,;\,r\,;\,r-p)$ and $(r-q\,;\,r-p\,;\,r)$

Hence in the square octahedron we have co-existent planes which may be included in this symbol

$$\left\{(p\,;\,q,\,r)\,(p\,;\,p-r,\,p-q)\,(q-r\,;\,q,\,q-p)\,(r-q\,;\,r,\,r-p)\right\}$$

All which are implied in $(p\,;\,q\,;\,r)$.

33. Prop. Having given the symbol of a plane derived from the tetrahedron, to find the manner in which it cuts the octahedron, Fig. 13.

Let PQR be any plane at the angle A; and let PQ meet DK and DE in S and T $\therefore \mathrm{DS} = \frac{\mathrm{DP}\,.\,\mathrm{AQ}}{\mathrm{AP}} = \mathrm{DP}\,.\,\frac{kc}{ha} = \mathrm{DP}\,.\,\frac{p}{q}\,.\,\frac{c}{a}$.

And drawing QL parallel to DE, $\mathrm{DF} = \frac{\mathrm{QL}\,.\,\mathrm{DP}}{\mathrm{PL}}$

Also $\mathrm{QL} = \mathrm{AQ}$ and $\mathrm{PL} = \mathrm{AP} - \mathrm{AL} = \mathrm{AP} - \mathrm{AQ}\,.\,\frac{\mathrm{A}x}{\mathrm{A}y} = ha - kc\,.\,\frac{a}{c} = (h-k)a$

$$\therefore \mathrm{DT} = \mathrm{DP}\,.\,\frac{\mathrm{AQ}}{\mathrm{PL}} = \mathrm{DP}\,.\,\frac{kc}{(h-k)a} = \mathrm{DP}\,\frac{p}{q-p}\,.\,\frac{c}{a}.$$

In the same way we find the portions cut off from DH and DF: and hence it appears that a plane $(p; q; r)$ cuts off from the four edges, which meet at the vertex D of the pyramid, lines which, parallel to the edges in the directions Ay, Az, xy, xz, are as

$$\frac{1}{q},\ \frac{1}{r},\ \frac{1}{q-p},\ \frac{1}{r-p}.$$

In whatever manner the plane DEF is cut by the plane PQR, the plane DHK will be similarly cut by the co-existent plane at x.

34. Hence, knowing the law by which a secondary face is derived from the octahedron, we can find its symbol.

The primary form is a square octahedron; to find the symbol of the face ${}^2\mathrm{E}^2$ (*Ex. Zircon unibinaire*, Haüy).

This plane is drawn cutting off the angle E, in such a manner that the portions cut from EF, EG are double of those from EK, ED respectively; and the section on the face EFG parallel to FG or to Ay.

Since the part cut from EG, parallel to Az, is double of that

from ED, parallel to xy, and is in the negative direction, $\frac{1}{r} = -\frac{2}{q-p}$ or $p - q = 2r$.

Also since the section is parallel to Ay we must have $q = 0$. Hence $(2 ; 0 ; 1)$ is the symbol required. And the co-existent planes are

$$(2 ; 0, 1)\ (2 ; 1, 2)\ (-1 ; 0, -2)\ (1 ; -1, 1)$$

each of the parentheses gives two planes, and hence we have 8 arising from this law.

35. To find the angles which these planes make with the planes of the octahedron.

Example. *Zircon unibinaire*, HAUY.

In the square octahedron, which has been considered as the primary form of zircon, the angle of two adjacent faces of a pyramid is 123° 15′, and the angle of two opposite faces measured over the summit is 95° 40′. (PHILLIPS).

Hence the dihedral angle at Ax, which is (α) the angle of the planes EFK, FDH, is 95° 40′. And (β,) the angle at Ay is the angle of DEK, FEG, and is therefore the supplement of the angle of HFG, EFG, and it is therefore = 56° 45′. In the same manner (γ) the dihedral angle at Az is 56° 45′.

In order to apply the formulæ of Art. 8, we must find the values of d, e, f. Let XYZ, fig. 15, be a spherical triangle made by describing a sphere with center A, meeting Ax, Ay, Az in X, Y, Z. Then if XD be drawn perpendicular to YZ, $d =$ sin. XD, similarly if YE be perpendicular on ZX, $e =$ sin. YE, and $f = e$.

Now by NAPIER'S rules, since XYD = 56° 45′, and YXD $= \frac{1}{2}$(95° 40′) = 47° 50′, r . cos. 56° 45′ = cos. XD . sin. 47° 50

$\therefore d =$ sin. 42° 11 ; $d = .6730125$.

Also $r.\cos.\,XY = \text{cotan}.\,56°\,45'.\,\text{cotan}.\,47°\,50' \therefore xy = 43°\,37'$
and $YXE = 180 - 90°\,40' = 84°\,20'$
$\therefore r.\sin.\,YE = \sin.\,XY.\sin.\,84°20' \therefore e = \sin.\,43°\,21'; e = .6864532$

The two planes of which we have to find the angle, are (2 ; 0 ; 1) (1 ; 0 ; 0)..

Hence by the formula, Art. 8,

$$-\cos.\,\theta = \frac{\frac{2}{d} - \frac{\cos.\,\beta}{f}}{\sqrt{\left\{\frac{4}{d^2} + \frac{1}{f^2} - \frac{4\cos.\,\beta}{af}\right\}}} = \frac{2f - d\cos.\,\beta}{\sqrt{\left\{4f^2 - 4fd\cos.\,\beta + a^2\right\}}}$$

To find θ, let $\tan.\,\omega = \frac{2f - d\cos.\,\beta}{d\sin.\,\beta} = \frac{2f}{d\sin.\,\beta} - \text{cotan}.\,\beta$; and we shall have, $-\cos.\,\theta = \frac{\tan.\,\omega}{\sec.\,\omega} = \sin.\,\omega\,.\,\therefore \theta = 90° + \omega$.

By the values above given, we shall find $\omega = 60°\,43'$ and $\therefore \theta = 150°\,43'$. The value given by Mr. PHILLIPS is $150°\,12'$.

It may be observed, that (2 ; 0 ; 1) is the side adjacent to the primary plane (1 ; 0 ; 0); and that we obtain sides adjacent to other faces by taking *corresponding* co-existent planes from the formulæ in Art. 32.

Thus the primary faces (1 ; 0 ; 0) have adjacent secondary faces (2 ; 0 ; 1) and (2 ; 1 ; 0).
The primary faces (0 ; 1 ; 0) have adjacent (1 ; 2 ; 0) and (1 ; — 1 ; 1)
The primary faces (0 ; 0 ; 1) have adjacent (1 ; 0 ; 2) and (1 ; 1 ; — 1)
The primary faces (1 ; 1 ; 1) have adjacent (2 ; 1 ; 2) and (2; 2; 1)

Here instead of (—1 ; 0: —2) &c. we have written (1 ; 0; 2) &c. which represents the same plane.

§ 5. *Inverse symmetrical Tetrahedron and rhombic Octahedron.*

36. Let A $x\,y\,z$, fig. 16, be a tetrahedron; and let its edges be bisected, and an octahedron formed as before. In this octahedron, let EFHK be the rhombic base; and the two

pyramids which compose the octahedron being right ones and equal, it is evident that the four lines DE, EG, GH, HD will be equal, and the four lines DF, FG, GK, KD. Now $A\,x$ is double of FH, and $x\,y$ of HK. Hence $A\,x = y\,z$. Similarly $A\,y = x\,z$, and $A\,z = x\,y$. Hence it appears that the four triangles which form the sides of the tetrahedron have their sides equal respectively, and are therefore equal and similar. Hence the four solid angles A, x, y, z, are contained by equal angles, and are symmetrical. Thus the angles $x\,A\,y$, $y\,A\,z$, $z\,A\,x$ are equal to $A\,x\,z$, $y\,x\,z$, $A\,x\,y$. And this tetrahedron may be called an inverse symmetrical tetrahedron.

From the law of symmetry, whatever plane is formed at the angle A, we must have a coexistent plane at each of the angles x, y, z, the equal and opposite edges being similarly affected.

37. Prop. A plane $(p\,;q\,;r)$ being known, to find the coexistent planes. Fig. 17.

Let $A\,x$, $A\,y$, $A\,z$ be $n\,a$, $n\,b$, $n\,c$.

$A\,p$, $A\,q$, $A\,r = h\,a$, $k\,b$, $l\,c$; and $p = \frac{1}{h}, q = \frac{1}{k}, l = \frac{1}{r}$.

$\therefore x\,P$, $x\,Q$, $x\,R$ are $h\,a$, $k\,b$, $l\,c$.

Draw $y\,O$, $x\,M$ parallel to PR.

$$x\,O = x\,y \,.\, \frac{x\,P}{x\,R} = n\,c\,\frac{h\,a}{l\,c} = \frac{n\,h\,a}{l}\,;\ AO = n\,a\left(1 - \frac{h}{l}\right)$$

$$AM = A\,x \,.\, \frac{A\,y}{AO} = \frac{n\,b}{1 - \frac{h}{l}} = \frac{n\,b}{1 - \frac{r}{p}}$$

Similarly if $x\,N$ be parallel to PQ, $AN = \dfrac{n\,c}{1 - \frac{h}{k}} = \dfrac{n\,c}{1 - \frac{q}{p}}$.

Hence the equation of the plane $x\,NM$, which is parallel to PQR, is

$\frac{x}{na}+\left(1-\frac{r}{p}\right)\frac{y}{nb}+\left(1-\frac{q}{p}\right)\frac{z}{nc}=1$ or $p\frac{x}{a}+(p-r)\frac{y}{b}+(p-q)\frac{z}{c}=np$
and its symbol is $(p\,;\,p-r\,;\,p-q)$.

In the same manner the angle y gives a plane $(q-r\,;\,q\,;\,q-p)$ and the angle z a plane $(r-q\,;\,r-p\,;\,r)$.

Hence the co-existent planes are

$(p\,;\,q\,;\,r), (p\,;\,p-r\,;\,p-q), (q-r\,;\,q\,;\,q-p), (r-q\,;\,r-p\,;\,r)$.

These four planes would truncate symmetrically the four faces of one of the pyramids which compose the octahedron, and planes parallel to them would truncate similarly the planes of the other pyramid.

38. PROP. To find the portions cut from the edges of the octahedron by the plane $(p\,;\,q\,;\,r)$.

Let the plane P, Q, R, fig. 16 and 18, meet DK, DE, DF, DH in S, T, U, V. Draw QL parallel to DE. Then

$$DS=DP\,.\frac{AQ}{AP}=DP\,.\frac{kb}{ha}=DP\,.\frac{p}{q}\frac{b}{a}$$

$$QL=AQ\frac{xy}{Ay}=kb\,.\frac{c}{b}=kc,\ AL=AQ\frac{Ax}{Ay}=kb\,.\frac{a}{b}=ka\,;\ PL=(h-k)a$$

$$DT=DP\,.\frac{QL}{PL}=DP\frac{kc}{(h-k)a}=DP\,.\frac{p}{q-p}\,.\,\frac{c}{a}$$

Similarly DV and DU would be $DP\,.\frac{p}{r}\,.\,\frac{c}{a}$ and $DP\,.\frac{p}{r-p}\,.\,\frac{b}{a}$

Hence DS, DT, DU, DV are as $\frac{1}{q}\,.\,b, \frac{1}{q-p}c, \frac{1}{r-p}b, \frac{1}{r}c$.

Hence for the four co-existent planes the edges cut off are respectively as

$$\frac{b}{q}, \frac{c}{q-p}, \frac{b}{r-q}, \frac{c}{r}\,;$$
$$\frac{b}{r-p}, \frac{c}{r}, \frac{b}{q}, \frac{c}{q-p}\,;$$
$$\frac{b}{q}, \frac{c}{r}, \frac{b}{r-p}, \frac{c}{q-p}\,;$$
$$\frac{b}{r-p}, \frac{c}{q-p}, \frac{b}{q}, \frac{c}{r}.$$

The calculations would be nearly the same as in the case of the square octahedron, article 35. We should have to calculate d, e, f from the angles of the octahedron. Thus in sulphur, according to Mr. Phillips (p. 361) we have incidence of

GEF on GEK $= 106° 30$; $\therefore$ angle at A $x = 73° 30 = \gamma$
GFH on GFE $= 85° 5$; $\therefore$ angle at A $y = 94° 55 = \beta$
GHF on DHF $= 143° 25$; $\therefore$ angle at A $z = 36° 35 = \alpha$

And if we construct a triangle, of which the three angles are α, β, γ, and draw arcs from these angles perpendicular on the opposite sides, the sines of these arcs will be respectively d, e, f. And by first finding the sides of the triangle by spherical trigonometry, these may be calculated.

§ 6. *The regular triangular Prism.* Fig. 19.

39. This is a right prism, having for its base an equilateral triangle. It includes the regular hexagonal prism by repeating the lateral faces.

Prop. To find the co-existent planes.

By the law of symmetry, for every plane on one angle A, we must have co-existent planes on x, y. Let $p\,q\,r$ be any plane whose symbol is $(p; q; r)$, and the lines $\mathrm{A}p = h$, $\mathrm{A}q = k$, $\mathrm{A}r = l$, when $p = \frac{1}{h}$, $q = \frac{1}{k}$, $r = \frac{1}{l}$. Then we shall have a plane PQR where $x\mathrm{P} = h$, $x\mathrm{Q} = k$, $x\mathrm{R} = l$. Draw x M, y O parallel to PQ.

$$\therefore x\mathrm{O} = xy \cdot \frac{x\mathrm{P}}{x\mathrm{Q}} = \frac{h}{k} \text{ if } \mathrm{A}x = xy = \mathrm{A}y = 1.$$

$$\mathrm{AM} = \mathrm{A}x \cdot \frac{\mathrm{A}y}{\mathrm{AO}} = \frac{1}{1 - \frac{h}{k}} = \frac{1}{1 - \frac{q}{p}} = \frac{p}{p - q}.$$

Similarly if x N be parallel to RP, $\mathrm{AN} = \mathrm{A}x \cdot \frac{x\mathrm{R}}{x\mathrm{P}} = \frac{l}{h} = \frac{p}{r}$. Hence the equation of the plane x MN is

$$x + \frac{p-q}{p} y - \frac{r}{p} z = 1 \text{ or } p x + (p-q) y - r z = p.$$

$\therefore$ its symbol, or that of PQR, is $(p; p-q; -r)$.

Similarly, at y, we shall have a plane $(q-p; q; -r)$.

Also, since the edges A x and A y are symmetrical, we have a plane $(q; p; r)$. And hence the co-existent planes are $(p; q; r)\ (p; p-q; -r)(q-p; q; -r)(q; p; r)\ (q; q-p; -r)$ $(p-q; p; -r)$. Which may be included in the symbol

$$\{(p, q; r)\ (p, p-q; -r)\ (q, q-p; -r)\}$$

§ 7. *The rhombic Dodecahedron.*

40. If we take a regular tetrahedron $w\,x\,y\,z$, fig. 20, and from its centre of gravity A draw lines A w, A x, A y, A z, the angles made by any two of these lines will be the same. And by taking planes passing through any two of these lines we shall have six planes symmetrically disposed, each of which will make an angle of 120° with four others. A figure bounded by planes parallel to these planes, each taken twice, and symmetrically disposed, will be the rhombic dodecahedron.

We may consider the three lines A x, A y, A z as axes of co-ordinates; and any plane $p\,q\,r$ which cuts them must have co-existent planes cutting any two of them and A w. Also, as the lines Ax, Ay, Az are similar, in a plane (p, q, r) we may present the indices in any manner.

41. Prop. To find the symbols of co-existent planes in the rhombic dodecahedron.

Let a plane $p\,q\,r$ cut w A produced in O. Let x, y, z be the co-ordinates of the point O. The equations of the line A w are $y = x$, $z = x$. And if the equation to the plane

$p\,q\,r$ be $p\,x + q\,y + r\,z = m$, we shall have the co-ordinates of the point O by combining these equations. Hence we have $p\,x + q\,x + r\,x = m$, or $x = \frac{m}{p+q+r}$.

But if the co-ordinates x, y, z be projected upon AO, we shall have $AO = A\,x \cos. x\,AO + A\,y \cos. y\,AO + A\,z \cos. z\,AO$. And since $\cos. x\,AO = \cos. y\,AO = \cos. z\,AO = \frac{1}{3}$, $AO = \frac{x+y+z}{3} = x$. $\therefore AO = \frac{m}{p+q+r}$.

Now let $p'x + q'y + r'z = m$ be the equation to a plane which cuts $A\,x$, $A\,y$, $A\,w$ in the same manner in which $(p\,;q\,;r)$ cuts $A\,x$, $A\,y$, $A\,z$. Therefore the portion cut off from $r\,y$ A produced will be $\frac{m}{p'+q'+r'}$. Also the portions from $A\,x$ and $A\,y$, are $\frac{m}{p'}$, $\frac{m}{q'}$.

Hence $\frac{m}{p'} = \frac{m}{p}$, $\frac{m}{q'} = \frac{m}{q}$, $-\frac{m}{p'+q'+r'} = \frac{m}{r}$

$\therefore p' = p$, $q' = q$, $p' + q' + r' = -r$.; $r' = -\overline{p+q+r}$.

Hence if $(p\,;q\,;r)$ be a plane $(p\,;q\,;-\overline{p+q+r})$ is a co-existent plane.

Also the axes of x, y, z being symmetrical, $(p\,;q\,;r)$ has co-existent planes (p, q, r). And making $-\overline{p+q+r} = s$, we have the planes

$$(p, q, r)\ (p, q, s)\ (p, r, s)\ (q, r, s).$$

Each of these symbols gives six permutations, so that we have in all 24 co-existent planes.

§ 8. *On the arrangement of secondary faces.*

42. When crystals have faces determined by the laws considered in the preceding pages, they will have the form of polyhedrons bounded by polygons; and in order to determine the dihedral angles, &c. it will be necessary to know

in what order the faces occur, and which are adjacent. This may be done in the following manner:

Let AI fig. 21, be any parallelepiped of which the edges Ax, Ay, Az are a, b, c. Let an ellipsoid be described, of which the center is I, touching three planes of this parallelepiped in D, E, F. If we suppose any secondary plane, deduced from this parallelepiped, to be drawn so as to touch the ellipsoid in P, the situation of the points P will determine the position of the planes. Let $Ax + By + Cz = m$ be the equation to the plane. The equation to the ellipsoid will be

$$\frac{(a-x)^2}{a^2} + \frac{(b-y)^2}{b^2} + \frac{(c-z)^2}{c^2} = 1.$$

And that the plane may touch the ellipsoid, the differential co-efficients $\left(\frac{dy}{dx}\right)$ and $\left(\frac{dz}{dx}\right)$ must be the same in both. Hence

$$\left(\frac{dy}{dx}\right) = -\frac{A}{B} = -\frac{b^2}{a^2}\cdot\frac{(a-x)}{(b-y)};\quad \left(\frac{dz}{dx}\right) = -\frac{A}{C} = -\frac{c^2}{a^2}\,\frac{(a-x)}{(c-z)}.$$

Therefore $\frac{a-x}{Aa^2} = \frac{b-y}{Bb^2}$; $\frac{a-x}{Aa^2} = \frac{c-z}{Cc^2}$.

And substituting in the equation to the ellipsoid we have

$$\frac{1}{a^2}(a-x)^2 + \frac{B^2b^2}{A^2a^4}(a-x)^2 + \frac{C^2c^2}{A^2a^4}(a-x)^2 = 1$$

$$\therefore\ a - x = \frac{Aa}{\sqrt{(A^2a^2 + B^2b^2 + C^2c^2)}}$$

$$\therefore\ b - y = \frac{Bb}{\sqrt{(A^2a^2 + B^2b^2 + C^2c^2)}}$$

$$\text{and } c - z = \frac{Cc}{\sqrt{(A^2a^2 + B^2b^2 + C^2c^2)}}$$

Knowing the position of the points P for all the planes, we have the polyhedron, on the supposition that it is made such that the ellipsoid can be inscribed in it; which is always possible by supposing the planes to move parallel to themselves till they touch it.

We shall see more clearly the position of the points P if

we suppose it to be determined by angular distances like the longitude and latitude on a globe, assuming as the axis of the ellipsoid that about which the figure is symmetrical.

43. (1) In the rhomboid. Here $Ax = Ay = Az = 1$, suppose IA be taken as the axis; and a plane API being drawn, let the angle between this plane and IA x be called the longitude (λ) of the point P; and let the complement of AIP be called the latitude (μ) of P.

Let the co-ordinates of P be called X, Y, Z. Then the plane API has a point A, of which the co-ordinates are 0, 0, 0; a point I, of which the co-ordinates are 1, 1, 1; a point P, of which the co-ordinates are X, Y, Z. Hence its equation is $(Y-Z)x + (Z-X)y + (X-Y)z = 0$. And the equation to IA x is $y - z = 0$. Therefore by the formula for the angle of two planes, Art. 8,

$$-\cos.\lambda = \frac{-2X+Y+Z-(2X-Y-Z)\cos.\alpha}{\sqrt{\left\{2\left((Y-Z)^2+(Z-X)^2+(X-Y)^2+2(X^2+Y^2+Z^2-XY-XZ-YZ)\cos.\alpha\right)\right\}}}$$

If the symbol of the plane be $(p; q; r)$ its equation is $px + qy + rz = m$; and hence

$a - X = \frac{pa}{\sqrt{(p^2+q^2+r^2)}}$; and similarly for Y and Z. Hence

$$\cos.\lambda = \frac{(2p-q-r)(1+\cos.\alpha)}{2\sqrt{\left\{(p^2+q^2+r^2-pq-pr-qr)(1+\cos.\alpha)\right\}}};$$

$$= \frac{2p-q-r}{2\sqrt{(p^2+q^2+r^2-pq-pr-qr)}}\cdot\sqrt{1+\cos.\alpha}).$$

To find μ; if we draw PM perpendicular in AI, and call IP, r, we shall have $IM = r\sin.\mu$, and μ will be greater as IM is greater. Now if IM, NO, OP to the co-ordinates of P measured from I, and if we draw perpendiculars from N and O on IA, we shall see that $IM = (a-X)\cos.\zeta + (a-Y)\cos.\zeta + (a-Z)\cos.\zeta$ where ζ is the angle which AI makes with A x, A y or A z.

$$r \sin. \mu = \frac{p+q+r}{\sqrt{(p^2+q^2+r^2)}} . \cos. \zeta.$$

By these formulæ we may determine the arrangement of any set or sets of secondary faces. Thus if we have a symbol (p, q, r) in which $p > q$, $q > r$; we have 6 faces. The expression for $r \sin. \mu$ is the same for all: hence they are all at the same distance from the summit B. And cos. λ will be greater as $2p - q - r$ is, or as $3p - (p+q+r)$ is so. Consequently the values of cos. λ taken in order of magnitude will correspond to $(p; q; r)(q; p; r)(r; p; q)$. The other three values be the same, viz. $(p; r; q)$ $(q; r; p)$ $(r; q; p)$; and indicate longitudes on the other side of Ax.

The arrangement of the planes is represented in fig. 22.

It is to be observed that as the order of the *first* index is p, q, r, beginning from x, the order of the *second* index is p, q, r beginning from y, and of the third p, q, r, beginning from z.

44. (2) In the Prism. Let the line IF, fig. 21, parallel to Az, be taken for the axis of the ellipsoid; and let the position of P be determined by (λ) the longitude which is measured by the angle between the planes FID and FIP; and by μ the latitude, the angle PIN.

It is evident that tan. λ will be greater as $\frac{\text{NO}}{\text{IO}}$ is greater. Let $(p; q; r)$ be the symbol of the plane, and its equation will be $\frac{px}{a} + \frac{qy}{b} + \frac{rz}{c} = 1$.

And the values of IO, ON, NP, will be

$$\frac{pa}{\sqrt{(p^2+q^2+r^2)}}, \frac{qb}{\sqrt{(p^2+q^2+r^2)}}, \frac{rc}{\sqrt{(p^2+q^2+r^2)}}.$$

Hence tan. λ will be greater as $\frac{qb}{pa}$ is greater; or as $\frac{q}{p}$ is greater; because a and b are constant for the same substance. Also sin. μ is greater as PN is greater; that is, as $\frac{rc}{\sqrt{(p^2+q^2+r^2)}}$ is so.

And hence we may arrange the faces in the order of their longitude and latitude.

We might in the same manner find the position of the planes for other primitive forms, but what has been done will generally be sufficient.

§ 9. *On the angles made by edges.*

45. If we have two lines referred to any co-ordinates, of which the equations are $y = Ax$, $z = Bx$; $y = A'x$, $z = B'x$; and if the plane angles of the faces be known; viz. the angle which x makes with $y = \phi$, the angle which x makes with $z = \psi$ and the angle which y makes with $z = \omega$; we shall find θ, the angle which the two lines make with one another, by the formula,

$$\cos.\theta = \frac{1 + AA' + BB' + (A + A')\cos.\phi + (B + B')\cos.\psi + (A'B + AB')\cos.\omega}{\sqrt{(1 + A^2 + B^2 + 2A\cos.\phi + 2B\cos.\psi + 2AB\cos.\omega)(1 + A'^2 + B'^2 + 2A'\cos.\phi + 2B'\cos.\psi + 2A'B'\cos.\omega)}}$$

(See Trans. of Camb. Phil. Soc. vol. ii; P. I; p. 202.)

When we know the symbols of the planes, the co-efficients A, B will be found by eliminating y and z in the equations of the planes where intersection is considered.

Ex. In a rhomboid it is required to find the angles made by the opposite edges of a pyramid formed of planes (p, q, r).

By referring to fig. 22. it will be seen that opposite edges are those which are produced by intersections of planes $(p; q; r)(q; p; r)$ and $(q; r; p)(r; q; p)$.

To find the equation to the first line we have

$$px + qy + rz = 0$$
$$qx + py + rz = 0$$

whence $y = x$, $z = -\frac{p+q}{r}x$.

In the same manner we should find for the second line

$$y = x, \; z = -\frac{q+r}{p}x.$$

Substituting for A, B, A′, B′ in the formula, we have, since $\varphi = \psi = \omega$

$$= \frac{1 + 1 + \frac{(p+q)(q+r)}{pr} + 2\cos.\varphi - \left(\frac{p+q}{r} + \frac{q+r}{p}\right)\cos.\varphi - \left(\frac{p+q}{r} + \frac{q+r}{p}\right)\cos.\varphi}{\sqrt{\left(1+1+\frac{(p+q)^2}{r^2} + 2\cos.\varphi - 2\frac{p+q}{r}\cos.\varphi - 2\frac{p+q}{r}\cos.\varphi\right)\left(1+1+\frac{(q+r)^2}{p^2} + 2\cos.\varphi - 2\frac{q+r}{p}\cos.\varphi - 2\frac{q+r}{p}\cos.\varphi\right)}}$$

$$= \frac{3pr + pq + q^2 + qr - 2(p^2 + r^2 + pq + qr - pr)\cos.\varphi}{\sqrt{((p+q)^2 + 2r^2 - 2(p+q-r)r\cos.\varphi))\,(q+r)^2 + 2p^2 - 2(q+r-p)p\cos.\varphi)}}$$

And if we take any other opposite pairs of planes, $(p\,;r\,;q)$ $(q\,;r\,;p)$ and $(q\,;p\,;r)(r\,;p\,;q)$; or $(r\,;q\,;p)(r\,;p\,;q)$ and $(p\,;r\,;q)$ $(p\,;q\,;r)$; we shall have the same value for θ. Hence this angle may be used as the characteristic of a pyramid produced by any such law from a rhomboid: and consequently of a dodecahedron resulting from repeating the faces of the pyramid. It is employed in this manner by Bournon in characterising the dodecahedrons of carbonate of lime.

§ 10. *Recapitulation.*

46. It may be useful to collect in one view the results of the foregoing investigations. If we take a solid angle of the primary form of a crystal for the origin, and the three edges for three co-ordinates, any secondary plane may be obtained by removing a pyramid, the edges of which consist of h, k, l, molecules respectively. If we make $p = \frac{1}{h}$, $q = \frac{1}{k}$, $r = \frac{1}{l}$, the secondary plane may be represented by $(p\,;q\,;r)$ which will express its position without determining its distance from the origin: p, q, r may be positive, o, or negative. By the law of symmetry with respect to the angles and edges of primary forms, if one secondary plane exist, certain others must also exist, which are hence called *co-existent* planes. Some of these are obtained by permuting the order of the letters in the symbol (p, q, r); and the instances where this

permutation is allowed may be distinguished from those where it is not, by separating the letters p, q, r in the former case by a comma, and in the latter by a semicolon. The other co-existent planes in each primary form will be seen in the following table.

Table of planes which exist if $(p\,;\,q\,;\,r)$ exist.

In the rhomboid - - - -	(p, q, r)
The doubly-oblique prism - -	$(p\,;\,q\,;\,r)$
The oblique rhombic prism - -	$(p, q\,;\,r)$
The oblique rectangular prism -	$(p\,;\,\pm q\,;\,r)$
The right oblique-angled prism -	$(\pm p\,;\,\pm q\,;\,r,$
The right rhombic prism - -	$(\pm p, \pm q\,;\,r)$
The right square prism - -	$\left(\substack{+\\+\\=} p, \substack{\pm\\\pm} q\,;\,r\right)$
The cube - - - -	$\left(\substack{+\\+\\=} p, \substack{\pm\\\pm} q, r\right)$
The regular tetrahedron and regular octahedron - - -	(p, q, r) $(p, p - q, p - r)$ $(q - p, q, q - r)$ $(r - p, r - q, r)$
The direct symmetrical tetrahedron and square octahedron - -	$(p\,;\,q, r)$ $(p\,;\,p - r, p - q)$ $(q - r\,;\,q, q - p)$ $(r - q\,;\,r - p, r)$
The inverse symmetrical tetrahedron, and rhombic octahedron. - -	$(p\,;\,q\,;\,r)$ $(p\,;\,p - r\,;\,p - q)$ $(q - r\,;\,q\,;\,q - p)$ $(r - q\,;\,r - p\,;\,r)$

The regular triangular prism ; - $(p, q; r)$
$(p, p-q; -r)$
$(q, q-p; -r)$

The rhombic dodecahedron ; - - (p, q, r)
$(p, q, -\overline{p+q+r})$
$(p, -\overline{p+q+r}, r)$
$(-\overline{p+q+r}, q, r)$

A crystal may be represented by uniting the symbols of the planes of which it is composed. And it will be convenient to represent by a figure in brackets thus (6), the number of faces which arise from each symbol. Also frequently the crystal has *parallel* planes; in which case one of them may be considered as a repetition of the other ; and the plane thus doubled may be indicated by writing a 2 before it. Thus the form of borate of magnesia, called by HAUY *magnésie borateé defective*, may be thus represented.

Primary ; a cube.

Secondary; $2(3)(1, 0, 0) + 2(6)(\pm 1, 1, 0) + (4)(\pm 1, 1, 1)$

Indicating — a cube $2(3)(1, 0, 0)$, formed by repeating each of the primary planes $(1, 0, 0)$;

Modified by 6 pairs of planes $(\pm 1, 1, 0)$; truncating the edges ;

And by 4 planes truncating angles, which are not repeated. Hence the opposite angles are not symmetrically affected.

The situation of planes with respect to each other, may be determined by assuming a certain point as the pole of the crystal, and measuring the latitude and longitude of the centre of the plane with respect to this pole. If we suppose an ellipsoid of which the three axes are as the three edges *a, b, c* of the primitive form, we may suppose secondary planes to

be in their natural position when they are drawn so as to touch the ellipsoid; and we may consider as the centre of the face, the point of contact. The latitude and longitude (μ and λ,) of this point, are given by the formulæ which follow.

In the rhomboid, the axis of the rhomboid being the axis of the crystal

$$\cos.\ \lambda \text{ varies with } \frac{2p-q-r}{\sqrt{(p^2+q^2+r^2-pq-pr-qr)}}$$

$$\sin.\ \mu \text{ ———— } \frac{p+q+r}{\sqrt{(p^2+q^2+r^2)}}$$

In the prism, the axis being the axis of the prism

$$\tan.\ \lambda \text{ varies with } \frac{q}{p}$$

$$\sin.\ \mu \text{ ———— } \frac{r}{\sqrt{(p^2+q^2+r^2)}}$$

And hence the situation of the planes is known. Also if any of the planes, instead of touching the ellipsoid, be nearer to or farther from the centre of the crystal, the *order* of the planes will not be altered.

Having thus determined what planes are adjacent, we find the angles which they make, by the formulæ given Art. 8.

In the rhomboid $(p; q; r)(p; q; r)$ being the planes, θ their angle, and α the dihedral angle of the primary form,

$$-\cos.\theta = \frac{pp'+qq'+rr'-(p'q+q'p+p'r+r'p+q'r+r'q)\cos.\alpha}{\sqrt{(p^2+q^2+r^2-2\,\overline{pq+pr+qr}\cos.\alpha)(p'^2+q'^2+r'^2-2\,\overline{p'q'+p'r'+q'r'}\cos.\alpha)}}$$

This is true also for the tetrahedron, and for the right rectangular prism, making $\cos.\alpha = 0$. In the other cases we have a formula involving the three dihedral angles of the primary form.

We can also find the angles contained between any two edges by first finding the equations to the edges, and then employing a formula given, p. 125.

The inverse problem, knowing two dihedral angles of the secondary figure to determine the symbols of the planes, is resolved by the same formulæ. In the case where the angles made with the primary planes are given, we have a direct solution. In the other cases we find the indices of the symbol of trial; and if the limits of the present paper allowed it, it might be shown how we might, after some trials, proceed directly to find the law.

P. S. The greater part of the formulæ in the preceding pages were calculated before my notice was directed to a paper by Mr. LEVY, in the Edinburgh Philosophical Journal for April 1822. Mr. LEVY there employs the principle which is the basis of the investigations now given, viz. the mode of expressing a secondary plane by means of its equation to three axes coinciding with the edges of the primitive form. From this principle he deduces, with great simplicity, the law of a secondary plane in a particular case; viz. when the intersections of that plane with two known planes, are parallel to *their* intersections with two others.* In order however to deduce the general formula, a new and different series of theorems is necessary, as appears in the course of this paper.

W. W.

* It may be observed, that the result in this case is easily obtained from the formula in Art. 14.

Fig. 1.

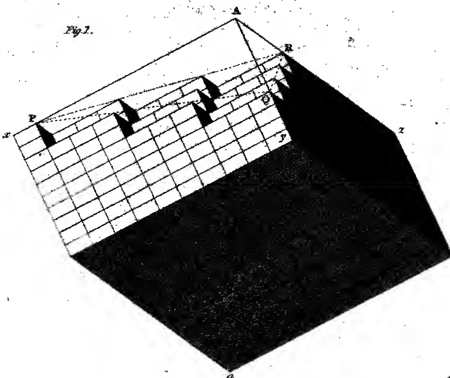

Fig. 2

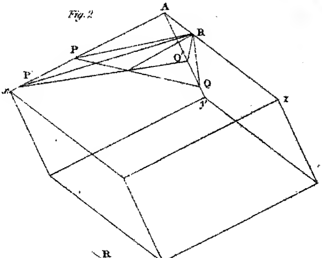

Fig. 3.

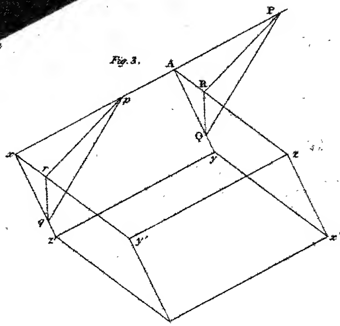

Fig. 4.

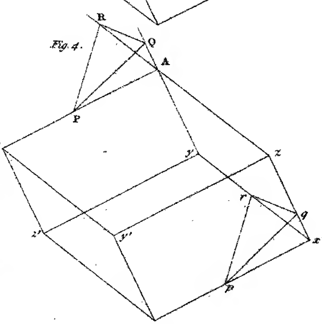

Fig. 5

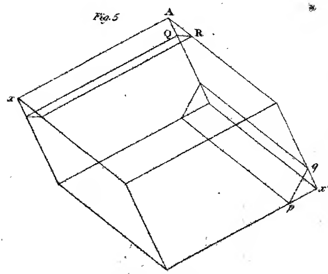

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Fig. 6.

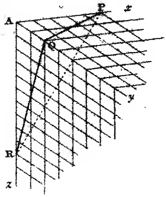


Fig. 7.

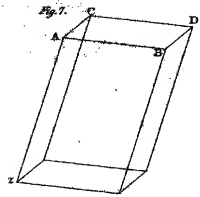


Fig. 8.

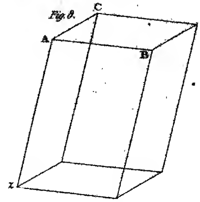


Fig. 9.

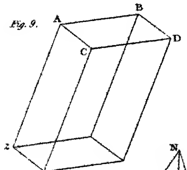


Fig. 10.

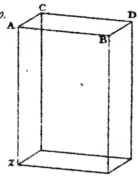


Fig. 11.

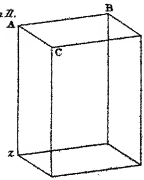


Fig. 12.

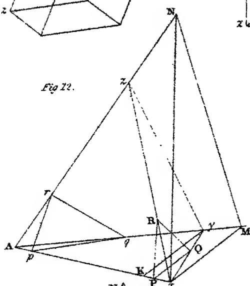


Fig. 15.

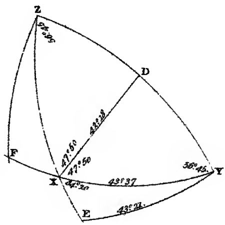


Fig. 14.

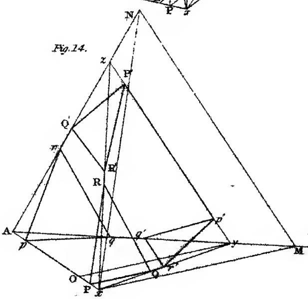
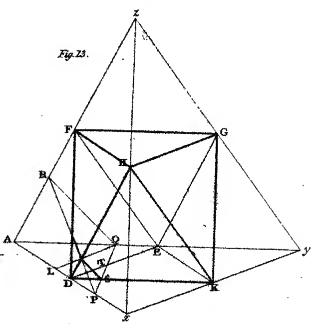


Fig. 13.



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V. *Explanation of an optical deception in the appearance of the spokes of a wheel seen through vertical apertures.* By P. M. Roget, *M. D. F. R. S.*

Read December 9, 1824.

A curious optical deception takes place when a carriage wheel, rolling along the ground, is viewed through the intervals of a series of vertical bars, such as those of a palisade, or of a Venetian window-blind. Under these circumstances the spokes of the wheel, instead of appearing straight, as they would naturally do if no bars intervened, seem to have a considerable degree of curvature. The distinctness of this appearance is influenced by several circumstances presently to be noticed; but when every thing concurs to favour it, the illusion is irresistible, and, from the difficulty of detecting its real cause, is exceedingly striking.

The degree of curvature in each spoke varies according to the situation it occupies for the moment with respect to the perpendicular. The two spokes which arrive at the vertical position, above and below the axle, are seen of their natural shape, that is, without any curvature. Those on each side of the upper one appear slightly curved; those more remote, still more so; and the curvature of the spokes increases as we follow them downwards on each side till we arrive at the lowest spoke, which, like the first, again appears straight.

The most remarkable circumstance relating to this visual deception is, that the convexity of these curved images of the spokes is always turned downwards, on both sides of the wheel; and that this direction of their curvature is precisely the same, whether the wheel be moving to the right or to the left of the spectator. The appearance now described is represented in Plate XI. fig. 1.*

In order to discover a clue to the explanation of this phenomenon, it was necessary to observe the influence which certain variations of circumstances might have upon it; and the following are the principal results of the experiments I made for this purpose.

1. A certain degree of velocity in the wheel is necessary to produce the deception above described. If this velocity be gradually communicated, the appearance of curvature is first perceptible in the spokes which have a horizontal position: and as soon as this is observed, a small increase given to the velocity of the wheel, produces *suddenly* the appearance of curvature in all the lateral spokes. The degree of curvature remains precisely the same as at first, whatever greater velocity be given to the wheel, provided it be not so great as to prevent the eye from following the spokes distinctly as they revolve: for it is evident, that the rapidity of revolution may be such as to render the spokes invisible. It is also to be noticed that, however rapidly the wheel revolves, each

* The appearance in question has been noticed by an anonymous writer in the Quarterly Journal of Science (Vol. X. p. 282) who gives, however, no explanation of the phenomenon. It would have been impossible, indeed, to reconcile the facts as they are there stated, with any theory that could be imagined for their solution.

individual spoke appears, during the moment it is viewed, to be at rest.

2. The number of spokes in the wheel makes no difference in the degree of curvature they exhibit.

3. The appearance of curvature is more perfectly seen when the intervals between the bars through which the wheel is viewed, are narrow; provided they are sufficiently wide to allow of the distinct view of all the parts of the wheel in succession, as it passes along. For the same reason, the phenomenon is seen to the greatest advantage when the bars are of a dark colour, or shaded, and when a strong light is thrown upon the wheel. The deception is, in like manner, aided by every circumstance which tends to abstract the attention from the bars, and to fix it upon the wheel.

4. If the numbers of bars be increased in the same given space, no other difference will result than a greater multiplication of the curved images of the spokes; but if a certain relation be preserved between the angles subtended at the eye by the whole intervals of the bars, and of the extremities of the spokes, this multiplication of images may be corrected. The distance of the wheel from the bars is of no consequence, unless the latter are very near the eye, as in that case the apertures between them may allow too large a portion of the wheel to be seen at once.

5. If the bars, instead of being vertical, are inclined to the horizon, the same general appearances result; but with this difference, that the spokes occupying positions parallel to the bars, are those which have no apparent curvature: while the curvatures of the other spokes bear the same relations to these straight spokes, and to each other, that they did in the

former case. When the inclination of the bars is considerable, however, the images become more crowded, and the distinctness of the appearance is thereby diminished. The deception totally ceases when the wheel is viewed through bars that are parallel to the line of its motion.

6. It is essential to the production of this effect, that a combination should take place of a progressive with a rotatory motion. Thus, it will not take place if, when the bars are stationary, the wheel simply revolves on its axis, without at the same time advancing: nor when it simply moves horizontally, without revolving. On the other hand, if a progressive motion be given to the bars, while the wheel revolves round a fixed axis, the spokes immediately assume a curved appearance. The same effect will also result if the revolving wheel be viewed through fixed bars by a spectator, who is himself moving either to the right or left; because such a movement on the part of the spectator produces in his field of vision an alteration in the relative situation of the bars and wheel.

It is evident from the facts above stated, that the deception in the appearance of the spokes must arise from the circumstance of separate parts only of each spoke being seen at the same moment; the remaining parts being concealed from view by the bars. Yet since several parts of the same spoke are actually seen in a straight line through the successive apertures, it is not so easy to understand why they do not connect themselves in the imagination, as in other cases of broken lines, so as to convey the impression of a straight spoke. The idea at first suggests itself that the portions of

one spoke, thus seen separately, might possibly connect themselves with portions of the two adjoining spokes, and so on, forming by their union a curved image made up of parts from different successive spokes. But a little attention to the phenomena will show that such a solution cannot apply to them: for when the disc of the wheel, instead of being marked by a number of radiant lines, has only one radius marked upon it, it presents the appearance, when rolled behind the bars, of a number of radii, each having the curvature corresponding to its situation; their number being determined by that of the bars which intervene between the wheel and the eye. So that it is evident, that the several portions of one and the same line, seen through the intervals of the bars, form on the retina the images of so many different radii.

The true principle, then, on which this phenomenon depends, is the same as that to which is referable the illusion that occurs when a bright object is wheeled rapidly round in a circle, giving rise to the appearance of a line of light throughout the whole circumference: namely, that an impression made by a pencil of rays on the retina, if sufficiently vivid, will remain for a certain time after the cause has ceased. Many analogous facts have been observed with regard to the other senses, which, as they are well known, it is needless here to particularize.

In order to trace more distinctly the operation of this principle in the present case, it will be best to take the phenomenon in its simplest form, as resulting from the view of a single radius, (fig. 2.) OR of the wheel VW, revolving steadily upon its axis, but without any progressive motion, and seen

through a single narrow vertical aperture which is moving horizontally in a given direction PQ. Let us also assume that the progressive motion of the aperture is just equal to the rotatory motion of the circumference of the wheel. It is obvious that if, at the time of the transit of the aperture, the radius should happen to occupy either of the vertical positions VO or OW, the whole of it would be seen at once through the aperture, in its natural position; but if, while descending in the direction VR, it should happen to be in an oblique position RO, terminating at any point of the circumference at the moment the aperture has, in its progress horizontally, also arrived at the same point R, the extremity of the radius will now first come into view, while all the remaining part of it is hid. By continuing to trace the parts of the radius that are successively seen by the combined motions of the aperture and of the radius, we shall find that they occupy a curve R*abcd* generated by the continued intersection of these two lines. Thus, when the aperture has moved to A, the radius will be in the position Oα; when the former is at B, the latter will be at Oβ, and so on.

Again; let us suppose that when the aperture is just passing the centre, the radius should be found in a certain position on the other side OY, and rising towards the summit. Then tracing, as before, the intersections of these lines in their progress, we shall obtain a curve precisely similar to the former. Its position will be reversed; but its convexity will still be downwards.

If the impressions made by these limited portions of the several spokes follow one another with sufficient rapidity, they will, as in the case of the luminous circle already alluded

to, leave in the eye the trace of a continuous curve line; and the spokes will appear to be curved, instead of straight.

The theory now advanced is in perfect accordance with all the phenomena already detailed, and is farther confirmed by extending the experiments to more complicated combinations.

It readily explains why the image, or spectrum, as it may be called, of the spoke, is at rest, although the spoke itself be revolving: a circumstance which might escape notice, if the attention were not particularly called to it.

Since the curved appearance of the lines results from the combination of a rotatory, with a progressive motion of the spokes, in relation to the apertures through which they are viewed, it is evident that the same phenomena must be produced if the bars be at rest, and both kinds of motion be united in the wheel itself. For, whether the bars move horizontally with respect to the wheel, or the wheel with respect to the bars, the relative motion between them, and its effects, in as far as concerns the appearance in question, must be the same. The attention of the spectator should in both cases be wholly directed to the wheel, so that the motions in question should be referred altogether to it. Thus, in fig. 4, the real positions, at successive intervals of time, of the spoke A *a*, when the wheel is rolling on the ground in the direction AZ, are expressed by the lines A*a*, B*b*, C*c* and D*d*. While the spoke is in these positions, the portions of it really seen through the fixed aperture VW, are the parts α, β, γ, δ, the impressions of which, being retained upon the retina, and referred to the wheel when in its last position,

form the series of points *m*, *n*, *p* and *q*, in the curved spectrum *m* D.

That the attention may the more easily follow the wheel in its progression, it is necessary that its circumference be distinctly seen, and its real situation correctly estimated. Hence, although it be true, that by a sufficient exertion of attention the phenomenon may be exhibited by means of a single aperture, it is much more readily perceived, when the number of apertures is such as to allow the wheel to be seen in its whole progress. For this reason the phenomenon is very distinct in the case of a palisade. Each aperture produces its own system of spectra; and hence, when the apertures occur at short intervals, the number of the spokes is considerably multiplied; but if the intervals be so adjusted as to correspond with the distances between the spokes at the circumference of the wheel, the images produced by each aperture will coalesce, and the effect will be much heightened.

A mathematical investigation of the curves resulting from the motion of the points of intersection of a line moving parallel to itself, with another line revolving round its axis, will show them to belong to the class of QUADRATRICES, of which the one which touches the circumference of the inner generating circle is that which is known by the name of the QUADRATRIX OF DINOSTRATES. Such a system of curves is represented in fig. 3, where MC, CN are the generating radii, A the outer, and B the inner generating circles, and PQ the common axis of the curves.

All these curves have the same general equation, namely,

$$y = (b - x).\text{tang.}\, x.$$

where the co-ordinates are referred to the axis at right angles to the vertical generating radii, and passing through the centre of their revolution: the basis b being measured on the axis from the point of its intersection with the curve to the centre: and x being the arc of the inner generating circle, as well as the abscissa.*

A wheel simply rolling on its circumference exhibits, when seen through fixed bars, only those portions of the curves which are contained within the inner circle; but when its motion of revolution is more rapid than its horizontal progression, as when it is made to roll on an axle of less diameter on a raised rail-way, then the remaining portions of the curves will be seen, and others, on the lower part of the wheel, having a contrary flexure, will also make their appearance. These are seen at FF in fig. 3.

If the spokes, instead of being straight, be already curved, like those of the Persian water-wheel, their form, when viewed through bars, will undergo modifications, which may readily be traced by applying to them the same theory. Thus, by giving a certain curvature to the spokes, as in fig. 5, they will at one part of their revolution appear straight, namely, where the optical deception operates in a direction contrary to the curvature.

The velocity of the apparent motion of the visible por-

* This equality between the arc and the abscissa is a necessary consequence of the progressive motion of the wheel being equal to the rotatory motion of its circumference: the former motion producing the increments of the abscissa; and the latter those of the arc of the circle. The equation $y = (b - x).\text{tang.}\, x.$ is deduced from a simple analogy of the sides of similar triangles.

tions of the spokes is proportionate to the velocity of the wheel itself; but it varies in different parts of the curve: and might therefore, if accurately estimated, furnish new modes of measuring the duration of the impressions of light on the retina.

Fig. 1.

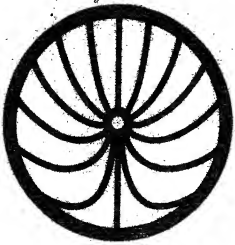


Fig. 2

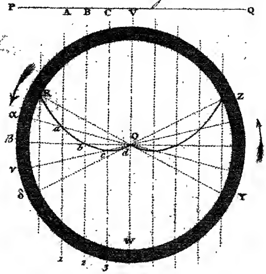


Fig. 3.

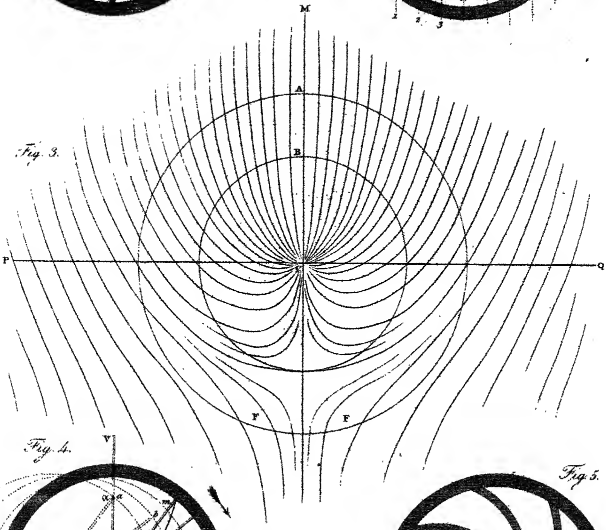


Fig. 4.

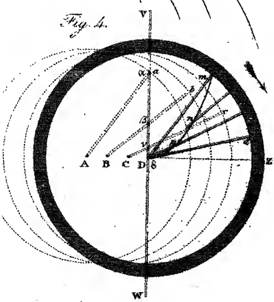
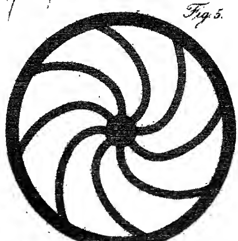


Fig. 5.



VI. *On a new photometer, with its application to determine the relative intensities of artificial light, &c. By* William Ritchie, *A. M. Rector of the Academy at Tain. Communicated by the President.*

Read December 16, 1824.

1. In a paper which I lately communicated to the Edinburgh Philosophical Journal, I endeavoured to show, that caloric flies off from the surface of a heated body by the repulsive energy existing between its own molecules, and consequently, that their velocity increases with the temperature of the body. This conclusion I deduced from the fact, discovered by Delaroche, that invisible caloric freely permeates very thin plates of glass, in the same manner as light, but that it is completely intercepted by thicker plates. If the temperature of the body be raised, the atoms of caloric will be brought nearer each other, their repulsive energy augmented, their velocity increased, and consequently, they will now find their way through a plate of glass which formerly intercepted them. If the temperature of the body be raised still higher, the molecules of caloric will acquire a velocity sufficient to permeate the various humours of the eye, and produce an impression on the retina, or in other words, they will become light. From this view of the subject, I was naturally led to the invention of an instrument which would be affected by visible caloric or light, whilst

it would not be sensibly acted upon by invisible coloric, or heat.

In short, I was led to the invention of a photometer, which appears to be the most accurate and delicate which has yet been described. But though such were the theoretical views which led to the invention of the instrument, its perfection does not depend upon any peculiar theory of light and heat. It is founded on the axiom, that equal volumes of air are equally expanded by equal quantities of light, converted into heat by absorption by black surfaces: and also on the well established principle that the quantity of light diminishes as the square of the distance of the luminous source from the object on which it is received.

2. The instrument consists of two cylinders of planished tin plate from 2 to 10 or 12 inches in diameter, and from a quarter of an inch to an inch deep. One end of each cylinder is inclosed by a circular plate of the same metal soldered completely air tight, the other ends being shut up by circular plates of the finest and thickest plate glass, made perfectly air tight. Half way between the plates of glass and the ends of the cylinders, there is a circular piece of black bibulous paper for the purpose of absorbing the light which permeates the glass, and instantly converting it into heat.

The two cylinders are connected by small pieces of thermometer-tubes which keep them steady with their faces parallel to each other, but turned in opposite directions, and also serve to make the insulation as complete as possible. The chambers are then connected by a small bent tube in

the form of the letter U, having small bulbs near its upper extremities, and containing a little sulphuric acid, tinged with carmine. The instrument is supported upon a pedestal, having a vertical opening through the stem to allow the glass-tube to pass along it, and thus secure it from accidents.

A small scale divided into any number of equal parts, is attached to each branch of the tube. In the annexed figure, Plate XII. ABCD and EFGH are the cylinders, AB and FG the plates of glass. CD, EFG the ends shut up by the circular tin plates, the blackened paper is represented by the lines between AB, CD and EH, FG. The other parts will be obvious from the mere inspection of the figure.

3. The accuracy of the instrument evidently depends upon the perfect equality of its two opposite ends. To ascertain, if it be accurately constructed, place it between two steady flames, and move it nearer the one or the other till the liquid in the tube remains stationary, at the division of the scale at which it formerly stood. Turn it half round without altering its distances from the flames, and if the liquid remains stationary at the same division, the instrument is correct. To show the extreme delicacy of the instrument, place it opposite a single candle, and it will be sensibly affected at the distance of 10, 20, or 30 feet, provided it be of sufficient diameter, whilst it will not be sensibly acted upon at the same distance by a mass of heated iron affording twenty times the quantity of heat. In order to cut off effectually the influence of mere radiant heat, I sometimes use screens composed of two plates of glass, placed parallel to each other, with a quantity of water interposed.

4. Place the instrument between any number of steady lights whose intensities are known, as for example, between four wax candles opposite one end, and one candle opposite the other, and move the photometer till the fluid remain stationary at the division where it formerly stood, and it will be found that the distances are directly as the square roots of the number of candles; or in other words, that the intensities of the lights will be inversely as the squares of the distances. If gas lights be employed, having burners capable of consuming known quantities of gas in equal times, and the photometer be placed between them, so that the effect upon the air in each chamber shall be the same; it will be found that the quantities of gas consumed by each, will be exactly proportional to the squares of the distances of their respective flames from the ends of the photometer.

5. This instrument seems well adapted for determining the relative quantities of light given out by the combustion of coal and oil gas. Place the instrument as before between the two burners, and ascertain the relative intensities of the two lights, by squaring their distances from the adjacent ends of the instrument; ascertain the quantities of gas consumed by each of the burners in the same time; multiply these quantities by the squares of the respective distances, and the product will be the relative quantities of light, afforded by the gases. Let d be the distance of the coal gas light and d' that of the oil gas light; and let q be the quantity of coal gas consumed in a given time, and q' the quantity of oil gas consumed in the same time, then the intensity of the coal gas will be to that of the oil gas $q\,d^2$ to $q'\,d'^2$.

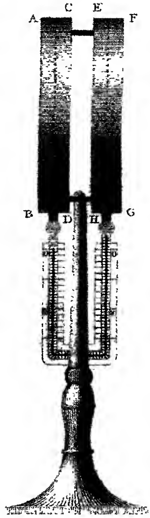
6. To find the ratio between the quantities of light given

out by the sun, and that afforded by a common candle, place one end of the instrument opposite the sun, and bring the candle opposite the other end, till the fluid in the stem remain stationary at the original division, and the light given out by the candle, will evidently be to that given out by the sun, as the square of a few inches to the square of the number of inches contained in 95,000,000 miles, provided none of the sun's light had been absorbed in its passage through the atmosphere. The delicacy of the instrument is such, that if it be placed opposite the sun without a counteracting force, the light absorbed from the body will be so great as to cause the liquid to move through a tube 20 or 30 feet long. By covering one end of the instrument, and directing the other to various quarters of the sky, we can ascertain the relation between the quantities of light reflected from the atmophere, and clouds floating in those regions.

I am just now constructing a photometer about two feet in diameter, and two or three inches deep, with which I hope to appreciate the effect of heat in the feeble rays of the moon.

7. Though this instrument has some resemblance to Professor LESLIE's photometer, yet it is founded on principles essentially different. The one depending on the *difference* of the temperatures of the two bulbs, whilst the perfection of the other results from the equality of the temperature of the air contained in both chambers. The one has a scale a few inches long attached to one branch of the bent tube, whilst the scale of the other is the distance between the two antagonist flames. The delicacy of the one is, from its very

nature, confined within very narrow limits, whilst that of the other may be increased at pleasure. There are numerous problems which this photometer is capable of solving, and which, if this paper be favourably received, may form the subject of future communication.



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VII. *The description of a floating Collimator.* By Captain HENRY KATER, *F. R. S.*

Read January 13, 1825.

THE line of collimation of a telescope is a line which passes through the centre of the object glass and the intersection of the cross wires placed in its focus. The apparatus which I am about to describe is intended to determine the situation of this line with respect to the horizon or the zenith, in some one position of an astronomical circle to which the telescope is attached.

A plumb line, a level, and an image reflected from the surface of a fluid, are the means which have hitherto been employed for this purpose. The defects and inconveniences of each of these have been felt, and the subject has for some years past engaged much of my attention, principally with a view of bringing instruments of portable dimensions into competition with those of a larger size.

Since a plumb line even of six feet in length would be subject to a deviation of only about three ten thousandths of an inch with an angular variation of one second, the application of the plumb line to small instruments becomes useless when great precision is required.

The difficulty of procuring a good and sensible level is well known, and though a very valuable instrument when

carefully fitted up, it is liable to so many errors from a variety of causes, that a single observation with the best level is little to be depended upon.

The method of observing by reflection, is perhaps by far the most perfect of any that has yet been practised; but it requires a union of favourable circumstances which rarely occurs. The fluid generally employed is mercury, which reflects a sufficient quantity of light to give a brilliant image, but is so easily deranged, that the slightest breath of air, not otherwise perceptible, or the distant passing of a carriage, the sound of which is scarcely heard, is sufficient to disturb its surface, and to render an observation either impracticable, or so erroneous as to be perfectly useless.

Nevertheless, by taking extreme care to protect the surface from wind, observations by reflection from mercury have been made with great success at the Royal Observatory at Greenwich, by means of the mural circle. But there are other objections besides those I have mentioned to all these methods. When an instrument with a plumb line is used, the observations may be conducted in two ways. In the first, the instrument being placed in the meridian, the plumb line is brought over a mark which is generally attached to the frame work carrying the microscopes. The star is taken as many nights in succession as may be thought necessary, and the instrument is then turned half round in azimuth, the plumb line brought over the same mark, and the star again taken, when the mean of the results of the readings in both positions of the instrument gives the altitude or zenith distance.

Here it is presumed, that the refraction remains the same

from night to night, and that the relative positions of the different parts of the instrument suffer no change, suppositions which are too gratuitous to be readily received.

In the second method, the plumb line being adjusted as before, the star is taken several times when very near the meridian, the time at each observation being noted. The instrument is then immediately turned half round in azimuth, readjusted if necessary, and the observations repeated, noting the time as before. The mean of the results of the readings in both positions of the instrument will be the altitude or zenith distance of the star at the mean of the observed times.

The time at which the star comes to the meridian being known, the difference between this and the mean of the times of observation gives the horary angle of the star with the meridian; whence the correction is computed which it is necessary to apply to the observed zenith distance in order to obtain the meridional zenith distance.

I know of no objection to this method except the length of time required for observing each star, the labour of computing the corrections, and the possibility of the instrument suffering a strain from being turned half round in azimuth.

The manner of using an instrument furnished with a level, is nearly the same as when it is constructed with a plumb lime; but as the level has a scale, the divisions indicated by the ends of the bubble are usually read off at the conclusion of each observation, and the resulting corrections applied to the observed zenith distance. This presupposes the value of the divisions of the scale to be well ascertained, and that they are equal, which last is not always the case.

In observing by reflection, the error which might arise

from turning the instrument in azimuth is avoided. The star is taken, and the telescope being afterwards directed to the reflected image, the very small movement in azimuth required to follow the star cannot well be supposed to occasion any strain; but the same observations of the time, and the same reductions to the meridian are necessary in this as in the preceding method.

If the instrument is fixed in the meridian, as is the case with the mural circle, the observation by reflection cannot be made on the same night as that by direct vision; and it may be supposed that unfavourable weather might occur for several nights and prevent the completion of the observation, during which interval a change in refraction, or in the relative positions of the circle and microscopes might take place, impossible to be detected, and which would vitiate the result. This inconvenience has been felt at Greenwich, and a second mural circle is nearly ready at the Royal Observatory, for the purpose of simultaneously observing the same star by reflection and by direct vision.

From the slight description that has been given of the different methods of observing, it must be evident, that the important desiderata are to keep the circle constantly fixed in the meridian, using no other motion than that of the telescope, and to possess the means of instantly determining and verifying at pleasure the place of the horizontal or zenith point with a degree of accuracy, limited only by the powers of vision assisted by the telescope.

If a telescope furnished with cross wires be adjusted to distinct vision upon a fixed star, the parallel rays proceeding from the object are converged to a focus, and an image of

the star is formed upon the cross wires. Conversely, if when the telescope is so adjusted, rays be supposed to pass from the cross wires through the object glass, they will emerge parallel, as if they had come from an object at an infinite distance. The cross wires are therefore similarly circumstanced with respect to vision, as a fixed star; and if another telescope, adjusted by a star, be employed to view these cross wires through the object glass of the former instrument, they will be seen with perfect distinctness, however near to each other the telescopes may be placed.

Professor GAUSS first published an account of this beautiful property, and he availed himself of it to measure the angular distance of the wires of a transit instrument by means of a theodolite placed near the object glass.

In the *Astronomische Nachricten, No.* 61, Professor BESSEL has given a "new method of determining the flexure of the telescope of astronomical instruments," which for elegance of invention can scarcely be surpassed. This he effected by means of the property described by Professor GAUSS; and at the end of his communication, M. BESSEL proposes a method of finding the zenith point of an instrument, of which the following is a translation.

"I may here be permitted further to remark, that the zenith point of an instrument may also be found without turning it in azimuth by a similar contrivance. For this purpose nothing more is required than a telescope, furnished with a sensible level, which may be placed on either side with respect to the axis. If this be placed alternately towards the north and the south, so that the bubble of the level may be similarly situated in both positions, then the mean of the

readings on the circle at each position of the telescope is the zenith point. This may be obtained in a manner most independant of all other corrections, if the level be fixed nearly at right angles to the axis of the telescope, suspending it in the zenith, and then repeating the observation after the telescope has been turned on its axis. This method, which presupposes only an arrangement very easily made in the slit of the observatory, or upon the pillars, will give a very exact result, since the cross wires appear at all times well defined and without motion, which is not always the case in the comparative observations of stars by a zenith sector and the circle itself."

By this method, the necessity of observing out of the meridian, or of waiting till another night for the completion of the observation is obviated, and the zenith point may be immediately determined with as much accuracy as can be attained by means of a level. But though this is by far the best mode of employing a level that has ever yet been devised, it is still subject to the objections which have been urged against that instrument.

It would require a level of very great delicacy and extent of scale to indicate the fraction of a second; and such a one would be readily deranged by a small inequality of temperature, or by the unavoidable elastieity of the parts necessary for its adjustment.

In the course of some former enquiries, I made many experiments to ascertain the degree of reliance that might be placed on the position of a body floating upon the surface of mercury, and fully satisfied myself that it might be so contrived as to have always when at rest the same

inclination to the horizon. I had thus a floating support to which I could attach a telescope, a support requiring no adjustment, offering the ready means of extreme accuracy, and precluding all fear of those errors which might arise from the use of a level.

For a preliminary experiment I procured a piece of oak, seven inches and a half long, four inches and a half wide, and one inch thick. Upon this, two wooden uprights in the form of Y s, were pinned and glued at the distance of five inches. Half way between these a small ring was screwed into the oak, and the telescope being laid upon the Y's, was firmly secured in its place by a string passing several times round it and through the ring.

In the middle of the longer sides of the oak support, and at right angles to its horizontal surface were inserted two pieces of brass, in which very smooth grooves were made, about one-tenth of an inch wide.

A deal box eight inches long, five inches wide, and an inch and a half deep, having its bottom just covered with mercury, received the float, which was kept in its situation in the middle of the box, and prevented from turning horizontally by two smooth iron pins passing through the sides of the box into the grooves. These were carefully regulated so as to allow the float to adapt itself with perfect freedom to the surface of the mercury.

The whole of the telescope was above the edges of the box, and a screen of black pasteboard, with an aperture equal to that of the object glass, was fixed to the end of the box. This is indispensably necessary, in order to exclude false light.

A fine achromatic telescope by DOLLOND, of thirty inches

focus, and two inches and three-quarters aperture, furnished with a wire micrometer, was placed upon a deal support with three legs. This support rested upon a flat stone laid upon sand which filled a pit of seven feet in depth, and afforded a perfectly steady foundation. The moveable wire of the micrometer was carefully placed in a horizontal position.

The floating collimator was put upon a table attached to the wall of the observatory, and was placed in the proper direction by looking through its telescope, and moving the box till the cross wires appeared upon the wire of the micrometer. The cross wires,* which formed an angle of about 15 degrees, were illuminated by a small lanthorn placed upon the table with a piece of oiled paper interposed.

My first trials were made with the telescope of a sextant, but as the object glass was not sufficiently perfect,† I did not conceive it worth while to register them, and I shall merely remark, that after deranging the float by moving it in a variety of ways, the cross wires returned, as nearly as the imperfection of the image would permit me to judge, to the same situation.

The telescope subsequently employed had an achromatic object glass of one inch and a quarter aperture, and seven inches and a half focus, and this gave a sufficiently good image.

In the experiments I am about to detail, every method I could think of was used that could fairly introduce error.

* The material which after numerous trials I found to answer best for cross wires, was the steel spring used in the balance of a watch.

† I may here observe, that I find this mode of examination to be a most severe test of the goodness of an object glass employed as a collimator.

The end of the box next the object glass of the telescope was raised so that the mercury ran from beneath the float to the other end of the box; it was then restored to its former position; this for the sake of brevity I shall designate by *O raised.*

The eye end of the box was elevated and restored to its place in like manner; this I have called *E raised.*

One side of the box elevated and replaced, *S raised.*

The other side of the box treated in like manner, *S s raised.*

The object end of the box raised, and the box carried in that position to the other side of the observatory, brought back and replaced, *O raised and carried.*

The eye end raised, and the box carried and replaced, *E raised and carried.*

S raised and carried, and *S s raised and carried,* signify similar operations, previously elevating one side or other of the box.

Out and replaced O first, signifies that the float was lifted quite out of the mercury and the box replaced, the end of the float next the object glass being brought first into contact with the mercury.

Out and replaced, E first, the same, with the eye end of the float first brought in contact with the mercury.

Out and replaced S first, the same with one side brought first into contact.

Out and replaced S s first, the same with the other side brought first into contact.

As soon as the cross wires appeared to be perfectly at rest, the angle was carefully bisected by the micrometer

wire, and the mean of seldom less than three readings registered.

The stability of the micrometer was severely proved by pressing upon the end of the telescope, and it was found that upon removing the finger, the micrometer wire always returned precisely to the same situation. The value of each division of the micrometer head is six-tenths of a second.

In the following tables, the first column contains the readings of the micrometer when the angle of the cross wires was bisected. The second column contains the difference between every two consecutive observations, and indicates the derangement of the position of the float from its having been moved between such observations. The third column contains the error in seconds, which would affect the determination of the horizontal point in consequence of the derangement of the float, and is equal to the half of that derangement.

Wooden Float.

	1st Set.	Divisions of Microm.	Difference.	Error in Sec^s. affecting the Horiz. point.
Dec. 5	Previously to moving - - -	83,9		
	O raised - - -	85,4	+ 1,5	+ 0,45″
	E raised - - -	82,6	— 2,8	— 0,84
	S raised - - -	80,9	— 1,7	— 0,51
	S s raised - - -	79,3	— 1,6	— 0,48
	O raised and carried - -	79,5	+ 0,2	+ 0,06
	E raised and carried - -	70,9	— 8,6	— 2,58
	S raised and carried - -	71,9	+ 1,0	+ 0,30
	S s raised and carried - -	70,3	— 1,6	— 0,48

The image of the cross wires not being perfectly distinct, I limited the aperture to three-quarters of an inch, and thus

obtained a much better image. These observations were made in extremely damp weather, and the support had been kept for a few days in a very dry place.

	2d Set.	Divisions of Microm.	Difference.	Error in Sec^s. affecting the Horiz. point.
Dec. 8	O raised - - - -	31,6		
	After an hour's interval - -	16,6		
	Lanthorn close to wires - -	19,9	+ 3,3	+ 0,99″
	After an hour - - -	8,8		
	Out and replaced, S first - -	13,7	+ 4,9	+ 1,47
	Ditto, - E first - -	13,1	— 0,6	— 0,18
	Ditto, - O first - -	13,1	0,0	— 0,00
	O raised and carried - - -	11,5	— 1,6	— 0,48
	S raised and carried - -	11,7	+ 0,2	+ 0,06
	Ditto, read again - - -	10,7	— 1,0	— 0,30
	Carried box, kept as level as possible	5,2	— 5,5	— 1,65
	S raised and carried - - -	3,5	— 1,7	— 0,51
	Out and replaced, S first - -	0,5	— 3,0	— 0,90

The wooden float being designed merely for preliminary experiments, and it not being my intention to introduce any errors but such as might arise from moving the float or agitating the mercury, I had a float made of cast iron eight inches long, four wide, 0,2 thick, and weighing 2 lb. 5 oz. troy. The telesccope was tied firmly to the Y s.

Experiments with the Iron Float.

	3d Set.	Divisions of Microm.	Difference.	Error in Sec^{s}. affecting the Horiz. point.
Dec. 9	Previously to moving	57,7		
	O raised	51,5		
	Not sufficient mercury; added more.			″
	Out and replaced S first	47,1	— 3,8	— 1,14
	O raised and carried	43,3	+ 1,0	+ 0,30
	S raised and carried, escaped from the grooves	44,3	+ 2,3	+ 0,69
	Out and replaced, the surface at once in contact	46,6	— 0,1	+ 0,03
	Out and replaced, O first	46,5	— 2,3	— 0,69
	Out and replaced, E first	44,2	— 0,4	— 0,12
	Out and replaced, S first	43,8	— 0,7	— 0,21
	Out and wiped the mercury, S first	43,1	— 1,0	— 0,30
	Out and replaced, E first	42,1	— 0,2	— 0,06
	Out and replaced, S first	41,9	— 4,4	— 1,32
	Out and carried, S first	37,5	— 0,1	— 0,03
	Out and wiped, S first	37,4		
	On returning after an hour's interval	42,4	— 1,7	— 0,51
	Out and carried, S first	40,7	+ 2,0	+ 0,60
	Out and carried, S first	42,7	+ 0,4	+ 0,12
	Out and carried, S first	43,1	— 1,0	— 0,30
	Out and carried, S first	42,1	— 3,0	— 0,90
	O raised and carried	39,1		
	Agitated by raising and depressing the end of the box	No Var	— 1,1	— 0,33
	Out and carried, S first	38,0		
	Left the lanthorn close to the wires for three quarters of an hour and returned	39,3	— 0,7	— 0,21
	Out and agitated mercury, S first	38,6	+ 2,7	+ 0,81
	O raised and carried	41,3	— 2,2	— 0,66
	Out and wiped mercury, S first	39,1		
	Lanthorn taken away and returned in an hour and a half	45,0		
	Lanthorn left close to the wires, returned in an hour	50,8		
	Agitated by tapping the box	51,6		
	Returned in an hour	No Var.	+ 4,2	+ 1,26
	Out and wiped mercury, S first	55,8		

Experiments with the Iron Float.

	4th Set.	Divisions of Microm.	Difference	Error in Secs. affecting the Horiz. point.
Dec. 10	Out and replaced, S first - - -	84,5		
	Ditto, and agitated the mercury, S first	90,3	+ 5,8	+ ″1,74
	Ditto - - - ditto	89,8	— 0,5	— 0,15
	Ditto - - - ditto	88,7	— 1,1	— 0,33
	Ditto - - - ditto	87,3	— 1,4	— 0,42
	Ditto - - - ditto	84,9	— 2,4	— 0,72
	5th Set.			
	Out and agitated mercury, O first -	84,3		
	Ditto - - ditto -	83,9	— 0,4	— ″0,12
	Ditto - - ditto -	82,9	— 1,0	— 0,30
	Ditto - - ditto -	83,2	+ 0,3	+ 0,09
	Ditto - - ditto -	85,0	+ 1,8	+ 0,54
	Ditto - - ditto -	79,2	— 5,8	— 1,74
	An accident suspected			

The surface of the mercury being very dirty, it was carefully strained through a paper funnel.

	6th Set.			
	Out and replaced S first - -	81,0		
	Ditto - - ditto - -	80,9	— 0,1	— ″0,03
	Ditto - - ditto . -	78,9	— 2,0	— 0,60
	Ditto - - ditto -	78,5	— 0,4	— 0,12
	Ditto - - ditto - -	78,0	— 0,5	— 0,15
	Ditto - - ditto - -	77,0	— 1,0	— 0,30

The mercury carefully strained through a paper funnel, and the support oiled and rubbed dry.

Experiments with the Iron Float.

	7th Set.	Divisions of Microm.	Difference.	Error in Sec". affecting the Horiz. point.
Dec. 11	Out and replaced, S first - -	17,3		
	Ditto - - ditto - -	12,4	— 4,9	— 1,47
	Ditto - - ditto - -	15,8	+ 3,4	+ 1,02
	Ditto - - ditto - -	16,7	+ 0,9	+ 0,27
	Ditto - - ditto - -	10,7	— 6,0	— 1,80
	Ditto - - ditto - -	15,8	+ 5,1	+ 1,53

Experiments to show the effect of turning the box round.

	8th Set.			
	Previously to moving - - -	16,7		
	Turned quite round - - -	16,4	— 0,3	— 0,09
	Back again - - -	19,0	+ 2,6	+ 0,78
	Turned round - - - -	15,8	— 3,2	— 0,96
	Back again - - - - -	13,7	— 2,1	— 0,63

The micrometer wire not being *perfectly* horizontal, the cross was brought precisely to the same part at each experiment.

	9th Set.			
	Previously to moving - - -	11,0		
	Turned round - - - -	10,3	— 0 7	— 0,21
	Back again - - - - -	10,2	— 0,1	— 0,03
	Turned round - - -	10,5	+ 0,3	+ 0,09
	Back again - - - -	8,4	— 2,1	— 0,63
	Turned round - - -	8,1	— 0,3	— 0.09

The side and subsequently the whole float, whilst replacing, pressed gently upon the mercury.

	10th Set.	Divisions of Microm.	Difference.	Error in Sec^s. affecting the Horiz. point.
Dec. 11	Out and replaced, S first - -	95,7		
	Ditto - ditto - -	96,7	+ 1,0	+ 0,30″
	Ditto - - ditto - -	99,5	+ 2,8	+ 0,84
	Ditto - - ditto - -	98,2	— 1,3	— 0,39
	Ditto - ditto - -	97,0	— 1,2	— 0,36
	Ditto - - ditto - -	97,7	+ 0,7	+ 0,21

The float pressed upon the mercury as before, and the mean of the readings of both angles of the cross wires taken.

	11th Set.			
	Out and replaced, S first - -	97,4		
	Ditto - - ditto - - -	98,1	+ 0,7	+ 0,21″
	Ditto - - ditto - - -	100,5	+ 2,4	+ 0,72
	Ditto - - ditto - - -	93,8	— 6,7	— 2,01
	Ditto - - ditto - - -	95,0	+ 1,2	+ 0,36
	Ditto - - ditto - - -	94,6	— 0,4	— 0,12

In the preceding experiments it may be seen that by far the greater number of the results are negative, or that the readings of the micrometer for the most part gradually decrease. I felt much at a loss to account for this, and at first supposed it to have been occasioned by the vicinity of the lamp to the Y supporting the cross wires; but I found on trial that this was not the fact; indeed in that case the effect would have been the reverse of what was observed. I can in no other way account for it than by supposing that as the weather was very damp and cold, my approach to the stand which supported the micrometer, caused the legs which were

next me to expand; a supposition which appears to be in some degree corroborated by the micrometer giving an increased reading on my return, after having been absent for some time from the Observatory. Whatever may be the cause, it constantly operates in one direction, and seems to be the principal source of the errors which are observable.

I now wished to ascertain whether by encreasing the length of the float, or by adding to its weight, the length being the same, I should attain greater accurary. I therefore procured two other cast iron floats, the one twelve inches long, four wide, and a quarter of an inch thick, and the other of the same dimensions as that before described, except that its thickness was half an inch, and its weight 4 lb. 8 oz. troy. Iron pins were fixed in the sides of these floats in place of the grooves, and grooves to receive the pins were attached to the sides of the box. The box in which both floats were used was fourteen inches long and six inches wide.

Before I made trial of the new floats, I browned that used in the preceding experiments by rusting it with nitric acid, and then rubbing it with oil; imagining that I might thus diminish any small affinity which the iron might have for the mercury. With the float thus browned, the following experiments were made.

Iron Float browned.

	12th Set.	Division of Microm.	Difference.	Error in Sec^s. affecting the Horiz. point.
Dec. 28	Out and replaced, S first - -	20,0		
	Ditto - - ditto - -	20,5	+ 0,5	+ 0,15
	Ditto - - ditto - -	19,9	— 0,6	— 0,18
	Ditto - - ditto - -	19,0	— 0,9	— 0,27
	Ditto - - ditto - -	15,9	— 3,1	— 0,93
	Ditso - - dttto - -	20,3	+ 4,4	+ 1,32
	13th Set.			
	Out and replaced, O first - -	22,2		
	Ditto - - ditto - -	15,8	— 6,4	— 1,92
	Ditto - - ditto - -	17,8	+ 2,0	+ 0,60
	Ditto - - ditto - -	23,5	+ 5,7	+ 1,71
	Ditto - - ditto - -	22,6	— 0,9	— 0,27
	Ditto - - ditto - -	16,4	— 6,2	— 1,86

I now made the following experiments with the long float, the rough surface of which had been made smooth by rubbing it with wax.

Long Float.

	14th Set.			
Dec. 29	Out and replaced, S first - -	31,0		
	Ditto - - ditto - -	32,0	+ 1,0	+ 0,30
	Ditto - - ditto - -	34,8	+ 2,8	+ 0,84
	Ditto - - ditto - -	31,6	— 3,2	— 0,96
	Ditto - - ditto - -	37,3	+ 5,7	+ 1,71
	Ditto - - ditto - -	33,1	— 4,2	— 1,26

Experiments to show the effect of turning the box round.

	15th Set.	Divisions of Microm.	Difference.	Error in Sec". affecting the Horiz. point.
Dec. 29	Previously to moving - -	33,3	— 0,8	— 0,24
	Turned round, (much agitation) -	32,5	— 0,9	— 0,27
	Back again - - -	31,6	— 0,6	— 0,18
	Turned round - - -	31,0	+ 0,2	+ 0,06
	Back again - - -	31,2	— 0,8	— 0,24
	Turned round - - -	30,4		

I now laid aside the long float to try the short heavy float.

Short heavy Float.

	16th Set.			
	Out and replaced, S first - -	57,2	+ 2,5	+ 0,75
	Ditto - - ditto - -	59,7	+ 0,9	+ 0,27
	Ditto - - ditto - -	60,6	— 4,5	— 1,35
	Ditto - - ditto - -	56,1	+ 1,9	+ 0,57
	Ditto - - ditto - -	58,0	— 0,4	— 0,12
	Ditto - - ditto - -	57,6		
	17th Set.			
	Out and agitated the mercury, S first	67,0	— 1,5	— 0,45
	Ditto - - ditto	65,5	+ 2,6	+ 0,78
	Ditto - - ditto	68,1	— 3,4	— 1,02
	Ditto - - ditto	64,7	— 4,0	— 1,20
	Ditto - - ditto	60,7	+ 4,7	+ 1,41
	Ditto - - ditto	66,0		

The utmost care taken. The mercury carefully strained.

Short heavy Float.

	18th Set.	Divisions of Microm.	Difference	Error in Sec^s. affecting the Horiz. point.
Dec. 30 A. M.	Out and replaced, S first - -	32,1	+ 2,8	+ 0,84″
	Ditto - ditto - -	34,9	— 0,2	— 0,06
	Ditto - ditto - -	34,7	— 1,5	— 0,45
	Ditto - ditto - -	33,2	+ 1,0	+ 0,30
	Ditto - ditto - -	34,2	— 2,4	— 0,72
	Ditto - ditto - -	31,8		

Experiments to show the effect of turning the box round.

	19th Set.			
	Previous to moving - -	31,0	+ 2,2	+ 0,66″
	Turned round - -	33,2	+ 0,7	+ 0,21
	Back again - - -	33,9	— 1,3	— 0,39
	Turned round - - -	32,6	— 1,9	— 0,57
	Back again - - -	30,7	+ 0,6	+ 0,18
	Turned round . - -	31,3		

Mercury carefully strained.

	20th Set.			
	Out and replaced, S first - -	21,1	+ 5,9	+ 1,77″
	Ditto — — ditto — —	27,0	— 4,9	— 1,47
	Ditto — — ditto — —	22,1	+ 3,6	+ 1,08
	Ditto - ditto —	25,7	— 1,3	— 0,39
	Ditto — ditto — —	24,4	— 0,8	— 0,24
	Ditto - ditto —	33,6		

Great care taken that the side of the float should be in contact with the mercury its whole length, previously to putting down the surface.

	21st Set.	Divisions of Microm.	Difference.	Error in Sec^s. affecting the Horiz. point.
Dec. 30 A. M.	Out and replaced, S first -	25,1		''
	Ditto - ditto - -	24,1	— 1,0	— 0,30
	Ditto - ditto -	24,8	+ 0,7	+ 0,21
	Ditto - ditto - -	20,6	— 4,2	— 1,26
	Ditto - ditto -	23,1	+ 2,5	+ 0,75
	Ditto - ditto - -	21,6	— 1,5	— 0,45
	22d Set.			
Jan. 1	Out and replaced, S first -	25,9		''
	Ditto - ditto - -	19,4	— 5,5	— 1,65
	Ditto - ditto - -	21,9	+ 2,5	+ 0,75
	Ditto - ditto -	21,4	— 0,5	— 0,15
	Ditto - ditto - -	23,5	+ 2,1	+ 0,63
	Ditto - ditto -	21,9	— 1,6	— 0,48

Having caused the long float to be ground smooth, and browned it by rusting it with nitric acid, and rubbing it with oil, the oil was very thoroughly cleaned off, and the following experiments made with the greatest care.

Long float browned.

	23d Set.			
Jan. 2	Out and replaced, S first - -	1,4		''
	Ditto - ditto -	2,7	+ 1,3	+ 0,39
	Ditto - ditto - -	2,1	— 0,6	— 0,18
	Ditto - ditto -	3,4	+ 1,3	+ 0,39
	Ditto - ditto - -	3,3	— 0,1	— 0,03
	Ditto - ditto -	5,7	+ 2,4	+ 0,72

Long float browned.

	24th Set.	Divisions of Microm.	Difference.	Error in Sec^s. affecting the Horiz. point.
Jan. 3	Out and replaced, S first - -	51,1		
	Ditto - - ditto - -	49,4	— 1,7	— 0,51″
	Ditto - - ditto - -	48,7	— 0,7	— 0,21
	Ditto - ditto - -	50,2	+ 1,5	+ 0,45
	25th Set.			
Jan. 3	Out and replaced, S first - -	57,5		
	Ditto - ditto - -	56,7	— 0,8	— 0,24″
	Ditto - ditto - -	60,3	+ 3,6	+ 1,08
	Ditto - ditto - -	57,4	— 2,9	— 0,87
	Ditto - ditto - -	59,0	+ 1,6	+ 0,48
	Ditto - ditto - -	58,1	— 0,9	— 0,27
	26th Set.			
	Out and replaced, S first - -	63,3		
	Ditto - - ditto - -	63,6	+ 0,3	+ 0,09″
	Ditto - - ditto - -	61,1	— 2,5	— 0,75
	Ditto - - ditto - -	59,6	— 1,5	— 0,45
	Ditto - - ditto - -	60,9	+ 1,3	+ 0,39
	Ditto - - ditto - -	61,2	— 0,3	— 0,09

Of the one hundred and fifty one results in the column indicating the errors affecting the determination of the horizontal point, only twenty-eight are found exceeding one second; viz. one of 2″,58, another of 2″,01, ten between two seconds and one second and a half, and sixteen between a second and a half and one second; the remaining one hundred and twenty-three errors being all less than one second.

But as it is not to be supposed that an observer would be satisfied with a single determination, when he has the power in a very few minutes of attaining far greater accuracy, I shall

now show the effect that would result from using the mean of a few of the preceding observations.

	Wooden Float.	Error in Seconds.
1st Set.	Mean of the first four - -	″ — 0,34
	Mean of the second four -	— 0,67
2d Set.	Mean of the first five - -	+ 0,36
	Mean of the second five -	— 0,66
	Iron Float.	
3d Set.	Mean of the first five -	— 0,16
	Mean of the second five -	— 0,40
	Mean of the third five - -	— 0,02
	Mean of the last six -	0,00
4th Set.	Mean of five - -	+ 0,02
5th Set.	Mean of five - -	— 0,31
6th Set.	Mean of five - -	— 0,24
7th Set.	Mean of five - -	— 0,09
8th Set.	Mean of four - -	— 0,22
9th Set.	Mean of five - -	— 0,17
10th Set.	Mean of five - -	+ 0,12
11th Set.	Mean of five - - -	— 0,17
	Iron Float browned.	
12th Set.	Mean of five - -	+ 0,02
13th Set.	Mean of five - -	— 0,35
	Long Iron Float.	
14th Set.	Mean of five - -	+ 0,13
15th Set.	Mean of five - - -	— 0,17
	Short heavy Float.	
16th Set.	Mean of five - -	+ 0,02
17th Set.	Mean of five - - -	— 0,10
18th Set.	Mean of five - -	— 0,02
19th Set.	Mean of five - -	— 0,02
20th Set.	Mean of five - -	+ 0,15
21st Set.	Mean of five - - -	— 0,21
22d Set.	Mean of five - -	— 0,18
	Long Float browned.	
23d Set.	Mean of five - -	+ 0,26
24th Set.	Mean of three - -	— 0,09
25th Set.	Mean of five - -	+ 0,04
26th Set.	Mean of five - -	— 0,16

On examining the above table, it appears that by taking the mean of a very few results, the greatest error, if the experiments with the wooden float be rejected, is four-tenths of a second, consequently the place of the horizontal point may be speedily determined by the use of the collimator, to the utmost degree of accuracy which the astronomical circle employed, is capable of attaining.

The results obtained by turning the collimator round without removing the float from the mercury, might have been expected to have been very nearly, if not wholly free from error; but as this does not appear to be the fact, and as the errors are all in defect, they seem to have been influenced by some constant cause, which, as before remarked, I believe to have been expansion of the stand of the micrometer in consequence of increased temperature.

When the float is removed in order to transport the box containing the mercury to the opposite side of the observatory, the manner of replacing it, so as to occasion the least error, seems to be that of bringing the edge of the side of the float first in contact with the mercury, and then gradually lowering it. This mode of removal can be necessary only when the collimator is used with a portable circle; but in a fixed observatory a plank should be laid, or a sort of railway contrived from one support to the other, on which the collimator should be either slid or passed along on rollers without removing the float from the mercury; by this arrangement the greatest, and perhaps the only source of error would be avoided.*

* It may perhaps be found preferable to have two boxes with mercury, and to carry the float from one to the other.

There appears to have been some advantage gained by using a longer float, and it certainly was improved by being browned, as previously to that operation, small particles of mercury were observed, occasionally to attach themselves to the float, which was not the case afterwards.

It may not perhaps be considered altogether superfluous to give in a few words the manner of using the collimator.

The instrument being placed on the north or south side of the observatory with its telescope pointed to the centre of the circle and nearly in its plane, it is to be directed, so that the wires of the telescope of the circle may be seen through it, when reciprocally the cross wires of the collimator will be visible through the telescope of the circle, and the collimator is to be so placed, that the cross wires may appear in the centre of the field of view. The place of the box should then be carefully marked, to ensure its being at once restored as nearly as possible to the same situation.

The collimator is then to be removed to the opposite side of the observatory, and the same process repeated, the situation of the box being here also carefully marked.

In observing, the star having been taken and the readings of the microscopes registered, the telescope is to be depressed to the collimator, and the angle formed by the cross wires carefully bisected. The collimator is then to be taken to the opposite side of the observatory, and the cross wires again bisected; the mean of the readings at the bisections will give the inclination of the collimator to the horizon, and the difference between this and the apparent inclination at either position of the collimator will be the correction to be applied to the mean of the readings registered at the bisection of the star.

For example, let the mean of the readings of the bisection of the cross wires when the collimator is to the south of the instrument be 7'.30" of altitude, and when it is to the north 8'.40". The mean of these readings 8'.5", is the true inclination of the collimator to the horizon, and the difference between this and 7'.30" (0'.35") must be added to all altitudes taken to the south, or subtracted from those to the north of the zenith.

The instrument I have described may be called *the horizontal collimator*, but another and in most respects a preferable arrangement may be employed, similar to that suggested by Professor Bessel. The telescope may be firmly fixed in a position perpendicular to the float, and I should then name it *the vertical collimator.** This must be placed directly under the telescope of the circle; and though not in a convenient position for observing, it yet possesses the very great advantage of obviating the necessity for carrying the collimator from one side of the observatory to the other, nothing more being requisite than to turn the float half round in azimuth, and to take the readings of the microscopes when the angle formed by the cross wires is bisected in each position of the collimator, the mean of which will be the place of the zenith point.

This is the construction which appears best calculated for a public observatory; but in addition, it would perhaps be adviseable that it should be furnished with a horizontal collimator, having a float of increased length. It is intended that

* The float of the *vertical collimator* should be circular, and an opening be made in the bottom of the tube of its telescope to throw light on its cross wires by means of an inclined plane mirror.

the horizontal collimator should remain stationary, and that the usual course of observations should be referred to it, its inclination to the horizon having been previously determined, and its permanency, when thought requisite, being examined by means of the vertical collimator.

As it is not necessary for the telescope of the collimator to have a tube, the object glass and the cross wires in the horizontal construction may be fixed in two uprights cast in one piece with the float. The distance of the object glass from the cross wires must be capable of the nicest adjustment. This may be effected by a screw cut on the outside of the tube in which the object glass is set, and a collar, by means of which after it is adjusted, it may be firmly secured in its proper place. There should be short pieces of tube screwed on each side of the upright, to protect the cross wires from injury, and also to contain the eye glass, which is convenient, as well for illuminating the wires, as for placing the collimator in the proper direction. This construction appears to promise the most perfect invariability of relative position between the line of collimation and the float. The box should be sufficiently deep to include the whole instrument, and should have apertures made in the ends, opposite to the object glass and to the cross wires. It is scarcely necessary to add, that it should also have a cover to exclude dust from the mercury, and a piece of ground glass or oiled paper should be placed between the cross wires and the lamp by which they are illuminated.

The accurate adjustment of the cross wires is a point of extreme importance. Upon whatever portion of an object glass parallel rays fall, they are converged precisely to the

same point in its focus, and consequently, whether the collimator be placed above or below the axis of the telescope of the circle, so long as the cross wires continue visible, the image will suffer no change of position. This affords an excellent method of discovering any want of parallelism in the rays; for if on placing the collimator as much above the axis of the telescope as possible, without losing sight of the cross wires, the image appears elevated above the horizontal wire, or if on placing it below the axis, the image appears to have descended, it is a proof that the rays falling upon the object glass of the circle are not parallel, but that they converge, and consequently that the cross wires of the collimator are too far from its object glass, and *vice versa.* It is necessary that this adjustment should be made with the utmost care.

It might possibly be supposed that the accuracy of the collimator would be augmented by increasing the length of its telescope, but this is not the case. It is the *direction* of a ray passing through the cross wires, and the centre of the object glass of the collimator, which is the subject of observation; and the direction of this ray is as definite in a telescope of an inch in length, as in one of ten feet focus. The degree of precision with which any variation in the horizontal inclination of this ray can be estimated, depends upon the length and power of the telescope employed to view the cross wires, and not upon the length of that of the collimator. There is an inconvenience however in using a telescope of too short a focus, as the cross wires are very much magnified, and consequently appear not so well defined; in addition to which, if a permanent point of reference be required, there might be some fear that the relative positions of the

float and telescope might suffer derangement, and the direction of the ray be consequently changed.

I may here point out an advantage, and not the least valuable, which this instrument presents, that of enabling the observer, by varying the inclination of the float, to bring a different part of the arc into use, and thus to check erroneous division of the circle: this may readily be done by securely fixing weights to either end of the float.*

I shall now proceed to give a description of the manner in which the floating collimator may be applied to the zenith tube.

The first zenith tube was I believe constructed for Dr. TIARKS. It was a telescope hanging in Y s upon pivots projecting from each side of the tube near the object end, and furnished with a wire micrometer: to this telescope a plumb line was attached.

When the star was upon the meridian, and of course sufficiently near the zenith to be seen in the telescope; it was bisected by the micrometer wire, and the divisions registered. The telescope was then inverted in the Y s, re-adjusted by means of the plumb line, and the following evening the star again taken, when the mean of the readings of the micrometer gave its zenith distance.

In the construction of the superb zenith tube 25 feet long, now making by Mr. TROUGHTON for the Royal Observatory at Greenwich, I understand it is intended that the axis of the tube shall be the centre of motion, and the plumb line be

* The above advantage may be considerably extended by the collimator being so constructed as to allow the inclination of the telescope to the float to be varied at pleasure.

suspended at the side. When the observations have been made a sufficient number of times with the plumb line on one side, the tube will be turned half round, and the observations repeated with the plumb line on the other side. The mean of both giving the zenith distance as before.

In this construction the zenith distance cannot be obtained in one evening; for were the telescope to be turned half round after the first observation, so much motion would be communicated to the plumb line, that there would not probably be time to re-adjust the instrument before the star would have passed out of the field of view.

As it is highly desirable that the completion of the observation should not be postponed, I endeavoured to effect this in a very fine zenith tube, which was constructed under my directions by Mr. Dollond for Colonel Lambton, and in another for Sir Thomas Brisbane, by placing the plumb line in the centre of motion; but these various forms are still subject to one or other of the inconveniences which have been detailed in the preceeding parts of this paper, and which it is the object of the floating collimator to remove.

The accuracy of the instrument I am about to describe, will depend upon the goodness of the telescope and of the wire micrometer employed. Exclusively of these it is within the reach of every observer, as the whole arrangement may be completed without difficulty and at a very trifling cost. Expence may contribute something in point of convenience, but can add nothing to its efficiency.

To a firm wall, at a sufficient height, let a shelf be fixed and supported by a bracket at each end. In the middle of this shelf let a circular aperture be made, rather larger than the object glass of the telescope. Precisely beneath, and at a

little distance from this aperture, the telescope is to be securely fastened to the wall in the direction of the zenith. This may perhaps be conveniently done by two irons driven into the wall, terminating in rings, into which the telescope may be passed and clamped. A box of sufficient size to contain the floating collimator being prepared with a circular aperture in the bottom of it, a very little less than that in the shelf, a piece of tube made of sheet iron, varnished brass, or even tinned plate, well painted or varnished, of a size to fit very tightly into the aperture of the box, must be passed into it and secured so as to project above the bottom on the inside an inch or two, and on the outside two or three inches more than the thickness of the shelf. The part of the tube outside the box must be passed through the hole in the shelf, and the box may then be readily turned about the axis of the tube as a centre. The side of the box being placed nearly in the direction of the meridian, its position must be determined by a pin driven perpendicularly into the shelf, so as to come in contact with a pin projecting from one corner of the box near the bottom, and in the direction of one of its sides. The box is then to be turned half round in azimuth, and a pin is to be fixed in the opposite end of the box in con-contact with that in the shelf. By this contrivance, the box may be turned at pleasure half round the azimuth.

The float should be of cast iron, with a hole in the middle an inch larger in diameter than the tube. It is to be furnished with pins, and the box with corresponding grooves to steady it, as before described. An arm of plate iron is to be fixed to the float, its edge being at right angles to the surface. This arm is to project over the aperture, and to terminate in a small tube at the centre, to receive a telescope not

larger than that of a sextant furnished with crossed wires,* and having its object glass next that of the zenith tube.

To any convenient part, either of the shelf or of the wall, a support must be fixed, to which a circular screen of blackened tin may be attached by a joint, so as to be elevated to the vertical or lowered to the horizontal position at pleasure. In the centre of this screen a hole is to be made rather smaller than the telescope of the collimator. The screen is intended when in use to occupy a horizontal position, just above the crossed wires of the collimator, and to exclude false light from the object glass of the zenith tube. In order to illuminate the wires of the collimator, a small plane reflector, which may be of planished tin, is to be attached at a convenient angle to the upper side of the screen over its aperture. This may be made to turn stiffly upon a hinge to vary its inclination.

Having put a sufficient quantity of mercury into the box to enable the float to act freely, the screen must be turned up and the micrometer adjusted, so that a star may pass along its moveable wire. The screen being then restored to its horizontal position, the crossed wires of the collimator will be distinctly seen when the arm carrying its telescope must be bent till they appear in the centre of the field of view, and the telescope of the collimator must be turned in its tube till the opposite angles of the crossed wires are bisected by the micrometer wire. These adjustments may be considered as permanent.

To determine the zenith distance of a star, it must be

* A small black dot upon mother-of-pearl forms a very neat object instead of the crossed wires, but from some trials, I fear it cannot be made sufficiently small.

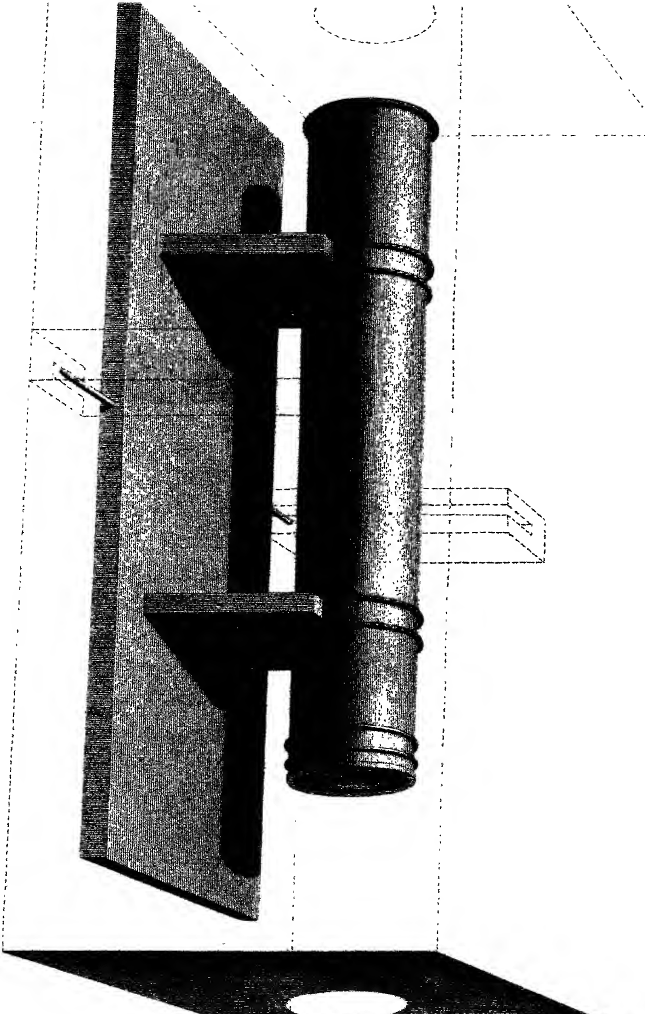
bisected by the micrometer wire at the time of its passing the meridian, and the division of the micrometer head read off and registered. The screen being then turned down, the angle formed by the crossed wires of the collimator is to be bisected, and the reading of the micrometer registered. The collimator is then to be turned half round, the angle again bisected by the micrometer wire, and the reading noted. The mean of these bisections is the place of the zenith, and the difference between this and the reading when the star was bisected, is the star's zenith distance.

It is evident that the operation for finding the place of the zenith may be repeated at pleasure, and consequently that the error, if any, in the zenith distance, may be ultimately referred to inaccurate bisection of the star, or imperfection of the screw of the micrometer.

I may remark, before I conclude, that a telescope,* similar to that I have used in the horizontal collimator, may be employed as a meridian mark for a transit instrument when a distant one cannot be obtained, and that the crossed wires afford an excellent object for the adjustment of the line of collimation. For this purpose, the telescope must be firmly fixed in the proper position. I attempted some years since to effect this by means of a convex lens, having cross wires in its focus, but as it did not occur to me to use an eye-glass, I was unable to place it in the proper direction, and after many unsuccessful trials I laid it aside.†

* The length of the telescope is here important to accuracy.

† Since this Paper was written, I have discovered that in the year 1785, such a meridian mark was actually used by Mr. RITTENHOUSE, who employed for the purpose the object glass of a telescope thirty-six feet long, in the focus of which was placed a metal plate, having several concentric circles drawn upon it. See Transactions of the American Philosophical Society, vol. ii.



VIII. *Notice on the Iguanodon, a newly discovered fossil reptile, from the sandstone of Tilgate forest, in Sussex. By* Gideon Mantell, *F. L. S. and M. G. S. Fellow of the College of Surgeons, &c. In a Letter to* Davies Gilbert, *Esq. M. P. V. P. R. S. &c. &c. &c. Communicated by* D. Gilbert, *Esq.*

Read February 10, 1825.

Sir,

I avail myself of your obliging offer to lay before the Royal Society, a notice of the discovery of the teeth and bones of a fossil herbivorous reptile, in the sandstone of Tilgate forest; in the hope that, imperfect as are the materials at present collected, they will be found to possess sufficient interest to excite further and more successful investigation, that may supply the deficiencies which exist in our knowledge of the osteology of this extraordinary animal.

The sandstone of Tilgate forest is a portion of that extensive series of arenaceous strata, which constitutes the iron-sand formation, and in Sussex forms a chain of hills that stretches through the county in a W. N. W. direction, extending from Hastings to Horsham. In various parts of its course, but more particularly in the country around Tilgate and St. Leonard's forests, the sandstone contains the remains of saurian animals, turtles, birds, fishes, shells, and vegetables. Of the former, three if not four species belonging to as many genera are known to occur, viz. the crocodile, megalosaurus, plesiosaurus, and the iguanodon, the animal whose teeth

form the subject of this communication. The existence of a gigantic species of crocodile in the waters which deposited the sandstone, is satisfactorily proved by the occurrence of numerous conical striated teeth, and of bones possessing the osteological characters peculiar to the animals of that genus; of the megalosaurus, by the presence of teeth and bones resembling those discovered by Professor Buckland in the Stonesfield slate; and of the plesiosaurus, by the vertebræ and teeth analogous to those of that animal.

The teeth of the crocodile, megalosaurus and plesiosaurus, differ so materially from each other, and from those of the other lacertæ, as be to identified without difficulty; but in the summer of 1822, others were discovered in the same strata, which although evidently referable to some herbivorous reptile, possessed characters so remarkable, that the most superficial observer would have been struck with their appearance, as indicating something novel and interesting. As these teeth were distinct from any that had previously come under my notice, I felt anxious to submit them to the examination of persons whose knowledge and means of observation were more extensive than my own; I therefore transmitted specimens to some of the most eminent naturalists in this country, and on the continent. But although my communications were acknowledged with that candour and liberality which constantly characterises the intercourse of scientific men, yet no light was thrown upon the subject, except by the illustrious Baron Cuvier, whose opinions will best appear by the following extract from the correspondence with which he honoured me.

"Ces dents me sont certainement inconnues; elles ne sont

point d'un animal carnassier, et cependant je crois qu'elles appartiennent, vu leur peu de complication, leur dentelure sur les bords, et le couche mince d'émail qui les revét, à l'ordre des reptiles. A l'apparence extérieure on pourrait aussi les prendre pour des dents de poissons analogues aux tetrodons, ou aux diodons; mais leur structure intérieure est fort différente de celles là. N'aurions-nous pas ici un animal nouveau, un reptile herbivore? et de même qu'actuellement chez les mammifères terrestres, c'est parmi les herbivores que l'on trouve les espèces à plus grande taille, de même aussi chez les reptiles d'autrefois, alors qu'ils etaient les seuls animaux terrestres, les plus grands d'entr'eux ne se seraient-ils point nourris de végétaux? Une partie des grands os que vous possidez appartiendrait à cet animal, unique, jusqu'à présent, dans son genre. Le tems *con*firmera ou *in*firmera cette idée, puisqu'il est impossible qu'on ne trouve pas un jour une partie du squelette réunie à des portions de machoires portant des dents. C'est ce dernier objet surtout qu'il s'agit de rechercher avec le plus de persévérance."

These remarks induced me to pursue my investigations with increased assiduity, but hitherto they have not been attended with the desired success, no connected portion of the skeleton having been discovered. Among the specimens lately collected, some however were so perfect, that I resolved to avail myself of the obliging offer of Mr. Clift, (to whose kindness and liberality I hold myself particularly indebted) to assist me in comparing the fossil teeth with those of the recent lacertæ in the Museum of the Royal College of Surgeons. The result of this examination proved highly

satisfactory, for in an Iguana which Mr. STUTCHBURY had prepared to present to the College, we discovered teeth possessing the form and structure of the fossil specimens.

In the annexed drawing, Plate XIV. examples of the recent and fossil teeth are represented, and the peculiar characters of each accurately shown; a description of it in this place will render the subsequent observations more intelligible.

Fig. 8 represents a portion of the upper jaw of the iguana viewed from within; it is magnified four diameters.

9 *a* shows the inner, and 9 *b* the outer surface of a tooth of the same, greatly magnified. It may be proper to remark, that the teeth differ considerably in the number of points, and that the eminence at *f*, fig. 9 *a*, is sometimes the first or second in the series, instead of being the third, as in the figure. In some teeth the points vary but little in size; they are more distinct on the edges of the teeth occupying the centre of the jaw, than in the anterior and posterior ones. The skeleton from which the drawings were made is three feet six inches in length. It is said to be the common edible iguana of the West Indies, but I have not been able to ascertain its species with certainty. The remaining figures represent different examples of the fossil teeth.

Fig. 1. *a* represents the outer, and fig. 1. *b* the inner surface of one of the largest and most perfect specimens of the teeth of the iguanodon. As the letters of reference in each figure indicate the same parts, they are explained here to avoid repetition.

a. Surface worn by mastication.

b. The serrated edges.

c. Fang broken; the cavity filled with sandstone.

d. Cavity or depression in the base of the fang, the effect of absorption caused by the pressure of a secondary tooth.*

e. Ridge extending down the front of the tooth.

Fig. 2. This tooth evidently belonged to a young animal; yet even in this example the apex is worn away, (see *a.* fig. 2 *c.*) The ridge extending down the front (see *e* fig. 2 *a.*) is more or less distinct in every specimen.

Fig. 3. A tooth much worn by mastication. The serrated edges and other characters are obliterated, the tooth being worn down to the point marked by the line at *g.* fig. 1. *a.* The fang has been removed by absorption; and the cavity formed by the pressure of the new tooth is very deep.

Fig. 4. In this specimen the point is perfect, and it therefore more closely resembles the recent tooth (fig. 9.) than those above described.

5. Is another example, where the point is but little worn.

6. A large strong tooth less curved than fig. 1 and 2. It probably occupied a place in the posterior part of the jaw.

7. In this figure, the cavity of the base of the fang for the reception of the new tooth is remarkably distinct.

The teeth above described, although varying from each other in some particulars, do not present greater dissimilarity than the differences arising from age, and the situation they respectively occupied in the jaw, would be liable to produce. Like the teeth of the recent iguana, the crown of the tooth is accuminated; the edges are strongly serrated or

* The hollow here described is so constantly found in every example, that it cannot be accidental. From the close resemblance it bears to the cavity formed in the base of the fangs of the recent iguana, by the secondary teeth; (Vide *d,* fig. 8) it may be confidently presumed that it is the effect of a similar cause.

dentated; the outer surface is ridged, and the inner smooth and convex; and as in that animal the secondary teeth appear to have been formed in a hollow in the base of the primary ones, which they expelled as they increased in size. From the appearance of the fangs in such fossil teeth as are in a good state of preservation, it seems probable that they adhered to the inner side of the maxillæ, as in the iguana, and were not placed in separate alveoli, as in the crocodile. The teeth appear to have been hollow in the young animals, and to have become solid in the adult. The curved teeth (figs. 1, 2.) probably occupied the front of the jaw; and those which are nearly straight, (fig. 3.) the posterior part.

It appears unnecessary to dwell longer on the resemblance existing between the recent and fossil teeth. Whether the animal to which the latter belonged, should be considered as referable to existing genera, differing in its specific characters only; or should be placed in the division of enalio-sauri of Mr. CONYBEARE, which includes marine genera only, cannot at present be determined. If however any inference may be drawn from the nature of the fossils with which its remains associated, we may conclude, that if amphibious, it was not of marine origin, but inhabited rivers or fresh-water lakes; in either case the term IGUANODON, derived from the form of the teeth, (and which I have adopted at the suggestion of the Rev. W. CONYBEARE) will not, it is presumed, be deemed objectionable.

It has already been mentioned, that of the bones of oviparous quadrupeds found in the sandstone of Tilgate forest, some are decidedly referable to the crocodile, and others to the megalosaurus and iguanodon; but our knowledge of the

osteology of the latter is at present so limited, that until some connected portion of the skeleton shall be discovered, it is impossible to distinguish the bones of the one from those of the other. Since, however, the teeth of the iguanodon are not known to occur in the Stonesfield slate, perhaps such of the bones from Tilgate forest as resemble those figured and described by Professor BUCKLAND, in Vol. I. Second Series of the Geological Transactions, may be attributed to the megalosaurus; while others not less gigantic may be assigned to the iguanodon. That the latter equalled, if not exceeded the former in magnitude, seems highly probable; for if the recent and fossil animal bore the same relative proportions, the tooth, fig. 1. must have belonged an individual upwards of sixty feet long; a conclusion in perfect accordance with that deduced by Professor BUCKLAND from a femur,* and other bones in my possession.

The vertebræ, as in the greater part of the fossil saurians, differ very materially from those of the recent iguana, crocodile, &c. They are not concave anteriorly, and convex posteriorly, but have both faces slightly depressed, resembling in this respect the vertical column of one of the fossil crocodiles of Havre and Honfleur. But among the recent lacertæ there are some, as the Proteus of Germany, the Syren of Carolina, and the Axolotl of Mexico, in which the vertebræ are deeply cupped at both extremities; and since the fossils in question are clearly of the saurian type, having the annular part united to the body of the vertebra by

* Vide Professor BUCKLAND's notice on the Megalosaurus. Geol. Trans. Vol. I. Second Series, p. 391.

suture, the discrepancy alluded to does not appear to be sufficiently important to invalidate the accuracy of the opinions which I have attempted to establish.

I have the honor to be,

SIR,

your most obedient Servant,

GIDEON MANTELL.

Castle Place, Lewes,
Jan. 1, 1825.

Teeth of the IGUANODON a newly discovered FOSSIL ANIMAL, from the Sandstone of TILGATE FOREST, in SUSSEX.

Portion of the Jaw of the Iguana, four times magnified.

J. Basire sculp.

IX. *An experimental enquiry into the nature of the radiant heating effects from terrestrial sources. By* Baden Powell, *M. A. F. R. S. of Oriel College, Oxford.*

Read February 17, 1825.

(1.) The nature of the heating effect emanating from *luminous* hot bodies has been distinctly shown to be, in many particulars, very different from that evolved from *non-luminous* sources; but the ideas commonly entertained on the subject, are far from being precise and distinct. To gain if possible some ground for establishing more clear views, is the object of the following enquiries.

(2.) Professor Leslie, in his well known and elegant experiments, (Inquiry concerning Heat, &c. chap. iii.) has fully established the theory of the effect of screens on radiant heat; and these effects give some of the most important criteria for examining the nature of radiating agents.

Those experiments apply only to the heat evolved from a non-luminous source. It therefore naturally becomes the subject in question, whether the interceptive power of glass is not limited to a certain temperature, or state, of the radiating source; and to this point accordingly the attention of several eminent observers has been directed in many well known investigations, among which those of M. De La Roche are justly regarded as the most important and complete. In these experiments it appears, that a greater effect

is produced on a blackened thermometer when a glass screen is interposed, in proportion as the body under trial approaches nearer its point of luminosity, or becomes more intensely luminous. (BIOT, Traité de Phys. tom. iv. p. 638.) (Ann. of Philos. O. S. vol. ii. p. 163.)

Both M. DE LA ROCHE and M. BIOT (See BIOT, iv. 612.) seem disposed to view the results obtained by the former upon the supposition of one simple agent, the principle both of light and heat. This is at first radiated as heat; at a certain point it begins to assume the form of light, when the interceptive power of glass decreases in proportion to the increase of luminosity.

(3.) As long as the hot body continues below the temperature of luminosity, the partial or total interception of the effect is precisely the same phenomenon as that described by Professor LESLIE in his experiments on screens and explicable in the same way; (Phil. Trans. 1816, Part I. On new Properties of Heat, Prop. 40.) And the apparent transmission of a portion of the effect must be referred to the same principle, as is clearly shown by Dr. BREWSTER, who has established, apparently beyond contradiction, the impermeability of glass to simple radiant heat upon quite independent principles.

(4.) Above the temperature of luminosity we must have recourse to further considerations. The hypothesis of M. M. DE LA ROCHE and BIOT appears to be nearly the same as that of Professor LESLIE (Inquiry, p. 162). And it certainly has the merit of simplicity and satisfactory explanation of the phenomena. But it is an opinion which has not received direct proof; and it is also obvious, that the phenomena may

be explained without it; for we may just as well account for the facts, by supposing two distinct heating influences, one associated in some very close way with the rays of light, carried as it were by them through a glass screen without heating it; the other being merely simple radiant heat, affected by the screen exactly as the radiant heat from a non-luminous body.

(5.) In order to ascertain which of these suppositions is true, it will not be sufficient to observe the effects produced by the intervention of a *screen alone*. We must combine this method with an examination of the relations of different sorts of heat to *surfaces*. These relations have been shown to differ according as the body is luminous, or not; in the one case, the direct heat affects bodies in proportion to the darkness of their *colour*, without regard to the texture of their surface: in the other, the magnitude of the effect depends solely on the *absorptive texture* without reference to colour. I use the term "absorptive texture," to signify that peculiar state of division in the particles of the surface, which has been shown, by Professor LESLIE and others to be most susceptible of the influence of simple radiant heat, and always to give a proportionally greater radiating power.

The question then is entirely one of facts; and involves no hypothesis as to the nature either of light or of heat. The object is simply to ascertain by experiment, whether, of the total heating effect radiated from a luminous hot body, the portion intercepted by a transparent screen is of the same nature as, or different from the part transmitted in its relations to the surfaces on which it acts.

(6.) In conformity with this view of the object proposed,

the general principle of the following experiments is this: taking different luminous hot bodies, to expose to their influence two thermometers presenting, one, a smooth black surface, the other an absorptive white one: thus obtaining the ratio of the total direct effect on the two, we may compare it with the ratio similarly observed, when a transparent screen is interposed.

(7.) This principle of experimenting was applied with one or two variations: and though in the abstract sufficiently simple, it will in practice require an attention to several considerations. I shall therefore proceed in the first instance to the detail of the different particulars; then give the results of the experiments in a tabular form; and lastly, recapitulate the conclusions, and make a few general remarks.

I. (8.) In the following set of experiments two common thermometers were employed. The diameters of their bulbs were, thermometer A, 0.6 inch,; B, 0.55. A, was coated with a wash of chalk and water, and B, with indian ink.

In order to compare the effects to be observed with those of simple radiant heat, I ascertained the ratio of the effects of the latter on the two bulbs thus coated, by a few preliminary trials; and found it to be very nearly one of equality, or perhaps the effect of the white rather greater than that of the black.

The two thermometers were graduated to quarters of centigrade degrees; and were both fixed on one mounting with their bulbs detached about one inch from its lowest part, and at the distance of about three-quarters of an inch from each other.

(9.) In the 2d set of experiments they were fixed into the

top of a box, the front of which was open, so that the glass screen could be applied to it or not, as required. When the screen was not used the box would acquire more heat, and radiate it to the bulbs in a small degree; which affecting them in the inverse ratio of their diameters, would diminish the ratio of their risings. That this diminution was very trifling, and not at all sufficient to account for the observed difference of ratio will be evident, because the 1st set was made without employing the box, the thermometers being suspended at a distance from any object which could radiate heat to them; and in this set the difference of ratio is quite as conspicuous. This remark applies likewise to the possible communication of heat by the air.

(10.) We must also take into consideration the effect due to the glass screen. When we consider the two bulbs as heated only by that part of the radiation which is transmitted through the screen, the screen may be regarded simply as a third body placed near the two bulbs; and whether it possesses a higher or a lower temperature, there will be a tendency to bring all three to an equality in proportion to the difference of temperature, and in the bulbs, dependent on their diameters modified by the state of their surfaces. This effect arises from simple radiant heat, whilst that derived from the luminous hot body, is evidently following a different law with regard to the surfaces. It will easily follow from what has been already shown, that such a secondary heating effect will be of a kind tending to *diminish* the ratio otherwise obtaining between the effects on the two bulbs. If the effect were of a cooling nature, the same thing would also take place: for I ascertained that the radiating powers of the

coatings employed, deduced from the observed rates of cooling, were in a ratio which happened to be almost exactly the inverse of that of the diameters; but this effect is probably always small, and I have roughly allowed for it, as will be seen immediately; taking the temperature of the screen by a small thermometer having its bulb in contact with the central part of the surface.

II. (11.) I now proceed to state the results, which will be most conveniently exhibited in a tabular form.

Ist Set. Incandescent iron. Distance 7 inches.

Glass screen.

Experiment.	Rise of Thermometer in 1 min. centigrade. A. White.	B. Black.
1	1.25	2.5
2	1.25	3.
Mean	1.25	2.75
Allowing for the screen (as below.	1.	2.5

No screen.

1	7.5	9.75
2	6.5	7.75
Mean	7.	8.75
Difference of exposed and screened results.	6.	6.25

(12.) Argand lamp without its chimney. Distance 3 inches.

Glass screen.

Experiment.	Rise of Thermometers in 1 min. centigradge.	
	A. White.	B. Black.
1	.75	1.75
2	.5	2.25
3	.75	2.25
Mean	.66	2.08
Allowing for the screen.	.41	1.83

No Screen.

1	1.75	3.5
2	1.75	3.25
3	2.	3.5
Mean	1.83	3.41
Difference of exposed and screened results.	1.42	1.58

(13.) II^d. Set. Incandescent iron. Distance 6 inches.
Glass screen 2 inches from bulbs.

Experiment.	Temp. of screen before experiment by thermometer in contact.	Rise of Thermometers in 1 min, (centigrade.) A. White.	Rise of Thermometers in 1 min, (centigrade.) B. Black.	Temperature of screen after experiment.
1	16.5	1.	1.5	25.5
2	16.5	.5	1.25	23.75
3	17	.5	1.5	24.5
4	17	.5	1.	22.25
5	17	.5	1.	22.25
Mean		.6	1.25	
Effect of the screen alone, heated above 25°.				
1		.25	.25	
2		.25	.25	
The former result diminished for this effect.		.35	1.	

Incandescent iron. No screen.

1		3.	3.5	
2		3.	4.	
3		2.75	3.5	
4		3.3	3.75	
5		3.3	4.	
Mean		2.95	3.75	
Difference of the exposed and screened results.		2.6	2.75	

(14.) Flame of an Argand lamp without its chimney.
Distance 3 inches.

Glass screen 1.5 inch from bulbs.

Experiment.	Temperature of screen before experiment.	Rise of Thermometers in 1 min. (centigrade.) A. White.	Rise of Thermometers in 1 min. (centigrade.) B. Black.	Temperature of screen after experiment.
1	17	1.25	2.25	23.
2		1.25	2.25	
3		1.75	2.25	
4		1.25	2.75	
5		1.	2.25	
	Mean	1.3	2.35	
Effect of the screen alone, heated above 25°.				
		.25	.25	
The former result diminished from this effect.		1.05	2.10	

Lamp. No screen.

Experiment.		A. White.	B. Black.	
1		2.25	3.	
2		2.5	3.25	
3		2.25	3.	
4		2.25	3.25	
5		2.5	3.5	
	Mean	2.35	3.2	
Difference of the exposed and screened results.		1.3	1.1	

(15.) In these experiments it will be evident upon inspection, that the ratio of the effects produced on the white and black bulbs, is in every instance considerably greater when they were affected only by that part of the total heating influence which is transmitted through a transparent screen, than when they were exposed to the whole. This then would indicate, that on the removal of the screen some new heating power was brought into action which affected the ratio by the addition to each of its terms, of quantities in a ratio expressed by that of the difference of the exposed and screened results above given. This ratio is evidently one differing a little from equality, and agreeing nearly with that of the diameters of the bulbs inversely.

(16.) The experiments now detailed will probably be considered sufficient to substantiate the conclusion; but in researches of this kind, where great numerical precision is unattainable; it seemed desirable to give the experiments that confirmation which they wanted in point of intrinsic accuracy, by frequent repetition and variation. With this view I made a great number of trials with a large differential thermometer; the bulbs were about one inch in diameter and nearly three inches apart. The bore of the tube was about $\frac{1}{10}$ of an inch. Many of the experiments made with this instrument I shall not mention, as, although all agreeing to confirm my former conclusions, they were complicated by several unnecessary conditions.

(17.) In order to obtain results in the most simple manner, it was desirable to get rid of any action on one of the bulbs, and to expose only the other; the instrument thus acting simply as an air thermometer. The effects on each bulb, one

being painted with indian ink, and the other coated with white silk pasted on, when exposed, might thus be compared with those through a glass screen. I first tried the experiment by placing the bulb in the focus of a spherical tin reflector about six inches diameter; by this means the source of heat could be placed at a sufficient distance to preclude any effect from the glass screen.

(18.) The experiment was again varied by placing a large opake screen before the instrument, in which was an aperture through which one bulb might be exposed. To this aperture a piece of glass could be applied; each bulb was presented both with and without the glass.

(19.) In all these experiments it is evident, that any heating effect arising from the screen, would tend to diminish the ratio of the black and white effects; and this not being allowed for in the statement of the result, the difference between this ratio and that of the exposed effects will be in reality greater than appears.

The result are comprised in the following table.

(20.) Lamp. Bulb in the focus of a reflector.

Coatings white silk, and indian ink.

Experiments.	Screened.		Exposed.	
	White.	Black.	White.	Black.
1	5	8	11	15
2	4	9	13	15
3	6	11	12	14
Mean in 30 sec^s^.	5	9.3	12	15

(21.) Incandescent iron.

1 2	4 4	7 6	11 10	13 10
Mean	4	6.5	10.5	11.5

(22.) Lamp. One bulb covered by an opake screen, the other exposed at an aperture. Dist. 5 inches. Screen 1.5 inch from bulb.

1 2 3 4	4 3.5 4 3	6 6 6 6	8 9.5 8 8	11 12 10.5 12
Mean in 1 min.	3.6	6	8.3	11.3

(23.) Incandescent iron.

in 30 sec^s.	3	4	12	9

(24.) It is perhaps not worth while to make any formal deductions from these results as to the ratios subsisting in the different cases. It will be sufficiently evident upon inspection, that when all due allowances are made, the ratio of the effects upon the white and black bulbs is considerably greater when they were affected only by the transmissible part of the heating effect, than when they were exposed to the whole. The part then which is added on the removal of the screen, is of a nature tending to add to the terms of the former ratio quantities in a ratio much nearer equality: quantities in a

ratio very nearly that which the effects of simple radiant heat would give.

III. (25.) I have above adverted to all the sources of error which occur to me as likely to have affected these results; and when these are taken into consideration, as well as the nature of the experiments and apparatus, the accordance which the different results exhibit, is perhaps as close as we can expect; and it appears that all the different sets of experiments agree in showing a very considerable difference in the *ratio* of the effects produced on a smooth black, and on an absorptive white surface, by that part of the radiant effect transmitted through glass, and by the total effect. If the total direct effect were the result of one simple agent, the intervention of the glass would, by intercepting some part of it, produce no other alteration than a diminution of intensity; the *ratio* of the two effects would remain unchanged. This distinction appears to me of some importance towards clearing our ideas respecting the nature of the phenomena, and thus affording an answer to the question originally proposed in reference to some theoretical views, which, though boasting the sanction of high authority, will be untenable if the validity of these results be admitted.

(26.) The general conclusions from all these experiments may be thus recapitulated:

1st. That part of the heating effect of a luminous hot body which is capable of being transmitted in the way of direct radiation through glass, affects bodies in proportion to their *darkness of colour*, without reference to the texture of their surfaces.

2d. That which is intercepted produces a greater effect in

proportion to the *absorptive nature of texture* of the surface, without respect to colour. These two characteristics are those which distinguish simple radiant heat at all intensities.

Thus then when a body is heated at lower temperatures, it gives off only radiant heat stopped intirely by the most transparent glass, and acting more on an absorptive white surface than on a smooth black one.

At higher temperatures the body still continues to give out radiant heat, possessing exactly the same characters.

But at a certain point it begins to give out light: precisely at this point it begins also to exercise another heating power distinct from the former; a power which is capable of passing directly through transparent screens, and which acts more on a smooth black surface than on an absorptive white one.

(27.) This last sort of heat, whatever its nature may be, is essentially different from simple radiant heat. It appears to agree very closely with what the French philosophers term "Calorique lumineux," and is, according to Professor LESLIE'S theory, a conversion of light into heat. These views of the subject are certainly gratuitous assumptions. We have no right whatever to identify those two agents, or to suppose that, because a heating effect very closely accompanies the course of the rays of light, the light is therefore converted into heat; but the theories above alluded to, seem to regard the *whole* heating effect of a luminous body as of this latter character. In this particular, the present inquiry has led us to an essential distinction; and if the experiments are to be relied upon, this peculiar sort of heat constitutes only *a part* of the total effect. These results do not indeed

present so simple a theory as that alluded to, but they apply very obviously to the explanation of many phenomena recorded by various experimenters.

(28.) The peculiar heat above spoken of, and which, for the sake of distinction and brevity, we may call "transmissible heat," is similar to that which acts in the solar rays, and which there constitutes the total effect. It is this kind of heat which has been employed as a principle of photometry, on the assumption that it is precisely proportional to the intensity of light. Within certain limits this may be the case; but there are unquestionably circumstances under which the relation is very different; such for example, as difference of colour in the light: and in general it cannot be assumed to hold good in light from different sources. To show this, there is a remarkable instance in incandescent metal, which produces but very faintly illuminating rays, yet its "transmissible heat" is very considerable. I have repeatedly tried the experiment with a small "photometer," having one bulb painted with indian ink and the other plain; the bulbs being in a vertical line; this instrument whether employed with or without its case, or a glass screen, always gave an effect of about 10° in 30″ at eight inches distance from a ball of iron heated to the brightest point in a common fire.

(29.) In making these last experiments, the effect was always greater when the instrument was used without its case, or a glass screen. This was no doubt in part owing to the greater action of the simple heat now admitted to the instrument on the coated, than on the plain bulb; but it was also in part occasioned by the circumstance, that the stem going to the upper bulb passes in contact with the lower,

and being a solid mass compared with the thin bulb, is slower in acquiring heat, and therefore cools it, thus increasing the apparent effect on the other.

(30.) In a variety of other experiments which I have tried, using either this "photometer," or another having the bulbs at equal heights, various apparent anomalies presented themselves, all which I found easily explained on the principles here established of two radiations, when connected with the various other considerations to which it is necessary to refer when employing instruments of this description; but I do not conceive it necessary to enter into any further details.

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APPENDIX.

ERRATA.

Page	Line	
317	0	*for* " [117] " *read* " [317] " at the top of the page.
375		Running title, *for* " prevention" *read* " preservation."
439	4	*for* " Plate XXVII." *read* " Plate XXIX."
445	3	from bottom, *for* " having " *read* " leaving."
449	18	*for* " 1092 " *read* " 8092."
457	1	from bottom, *for* " there were mixtures " *read* " these were mixtures."
463, 464		In the column of differences, the numbers should be placed to stand intermediate between those of the column of parts. Also, *dele* " 23.4" in line 8 of the former column, and bring up the remaining differences each a line higher.
492	2	*for* " place " *read* " plane."

PHILOSOPHICAL

TRANSACTIONS.

X. *On the Anatomy of the Mole-cricket.*

By J. Kidd, *M. D. and F. R. S. Reg. Prof. of Medicine in the University of Oxford.*

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The following observations contain the principal points of a laborious examination of the anatomical structure of the gryllotalpa, or mole-cricket; and if I dare hope that that examination has been conducted with any thing like adequate accuracy, I need not apologize for the length of the details with which the account of it is accompanied, since Cuvier has affirmed of an entire volume written by Lyonnet on the anatomy of a single species of caterpillar, that it contains not one word that is useless.

Natural science indeed has now arrived at that point, in which individual detail is requisite for the acquisition not only of a surer basis of classification of species, but also of more correct principles of general physiology. Independently however of these considerations, the insect, which is the subject of the present communication, is so singular in its

structure and habits, and is in some parts of the world so formidable to the agriculturist, as to render its history peculiarly interesting.

It is described under various names; as the earth-crab, from its general appearance; vermis cucurbitarius, from the mischief it does to cucumber-beds. By the French naturalists it is called courtilière.

The best account of the mole-cricket with which I have met is in a well known etymological work by Rösel, published at Nuremberg in 1749. This account is accompanied by the best engravings also of the external characters of the animal in its different states: and the value of these engravings is greatly enhanced by the accuracy with which they are coloured.

Rösel says that about the month of June or July, rarely later, the gravid female gryllotalpa excavates a cavity, from 4 to 5 inches beneath the surface of the earth, in which she deposits her eggs in one heap, to the number of three hundred or more; and dies within a few weeks afterwards. At the end of about a month the young mole-crickets are produced; and appear, on a hasty survey, to bear a general resemblance to the ant. Between the time of their birth and the commencement of winter, the young animals cast their skin three times; they lie dormant during the winter, deeper in the earth in proportion to the inclemency of the season; and during this period cast their skin for the fourth time. About May they leave their winter quarters, and at this time are furnished with the rudiments of their future wings, four in number; which differ remarkably in size and form and position from those of the perfect insect; in which the inferior

wings are folded in a very curious manner, while in the imperfect insect they are always open.

During the month of June or July they cast their skin for the fifth and last time; after which the wings acquire a permanent character, and the insect becomes capable of propagating its species.

RÖSEL says that he himself never dissected a mole-crcket; but reports, on the authority of others, that its stomach resembles that of the locust, represented in his ninth plate of the series of that tribe of insects. I may here add, from my own observation, that it very closely resembles that of the gryllus viridissimus, and also that of a species of gryllus preserved in the Ashmolean Museum, which answers to the pneumora of LAMARCK: it also somewhat resembles that of a locust, marked 614 in the Hunterian collection; and, still more, that of the Cape grasshopper, engraved in the 84th plate of the first part of Sir E. HOME's Comparative Anatomy.

It appears from RÖSEL's account, that while very young, these insects are gregarious, but not afterwards; that they are usually found in the vicinity of meadows and of fields of corn, particularly of barley; to which they are very detrimental by feeding on the roots, and thus intercepting the due nourishment of the plants themselves. I have no doubt of the general accuracy of the foregoing remarks of RÖSEL, and have little to add to his account of the natural history of this insect. I have hitherto met with the mole-cricket in one situation only; namely, in some peat-bogs, at the distance of a few miles to the west of Oxford. In the neighbourhood of these peat-bogs the insects are familiarly known by the name

of croakers, from the peculiar sound which they occasionally make; a sound not very unlike, but more shrill and more soft than that of the frog. This sound, even in the case of a single individual, may be heard at the distance of some yards; but when made by numerous individuals at the same time it may be heard, as I have reason to believe, at the distance of some hundred yards, provided the air be in a favourable state. I have usually found the insect within a foot and a half of the surface, and in parts where the peat was neither quite dry, nor very moist; of such a consistence indeed as is most favourable to the mining operations of the animal.

The accounts of different authors differ as to the food of the mole-cricket. Having kept several individuals in glass vessels during some weeks, I observed, that of all kinds of vegetable food they preferred the potatoe, while cucumber they hardly touched; but if raw meat were offered them they attacked it with great greediness, and in preference to every thing else. And, when they had been kept, though even but for a short time without any food, they did not hesitate to attack each other; in which case the victor soon devoured the flesh and softer parts of the vanquished. As I have not unfrequently found them in their native haunts maimed in various parts of the body, I have very little doubt that, although captivity may increase their ferocity, they are not, even in a natural state, free from each other's attacks. If they are carnivorous, they probably feed on worms, and various larvæ, which are abundant in the peat-bogs above-mentioned, for I have repeatedly found the horny and indigestible parts of insects within their stomachs. Similar relics I have found in the stomach of the pneumora and gryllus viridissimus. The

two following facts attest in the tribe of insects to which the mole-cricket belongs, a remarkable degree of voracity, and an equally remarkable power of abstaining from food. My friend Dr. Macartney, of Dublin, informs me that he has known a gryllus devour a portion of its own body: on the other hand, my friend Mr. Buckland, of this University, gave me, at the commencement of the present summer, a living gryllotalpa, which had been confined during nine or ten months in a tin case, containing a small quantity of garden mould, without the possibility of having met with any other nourishment than such as that portion of mould might be supposed to contain.

External characters of the perfect gryllotalpa.

In this, as in the case of every other animal with whose habits of life we are acquainted, we see a perfect accommodation in form and structure to the circumstances in which the individual is naturally placed. Destined like the common mole to live beneath the surface of the earth, and to excavate a passage for itself through the soil which it inhabits, the gryllotalpa is furnished like the mole, with limbs particularly calculated for burrowing; with a skin which effectually prevents the adhesion of the moist earth through which it moves; and with exactly that form and structure of body, by which it is enabled to penetrate the opposing medium with the greatest ease. At the same time, in order to prevent the necessity of its excavating a track so wide as to admit of the body being turned round in case of a desire to retreat, it is endued with the power of moving as easily in a retrograde as in a progressive direction; and, apparently to perform the

office of antennæ, which warn the insect of approaching danger in its progressive motions, it has two appendages, which might not improperly be called caudal antennæ, evidently calculated to serve a similar purpose during its retrogade motions; particularly as they are furnished with very large nerves. The indifference with which the insect is disposed to move in either direction is manifested by the following experiment: if you touch it towards the head, it retreats; if towards the other extremity of the body, it advances.

The general colour of the animal is such as indirectly to serve as a protection to it, being nearly of the same hue as the vegetable mould in which it lives; so that it is not very readily distinguished upon being first turned up to view; and its safety seems to be still farther insured by the appearance of death, which, in common with many other insects, it assumes when suddenly disturbed. This stratagem, for so it may be called, appears to be most decidedly practised by the animal while in captivity; and if thrown at random out of the vessel in which it has been confined, however unnatural the posture may be into which it has been thrown, it remains as it were in a state of catalepsy during half a minute or more; the first indication which it gives of recovery from this stupor, invariably consists in a motion of the extremity of the antenna.

The general colour of the insect is a dusky brown, passing either into a reddish brown, or into an ochry yellow; those parts being of the darkest colour which are most exposed to view when the animal is moving in the open air. Every part of the body is to a greater or less degree covered by a kind of down, which seems to be the efficient cause of its capa-

bility of repelling moisture; which capability is so remarkable, that when the insect is plunged under water, it appears as if cased in silver, or some bright metallic covering: this appearance being evidently derived from a stratum of air, interposed between its body and the surrounding liquid. This down not only serves to repel the adhesion of any moist substance to its body, but also facilitates the motion of the animal, by lessening the degree of friction which would otherwise take place; and it is owing to the same circumstance that there is an unusual degree of difficulty in retaining a sure hold of the insect, even when dead; but more especially when alive, and struggling against detention. The degree of force which it commonly exerts on such occasions is very remarkable; and, from the sensation produced, may easily be supposed to be what RÖSEL says it is, equal to the counterpoise of two or three pounds. The skin or covering of the insect is in some parts nothing more than a thin membrane; in other parts it resembles soft leather; and sometimes equals horn or even shell in its degree of hardness.

The mole-cricket is more distinctly divisible than most other insects into three separate parts, which I will call respectively the head, the thorax, and the abdomen; although I am aware that the anterior part of that which I call the abdomen is usually considered as a part of the thorax. Of the three parts above-mentioned, the head is not above one-twelfth the length of the whole body; the thorax three-twelfths; and the abdomen eight-twelfths.

The head is united to the thorax, as the thorax also is united to the abdomen, by means of a loose membrane, which envelopes the muscles that pass respectively from one to the

other; and it is in consequence of the looseness of these membranes that the animal is enabled either to separate the connected parts to a considerable distance from each other; or to contract them so closely together as to hide the interposed membranes from view; and, from the arched form of the anterior part of the thorax it can draw in its head under that part, much after the manner of a tortoise. The same flexibility of the connecting membranes enables the animal to place either its head or its thorax at a considerable angle with the rest of the body; a movement which is very characteristic of this insect, and gives it an air of intelligence; the attitude being apparently that of watching, or listening.

*The head.** All the upper part and the sides of the head form a hard, thick, horny case, containing the various muscles which move the jaws; and, in order to strengthen this case, two firm bars run transversely across the bottom both of the anterior and posterior margin; which bars are themselves united together by a still stronger bar or beam, which runs longitudinally from the middle of the one to the middle of the other. There is nothing very remarkable in the parts which constitute the mouth, excepting the maxillary and labial palpi. In the maxillary palpi there are five joints or parts; in the labial there are three; and the last of these joints in each of the palpi terminates in a rounded extremity, like a pestle; this extremity, which is of a honey-yellow colour, is perfectly smooth, while every other part of the palpi has a rough and hairy surface. In their natural position the palpi are bent and projected forward, so as to resemble the fore-legs of a horse in the act of cantering.

* Vide Fig. 1 and 2. Plate XV.

The antennæ, which are situated near the articulation of the mandibles, consist of a great number of minute segments, resembling beads of a circular form: the number of these beads, which varies in different instances, is usually from 100 to 110; rarely more or less: but it is worth noticing that in examining the two antennæ of the same individual, I sometimes found the number of beads greater in one than in the other; and as the terminal bead differs in its form from all the rest, the result of the examination is less open to doubt than it would otherwise have been. Each bead is united to the one that precedes and the one that follows it by means of a soft, white, very flexible membrane; in consequence of which, and of the number of the joints, the insect can move and bend the antennæ with great facility in every direction, excepting at the very root: there the motion is confined by a ridge that only admits of its being directed from behind, forwards, or *vice versâ.*

The anterior edge of each bead is fringed with bristly hair; which, surrounding the joint that connects it to the following bead, gives to the whole, when viewed by a magnifying lens, the appearance of a sprig of equisetum. The beads are upon the whole larger, in proportion as they are nearer to the origin of the antennæ: but here and there, and without any regularity in the variation, one of the beads is either much larger or much smaller than those in the vicinity.

Whatever be the primary use of the antennæ and palpi, on which subject entomologists are not agreed, their general importance is allowed by all; and is evinced in the particular instance now before us by the extraordinary attention bestowed upon them by this insect. Those who may be led

to watch its habits, will repeatedly observe the antennæ bent forwards and downwards, by a curious application of the fore-legs towards the mouth: and then by a regulated motion, not unlike that by which the resin is applied to the bow of a violin, they are passed between the maxillæ: in order, as it would appear, either to moisten the organs, or to disengage from their surface, particles of dust or other extraneous substances which may have accidentally adhered to it. With a more rapid motion the insect from time to time dresses, if I may use the expression, its palpi; bending them inwards and brushing the surface of their extreme parts by a frequent application of the maxillæ. A similar care of the antennæ and palpi is observable in the gryllus viridissimus; with the additional circumstance, that that insect very often passes between its maxillæ the curiously padded surfaces of its feet, much in the same manner as a cat licks its paws.

*The eyes.** The gryllotalpa has two compound eyes, as they are called, and two ocelli or stemmata. Latreille uses this expression "ocellus medius subobiteratus;" from which it may be inferred that he supposes the ocelli to be three in number; but after the most careful examination I have not been able to discover more than two. The compound eyes are situated immediately behind, but a little exteriorly to the antennæ: the corneæ of these eyes, which are large in proportion to the size of the head, are segments of a sphere; flattened however on the inner side so as to present a vertical plane surface to a similar plane surface in the opposite eye; and it is remarkable that this part of the cornea, and the mere margin of the rest of it, are the only parts

* Vide fig. 1 and 2.

capable of freely transmitting light: all the remaining portion is covered, on the interior surface, by an opaque pulpy membrane, or pigments of a mulberry colour; yet the portion obstructed by this pigment is in itself nearly as transparent as flint-glass: it is studded over on the interior surface with numerous depressions of a circular form, which, being very closely set together, give it a reticulated appearance.

The stemmata are placed between the middle of the compound eyes, so as to be rather further from each other than from the eye of the same side. They are not so large as a very minute pin's head, of a lenticular form, perfectly transparent, but not quite colourless, resembling particles of very pale cairngorum quartz. In two instances I have found only one of the stemmata, without any trace of the other. An anomaly somewhat of the same kind has been observed by the father of my friend Dr. Ogle, of this University, in the case of a man; on one side of whose breast the usual rudiments of a mamma were entirely wanting.

With respect to the small quantity of light admissible through the corneæ of the eyes of the mole-cricket, it is apparently sufficient for the purposes of an animal living almost constantly underground. The spherical form of that part of the corneæ which is itself incapable of transmitting light is probably intended, as was suggested to me by Mr. Whessel, to whom I am indebted for the principal drawing which accompanies this paper, as a protection for the vertical transparent portion.

*The thorax.** The form of this part is that of an irregular

* Vide fig. 3 and 4.

cylinder, passing into a cone towards the anterior part: the upper portion and the sides, which are covered with a remarkably smooth down resembling the finest velvet, form a horny case of considerable thickness and strength; which contains, or, more properly speaking, is almost entirely occupied by the very large and powerful muscles which move the fore-legs. It is divided longitudinally into two equal parts by an almost bony septum of a complicated form: this septum upon the whole bears an obvious resemblance, but in an inverted position, to the deep sternum, together with the furcular clavicle of birds, and is destined indeed to a similar use; to give attachment to the powerful muscles which are to move the anterior extremities. It differs however from the corresponding part in birds in two considerable points. It differs, first, in consisting of two laminæ instead of one: these laminæ are parallel to, but distinctly separated from each other, so as to give passage to the esophagus, and room for the attachment of muscles which assist in moving the adjacent parts. It differs again from the sternum of birds by having a very hard spine, which resembles a common thorn, attached to the inferior and posterior edge of the furcular bone, and passing rather obliquely downwards and backwards. This process serves for the attachment of numerous muscles which adhere very firmly to it, and are inserted on either side of the commencement of the abdomen; enabling the animal to bend its thorax to an angle with the abdomen, a posture which has already been described as very characteristic of this insect.

From the under part of the thorax and near its posterior extremity arise the two fore-legs; those singular instru-

ments which so peculiarly characterize the mole-cricket. Compared indeed with the other legs, and with the general size of the animal, they are as if the brawny hand and arm of a robust dwarf were set on the body of a delicate infant; and the indications of strength which their structure manifests, fully answer to their extraordinary size: but I shall describe them more particularly hereafter, and proceed now to the description of the abdomen.

*The abdomen.** In its general form and structure this part resembles the corresponding part of the hornet: but it consists of more segments, and is much less bright in colour. There are twelve segments in the abdomen of the gryllotalpa, of which the nearest to the thorax carries the upper pair of wings on its upper part, and the middle pair of legs on its lower part; the next segment carries the under pair of wings on its upper part, and the hind pair of legs on its under part. These two segments which are usually described in entomological systems as belonging to the thorax, are of a horny consistence and very hard on their upper side; while all the rest are merely membranous; they are also covered with much long and rough hair, while all the rest, excepting the last but one, are sparingly covered with short hairs. The last segment but one is furnished on each side of its upper surface with a row of red hairs or bristles, which are curved inwards in a direction towards each other; obviously for the purpose of preventing the folded extremities of the under wings from falling off the back on either side.

The under surfaces of all the segments are of a thicker

* Vide fig. 2.

substance than the upper, and are covered entirely with a coarse down, which probably gives the animal a more firm hold while in the act of burrowing. In the last segment is situated the vent, formed by three oval flaps, two below, and one above. This segment sends out from each side of its upper surface two caudal antennæ, as I have ventured to call them, of a tapering form, which differ essentially in structure from those of the head, inasmuch as they are not jointed in any part of their extent, excepting at their very commencement: they are furnished with short hairs set comparatively closely about every part; among which are interspersed long single hairs. These caudal antennæ are evidently very sensible, and serve probably to give the animal notice of the approach of any annoyance from behind; they are partially hollow throughout great part of their extent, and muscles may be traced into them from the inner and adjoining part of the abdomen.

The legs. The anterior legs passing out from under the hind part of the thorax, advance by the side of the head in a direction parallel to each other, which is their natural position while the animal is at rest. I should deem it a servile adherence to system were I to describe the parts composing these legs by the terms strictly indicative of the order of their succession; for, thus, that part which answers so eminently to the character of a hand, must be called the tibia. I shall beg leave therefore to state principally that the foreleg of this insect consists of three main parts, with a lateral appendage attached to the last of them. The two first of the three parts bear some general resemblance to the claw of the crab; being short and thick, for the purpose of affording

room for powerful muscles, intended to move the last part; which is the immediate instrument employed by the animal in burrowing.

It might I think be asserted, without the fear of contradiction, that throughout the whole range of animated nature, there is not a stronger instance of what may be called intentional structure, than is afforded by that part of the mole-cricket which I am now to describe.*

The natural and constant position of this member is worth noticing; the palm, as it may be called, facing outwards, and the claws ranging not in a horizontal but a vertical line, so that none of them but the lowermost, and not even this necessarily, touches the surface on which the animal is walking. Accordingly the insect does not make much use of its fore-legs in walking; and, if irritated, it advances towards you with these legs elevated, in a menacing attitude as it were; not unlike the corresponding attitude of the insect, called the mantis. The form of the hand is that of a triangle; the base of which is formed by the four claws, while the apex is situated at the joint connecting this with the preceding part; by which form and disposition, two important objects are gained; for the joint is thus capable of a much greater extent of motion than it could have possessed, had the articulating surface been more than a mere point; and at the same time, the greater extent of the base enables it to act with more powerful and more rapid effect than could have been otherwise produced. The four claws, which form this base, constitute the proper burrowing instrument; and their shape and structure are beautifully adapted to the pur-

* Vide fig. 5.

pose: for instead of being covered with down or hair, like all the rest of the limb, they are hard, and have a perfectly polished surface; doubtless in order to prevent as much as possible the adhesion of the earth through which the animal is to make its way; they have each of them sharp but strong points, which proceeding from a broad base are thus rendered more effectual. In each also of the claws one of the edges is sharp, while the other is comparatively blunt; and all the cutting edges, as also the terminating points, are directed downwards. Their outer surfaces are slightly concave both in the longitudinal and transverse direction; so that all together they form a scoop as it were, by which the earth that has been scraped off by the points is moved out of the way. They are also each of them divided longitudinally on their concave side by three or four slight ridges; so that, though highly polished, their surface is not absolutely smooth: and thus being concave and uneven, they are more apt to retain particles of the excavated earth; which, by filling up the indentations of the claws would necessarily impede their due action. To obviate this inconvenience, an exceedingly curious instrument is attached to the upper part of the concave surface of this member: this instrument consists of two claws, closely resembling those already described, having by their side a small brush as it were, which terminates in two spines. These two claws, together with the piece bearing the spines, arise from a single piece, or handle, which is articulated in such a manner, as to move in a plane parallel to that in which the four claws are placed; but in a direction opposite to that in which they are moved: they are also placed in such a manner that their points and cutting

edges are opposed to the points and cutting edges of the true claws; and hence the two parts, thus opposed to each other, act like the blades of a pair of shears. When first I considered this mechanism, and remembered that in the localities where I had found the animal, the earth was frequently traversed by fibrous vegetable roots, which must necessarily retard its progress, I supposed that it used this instrument as a pair of shears to cut through those fibres. It is Rösel's opinion, however, that the instrument is intended to clear the true claws of the dirt that may from time to time collect upon and clog them; and unless both opinions be true, Rösel's appears the more probable. But I have not yet concluded the account of the curious mechanism of this member: for the brush which has just been described, has only such an extent of motion as enables it to clear the two uppermost claws, or at most, the three uppermost; the two lowermost however may effectually be cleared by a kind of feathered spur, which, arising from the further extremity of the joint answering to the femur, proceeds directly towards the lowest part of the burrowing instrument, and is easily made to sweep over the surface of the two last claws by bending the intermediate joint, the only difference in its mode of action being, that it passes over their inner instead of their outer surface.

The middle pair of legs, which are the smallest of the three pairs, arises from the under part of the first segment of the abdominal division: they pass out from the body at right angles to the abdomen, and usually are seen in that direction whether the animal be in motion* or at rest. They consist each of four parts; a very short coxa, a femur and tibia

nearly equal in length to each other, and a tarsus, which consists of two long and an intermediate short joint; the last joint terminated by two curved spines. There are several sharp, hard, straight spines near the angle made by the union of the tibia with the tarsus; some of which being directed downwards, give the insect a firmer hold in walking.

The hind legs bear a general resemblance to the middle legs; but the coxa, femur, and tibia, the femur especially, are much larger and stronger: the relative position of the parts with respect to each other is the same as that of the middle legs; but their general direction, instead of being at right angles to that of the abdomen, is parallel to it. In addition to several sharp spines placed about the joint of the tibia and tarsus, and directed downwards as in the middle legs, there are four or five others placed at the back of the tibia near its lower extremity, and pointing slightly downwards. The structure of the tarsus scarcely differs from that of the middle leg. These hind legs are evidently the great instruments of progressive or retrogressive motion.

The wings. There are two pair of wings: the upper pair arising from each side of the first segment of the abdomen partially cover the lower pair, which arise from each side of the second segment. In several instances I found adhering to the body, in the vicinity of the roots of the wings, a minute parasytic insect of a light scarlet colour; the number of these parasytic insects rarely exceeded eight or ten in the same mole-cricket, but in one instance I counted nearly forty.*

The upper wings in the full-grown mole-cricket are not

* Vide fig. 5 *a*.

above one-fourth the size of the other pair; they are of an oval form and convex externally; and their nervures or wing-bones, as they are called by Dr. Leach, are remarkably thick and hard.

The under wings when expanded, measure full three inches from the outer extremity of one to the corresponding extremity of the other. They may be compared in form to a bivalve shell, contracted and elongated towards the hinge, at which point is the joint of the wing; from hence, as many as thirty nervures, almost all of which are remarkably delicate, radiate in straight lines to every part of the extremity. A very thin and nearly colourless and transparent membrane forms the medium through which these nervures radiate; and throughout the whole expanse of the wing, these nervures are mutually united by more delicate nervures, which cross at nearly regular intervals, and at right angles from one to the other, presenting altogether the appearance of a curiously checquered surface. These wings, though so broad when expanded, are scarcely the twelfth of an inch in breadth when folded; and appear at first view, in this state, any thing but what they really are. They have indeed been often mistaken for a mere caudiform appendage to the other wings, from under which they emerge. When folded, and they fold themselves longitudinally like a fan, their very delicate texture is protected by the following simple contrivance. In each wing the two exterior longitudinal nervures, with their intervening membrane, are comparatively strong and thick; and these form the lateral walls of the wings when folded.

In each wing also there are two other nervures not far from the former, and circumstanced like them with respect

to strength; which, when the wings are folded, close together so as to form a horizontal covering, or roof, of sufficient strength to protect the subjacent membrane from ordinary accidents. As the narrow case formed by the wings thus folded extends beyond the extremity of the abdomen, and might easily slip off so convcx and smooth a surface, such an accident is guarded against by the contrivance already described, namely, an apparatus of hairs or bristles placed on either side of the upper surface of the last segment but one.

*The digestive organs.** It is mentioned in the 48th Letter of WHITE's Natural History of Selborne, on the authority of Anatomists who have examined the intestines of the mole-cricket, that "from the number of its stomachs or maws, there seems to be good reason to suppose that it ruminates, or chews the cud like many quadrupeds." A cursory view of these parts however is enough to show, that such an opinion could only have been deduced from some very general points of resemblance, and the probability of its truth is entirely destroyed upon an examination of their internal structure.

In fact, the digestive organs of this insect resemble more closely those of a granivorus bird than of any other animal, as will appear from the following description. The esophagus, which on its upper side is blended with, and forms a continuation of the inner surface of the upper lip, commences on the lower surface in a loose corrugated tongue, as it were, which is attached at its base to the inner surface of the lower lip; from hence it is continued along the under part of the head and neck, and between the bony laminæ of the sternum,

* Vide fig. 6.

in the form of a distensible and longitudinally folded tube of a reddish brown colour; it then passes on among the muscles of the two hind pair of legs, and at length terminates in a very large crop of an oval form. In the vicinity of the mouth it is surrounded by muscles which arise from its outer coat, and are inserted at nearly right angles into the adjacent parts; these muscles of course serving to open and distend it.

In the crop itself two sets of muscular fibres are very easily discernible, some running in the direction of its length, others surrounding it in the opposite direction; and it is lined by a very thin membrane having a cuticular character.

The tube which passes from the crop towards the intestines commences so near the termination of the esophagus, that externally it appears to be a continuation of the latter; it is very thick and strong in comparison with its diameter, and consists of a coat of muscular fibres disposed circularly, lined by a membrane which has evidently a glandular character. This tube terminates at a short distance from its commencement in a small organ, scarcely larger than a hemp-seed, which may very properly be called a gizzard; though more complicated in its structure, and more effectual for the intended purpose than the gizzard of any bird.

The form of the gizzard is nearly spherical, and it consists of a thick external muscular coat, which is lined by a glandular membrane of very singular construction; the inner surface being divided longitudinally into six equal parts, separated from each other by two horny ridges of a dark brown colour; each division is furnished with three series of serrated teeth, of the consistence of tortoise-shell, and nearly

of the same colour, running from the top to the bottom; of which those of the middle series are twice as broad and more complicated in form than those of the lateral series. As there are fifteen teeth in each of the three series of the six divisions, the gizzard contains in the whole 270 teeth.* In separating the muscular coat of the gizzard from that which lines it, which may be easily done by maceration, the exterior surface of the glandular coat in which the teeth are inserted is exposed to view. The appearance of this surface is very singular, and may be compared to a piece of fine lace-work, of which the meshes represent the intervals of the inserted teeth, the parts of the membrane in which the roots of the teeth are inserted resembling the lace-work itself.

Four of the divisions above described are elongated so as to terminate in a tapering membranous appendage, consisting of a natural fold, which serves to convey onwards any fluid particles that may have been pressed out by the action of the gizzard; and these four appendages so collapse together as to form a point, as it were, which lies immediately in contact with the commencement of the common intestines. This apparatus is only discoverable by dissection; for it is contained in a large membranous cavity of the shape of a horse-shoe, the base of which passes across the lower extremity of the gizzard, while the sides form two enormous cæca, which ascend obliquely outwards on each side of the gizzard.

As the muscular compression of the gizzard must necessarily have a tendency to force a part of any expressed fluid back into the esophagus, we may expect this organ to be so constructed as to prevent such an effect; and it is pro-

* Vide fig. 6, 7, 8.

bably for this purpose, that its upper part is furnished with several projecting papillæ, each terminating in a small horny particle; which, like the sesamoid particles in the semilunar valves of the human aorta, may serve to complete the valvular action of the papillæ to which they are attached.

The cæca which have been above described, are traversed longitudinally by several very broad duplicatures of their internal membrane; and judging from their usual contents, these appendages of the intestine are destined to receive and to perfect the digestion of those particles of food from which the gizzard has pressed out the liquid contents; and while, by means of the membranous folds already described, the expressed fluid is conveyed immediately into the mouth of the intestinal canal that passes from the general cæcal cavity, the cæca themselves receive the solid compressed particles which are forced out laterally at the extremities of those two divisions of the gizzard, which, having no membranous fold attached to them, leaves thus a vacant interval for the passage of the undigested mass. That this opinion is correct may be presumed, not only from the very mechanism of the parts, but from the state of the contents of the cæca, which are of a less crude character than the contents of the crop, and of a more crude character than the contents of the portion of intestine immediately beyond them. A strong confirmation of the foregoing opinion is obtained from a comparison of this part of the anatomy of the mole-cricket, with that of the corresponding part in the ostrich; the stomach of which bird, acting like a gizzard by means of numerous pebbles which it takes into that organ, is aided by two enormous cæca, which, though they are not immediately in contact

with the stomach, are not far removed from it; and like the stomach, contain numerous pebbles, which are both smaller and smoother than those of the stomach itself, as being only destined to act on food already partially digested. The analogy on which I have just insisted, is strengthened by the fact, that there are very large duplicatures of the internal coat of the cæca of the ostrich, as in the corresponding parts of the mole-cricket. I either therefore misunderstand, or cannot agree with M. MARCEL DE SERRES, the author of a very interesting paper on the Intestinal Canal of Insects, published in the 76th vol. of the Journal de Physique; who seems to attribute to the æca above described, the office of an hepatic organ, and calls them "*Vaisseaux hépatiques superieures,*" in contradistinction to another organ situated lower down in the intestines, and acknowledged by all to be of an hepatic character.

From the common base of the two cæca a very narrow but powerfully muscular tube, which might with much propriety be called the jejunum, passes onwards for a very short space, and terminates in a large intestine; this intestine, which is eight or ten times the diameter of the jejunum, contracts very gradually as it proceeds, till, near the extremity of the rectum it swells out very considerably. This large intestine is slightly convoluted in its course, and is usually more or less distended with a black pasty matter resembling soft clay. Among the contents of the upper part of this large intestine were almost invariably found from ten to twenty worms, of a white colour, and of a shape resembling the lumbricus teres of the human intestines, but thicker in proportion to their length, and narrowing more suddenly

towards their caudal extremity. In all of these worms the common intestines were distinctly visible through the integuments; and in many of them were distinctly visible also from ten to fifteen ova.*

On opening and removing the contents of the upper portion of the great intestine, four rows of minute bodies of a glandular character,† and of nearly a black colour, are brought into view; ‡ two of which rows originate from the very commencement of the great intestine, and pass downwards through more than half its course: exteriorly to these two rows are two others, one on each side, which are parallel to the preceding, but originate at some distance from the commencement of the intestine. Immediately below the termination of this glandular apparatus is a small opening, very readily distinguishable on the inner surface of the intestine; which is the orifice of a cylindrical tube of a white colour, and of about the size of a horse hair. This tube, after having been traced a short distance in a direction towards the gizzard, is lost in a mass or brush of still smaller tubes of an exceedingly bright yellow colour; these tubes, which amount probably to 150 or 200, § are partially coiled round the contiguous viscera so as not to be very easily disentangled. A

* Vide fig. 9.

† The only doubt which I entertain as to the glandular character of these bodies, arises from a reliance on the authority of Cuvier, who says, that the glands of insects are in every instance nothing more than parcels of free tubes floating in the interior of the body, and held together by the tracheæ." Journ. de Phys. Tom. 49. p. 344. ‡ Vide fig. 9 *a*.

§ Cuvier states in the Journal de Physique, Tom. xlix, p. 346, that the number of these tubes in the gryllotalpa amounts to many hundred: but I feel certain that he greatly overrates the number.

similar organ is represented in Sir EVERARD HOME's Comparative Anatomy, vol. 1. pl. 84, as belonging to the Cape grasshopper; it was originally considered by Mr. HUNTER, and is considered generally at present, as answering to the liver of the higher classes of animals.

Each of these tubes springs out of a common cavity in which the white tube from the intestine terminates; but at their free extremity they are all impervious. Each tube appears partially filled with a granular pulpy substance which is almost universally of a bright yellow colour; though sometimes a particle is visible here and there of a clear light green colour, and I have seen similar green particles in the duct leading from the intestines.

The following peculiarity is observable in the individual structure of these tubes: their diameter for about one-third of their course from the closed extremity is very small, and they are colourless, and apparently empty; after which they suddenly undergo a considerable enlargement, become yellow, and are partially filled with the contents above described.

Maceration in water destroys the yellow colour in the course of a few minutes; from whence it may be inferred, that after death the colouring matter transudes through the tubes containing it—a circumstance observable also with respect to the biliary vessels of the higher orders of animals; but it seems certain that no such transudation takes place during the life of the animal; for, upon examination of the insect soon after death, I have never found the adjacent parts coloured, as they would have been by the escape of the contents of the tubes.

The portion of the intestine below the orifice of the hepatic duct, as it may be called, appears to be externally traversed in a longitudinal direction by several rows of small convex eminences resembling beads; these are the outer surfaces of so many corresponding internal sinuses, which are probably formed as the similar sinuses in the large intestines of man, and many other animals, by a pecularity in the disposition of the fibres of the muscular coat.

Near the termination of the intestine are two orifices, one on each side, communicating each with a duct which soon swells out into a vesicular bag; these bags may probably be glands that secrete the fetid matter which the insect ejects from the anus when irritated. In one instance I found, on the site of the orifices above-mentioned, two small bodies about the size of a pin's head, of a dark colour, and to the naked eye of a spherical form; my surprize was considerable when upon observing them with a magnifying lens, I perceived that they exactly resembled a crystallized rosette of brown pearl-spar. Upon being removed and submitted to the requisite experiments, they proved to be of considerable hardness, sparry in their structure, and insoluble either in boiling water or alcohol; but they were dissolved with rapid efferverence in diluted muriatic acid. These calculous concretions were probably the result of diseased action in the vesicular glands round the orifices of the excretory ducts of which they had been deposited.

The blood. Upon wounding the animal in almost any part of the body, even in cutting off a portion of the caudal antenna, there oozes out a very clear thin fluid of a bright honey-yellow colour; having sensibly alkaline properties,

and coagulating either by heat or by the addition of alcohol. A quantity of this fluid, weighing 1.85 grains, being evaporated under an exhausted receiver, in which was placed dry muriate of lime, left a solid residuum of a bright golden yellow colour, which weighed 0.25 grains; this residuum was brittle, and had the general properties of solid albumen. The foregoing characters render it highly probably that the yellow fluid distributed through the body of the insect, resembles in its nature the serum of common blood, and there can be no doubt, arguing physiologically, that this yellow fluid is the blood or nutrient juice of the animal. I wish I could as satisfactorily show the means employed by nature to distribute this fluid through the system of this and other animals of the same class; for, though I cannot hope to discover what more experienced and skilful anatomists have sought in vain, a heart, namely, and a system of circulating vessels; yet I cannot subscribe to their opinion, that the blood transudes through the the coats of the intestines, where of course it must be primarily formed, and thence passes, as through the pores of a sponge to every part of the body. Both Cuvier and M. Marcel de Serres completed a very elaborate set of experiments for the purpose of ascertaining whether the dorsal vessel of insects sends out any lateral branches which might serve the purpose of a circulating system, or whether any other distinct circulating system exists; but they have entirely failed in their endeavours; and I feel assured, that where such men have failed, others will not succeed; and yet their consequent supposition that the blood is diffused through the general substance of the body, appears to me very highly improbable. It accords not with

the general character of those means by which nature usually produces its effects; there is too little of art and contrivance, if I may use such terms, on such an occasion, in the mode supposed to be employed. Even in the formation of mineral crystals, which are unorganized bodies, the attraction by which the component particle are aggregated is regulated by laws, the most systematically framed and observed: and whoever has viewed with any attention that wonderful monument of human industry and sagacity, the Anatomical Museum of JOHN HUNTER, and has there seen the proofs of a sanguineous circulation in animals of an order so low, that they can hardly be said to have any specific form or substance, will almost necessarily be disposed to expect a similar provision in a class of animals, whose general structure is so elaborately and beautifully organized as that of insects. But I shall again advert to this subject after having described the tracheal system or respiratory organs of the insect under consideration.

The organs of respiration. As it is very generally known that the atmospherical air, so necessary for the existence of all animated beings, is admitted into the bodies of insects by certain apertures called stigmata, and is then distributed through the system by means of tracheæ or air tubes, I shall not dwell longer on the description of those organs in the gryllotalpa than is necessary for the elucidation of its particular history.

Omitting the questionable existence of two stigmata in the upper lip, and of two others in the vicinity of the caudal antennæ, there are ten stigmata very distinctly visible on each side of the body.* Hence, therefore, it is necessary to

* Vide fig. 10.

correct, though probably it has ere this been corrected by himself, a statement made by CUVIER in his Règne Animale, Tom. iii. p. 126, that in the myriapoda there are twenty stigmata and upwards; but in all other insects eighteen at most. He also asserts in the same place, that insects respire by two principal tracheæ extending longitudinally, one on each side of the body, from which other tracheæ ramify. Now certainly in the gryllotalpa, and, as I have reason to believe in many other insects also, the longitudinal tracheæ bear so small a proportion in their capacity to the aggregate capacity of the other tracheæ, that in such instances they cannot be called principal tracheæ. My own opinion is, that these longitudinal tracheæ serve as connecting channels, by which the insect is enabled to direct the air to particular parts, for occasional purposes.

Though not immediately bearing on the present point, I beg leave here to state a fact which I have not seen elsewhere noticed, that in the two segments of the body which carry the middle and hind pair of the true legs, in the larvæ of coleopterous and lepidopterous insects, there are no stigmata, discernible at least either to the naked eye, or a common magnifying lens.

But, to return to the stigmata of the gryllotalpa, the first in order beginning from the head, is situated very near the lower part of the posterior ridge of the thorax. This stigma, not to object to the term in the present instance, is apparently connected with all the tracheæ both of the thorax and of the head itself. It differs remarkably in size and form from all the rest; for instead of being a mere dot or point, it is an elongated fissure, bounded by two horny lips. The second stigma, which somewhat resembles in form, though of much

less extent than the preceding, is situated immediately behind the root of the middle leg; the third, which is still less than the second, is situated immediately behind the root of the posterior leg; near the termination of the dorsal part of the third abdominal segment; the fourth, fifth, and onwards to the tenth inclusive, are situated near the terminations of the corresponding dorsal segments of the abdomen.

I would here notice by the way, a peculiar appearance very constantly observable on the ventral surfaces of most of the abdominal segments between the hind pair of legs and the caudal antennæ. At either extremity of those segments there is a short line, not unlike that made by the stroke of a pen, passing obliquely downwards and inwards: it does not seem easy to conjecture the use of these lines.

I may state from repeated observations, that the stigmata, taken generally, are not the terminations of single tubes; very frequently two and often more than two tracheæ originate from the same stigma; and very soon after the commencent, one or even two of these tracheæ subdivide into numerous branches, which follow as nearly as may be the direction of the original tubes.

The distribution of many of the tracheæ may be very satisfactorily demonstrated by drying one of the insects under an exhausted receiver, containing muriate of lime: for after having been thus dried, the tracheæ become perceptible to the naked eye through the sustance of the integuments. The foregoing method of drying anatomical preparations may be successfully employed on many occasions; it answers particularly in the case of the human eye, or the eye of any sufficiently large animal; for, in the act of exhaustion, the air contained

in the vitreous humor of the eye becoming expanded, preserves the spherical form of the organ until the whole of the moisture has been evaporated; and it is then sufficiently firm to support itself. I have traced most of the tracheæ to the parts on which they are respectively distributed; but as no adequate object, nor indeed any object of importance, would be gained by the description of a distribution which is not marked by any physiological peculiarity, I shall only insist on such points as appear to me to be either new, or hitherto not sufficiently elucidated.

The tracheæ of insects are generally described as tubes constructed of a spiral thread, the successive coils of which are closely in opposition with each other; such a structure is represented in SWAMMERDAM'S plates, and I have no doubt from his acknowledged accuracy, that he represents what he observed. It has not however happened to me, with the exception of one equivocal instance, to perceive such a structure in the mole-cricket, the character of the tracheæ of which varies in different parts of the insect; for sometimes they resemble the pulmonary tracheæ of the higher classes of animals, in having an annulated structure; and sometimes they appear as tubes of a perfectly uniform substance like cuticle, or some very thin and unorganized membrane. It is generally understood, that the tracheæ of insects penetrate each organ and every part of the body: and certainly the case is such in the instance before us. Thus, in that brush of capillary yellow tubes supposed to constitute the hepatic system, the total number of which amounts to 150 or 200, there is reason to believe that each tube is accompanied by a distinct trachea coiled round it in a long spiral. Again; the

two medullary cords which connect the several ganglions of the nervous system, are in their natural state united together by means of the branches of a tracheal tube which runs between them; a similar tube being attached to the exterior edge of the cords; and the surface of what may be called the brain of this insect, is as beautifully characterized by the ramifications of the tracheæ which pervade it, as the surface of the pia mater of the human brain by the blood vessels which penetrate that membrane in every direction.

In meditating on the difficult problem of the sanguinous circulation of insects, it has forcibly occurred to me, that the tracheæ may possibly be the instruments of such a circulation; absorbing the blood or the chyle in the first instance from the internal surface of the alimentary canal, and thence conveying it to the various parts of the body; nor is this opinion, however improbable it may appear, entirely gratuitous. No difficulty, I apprehend, attaches to the supposition that such an an absorption may take place; seeing that innumerable minute ramifications of the tracheæ penetrate the intestinal canal in every part: nor does there seem any difficulty in admitting that the insect may, by the power of exhausting the air from individual tracheæ, draw on the absorbed fluid towards those two lateral tracheal tubes, which are apparently a general medium of communication between all the other tracheæ of the body. And, when once the blood has reached this supposed point of its course, it is manifest, that by whatever means the air itself is forwarded from the same point to the most distant parts of the body, by a modification of the same means, the blood may be forwarded to the same part; and the elegant proposition of CUVIER,

that "the blood being incapable of going in search of the air, the air goes in search of it," will still remain inviolate.

If it should be argued that the tracheæ are not found charged with blood after the death of the animal, it may be answered, that neither are the arteries in the higher orders of animals found charged with blood after their death. However, I have actually seen some of the ramifications of those tracheæ which are connected with the cæca distended with a fluid of the same colour as that found in those organs; and though I have only witnessed this fact in two instances; yet such a fact, even singly taken, must be allowed to be of considerable importance.

Of one thing I am certain, that, after careful observation, I have never found the abdominal viscera, I will not say bathed, as some authors of credit have expressed themselves, in the nutrient fluid which is supposed to have transuded through the coats of the intestines; but I have not even found them lubricated by a greater proportion of moisture than lubricates the intestines of the higher classes of animals.

There is another difficulty which occurs to the hypothesis of the transudation of the chyle through the coats of the intestines; for, if the blood be conveyed to the several parts by previous general diffusion through the interior of the body, and then by absortion into the substance of particular organs, as the hepatic tubes, the vesiculæ seminales and the ovaries; how does it happen that the bile, for instance, does not transude through the coats of the same vessels, the pores of which have admitted the blood from which it has been formed? It may be answered, that the alteration which the

blood undergoes in the several organs, changes its properties to such an extent, as to render it incapable of repassing through the pores which admitted it. I cannot of course presume to say that such is not the case; and I am aware that many entomologists will be surprised at, and perhaps disinclined to listen to the opinion here advanced with respect to a sanguineous circulation in insects; but I nevertheless hope that the opinion will not be rejected without some previous attention to it. With regard to the dorsal vessel of the gryllotalpa, which in this, as in other insects, has been supposed to stand in the place of an arterial heart, I have very few observations to offer. It does not agree in its form with the description commonly given of this mysterious organ; for though it diminishes in diameter as it approaches the head, this is by no means the case towards the other extremity of it. I have not yet completely succeeded in tracing this vessel to its anterior extremity; because as it approaches its termination in that direction, it becomes so delicate as to have hitherto broken under dissection before I arrived at the extremity of it. Towards the opposite extremity it gradually becomes larger from the centre of the body, and terminates apparently in a cul de sac about the last segment but two of the abdomen.

The muscles. In the gryllotalpa, as in insects in general, the muscles are exceedingly numerous, and usually very distinctly defined; but as their form and size in different parts of the body may, without difficulty, be conjectured from the form and size of the parts to which they are appropriate, I need not occupy the time of the Society by enumerating or particularly describing them. Those which move the fore

legs are remarkable for their size, and apparently fill nearly the whole of the interior of the thorax. Some muscles, as is the case with two belonging to each mandible, and with some of those that are situated within the thigh of the hind leg, have tendons attached to them of considerable extent and strength. I must not omit to mention several parallel muscular bands, which run in a longitudinal direction along the outer coat of the extremity of the great intestine, and are inserted into what may be called the sphincter of the rectum: these muscular bands may evidently assist, by their previous contraction and subsequent relaxation in discharging that foetid matter, which as has been already said, the animal usually emits when irritated. For the discovery of these muscles I am indebted to Mr. Whessell, whose name I have before mentioned on a similar occasion.

*The nerves.** In removing the integuments throughout the whole length of the lower surface of the body, we discover a series of nine ganglions, of a pale cream colour, distributed at unequal intervals from the commencement of the esophagus to the termination of the rectum; a double medullary cord being continued from one ganglion to another throughout the whole series. The ganglions and their connecting cords lie so nearly in contact with the common integuments, that great care is requisite, lest, in removing these integuments, the nerves themselves should be removed, or at least injured. The first of these ganglions, reckoning from the anal extremity of the abdomen, is globular in its form; and is situated between the intestine and the sexual organs, the latter being placed immediately under the ventral integuments. This

* Vide fig. 11 and 12.

ganglion gives off several pairs of nerves, of which by far the largest pair may be traced into the caudal antennæ. The second, third, and fourth ganglions are smaller than the first, and are of an oval rather than a globular form: they each send out from two to four or five pairs of nerves. The fifth and sixth ganglions of which the former is the smallest, the latter the largest ganglion, of the whole series, are situated so closely together, that it not always easy to demonstrate the connecting medullary cords. The sixth ganglion, which from its size and the number of nerves radiating from it might be called the solar ganglion, is situated between the roots of the posterior legs. The seventh and eighth ganglions are situated respectively between the roots of the middle and the fore legs.

From the eighth ganglion, which lies under the furcular bone of the sternum, two parallel medullary cords pass on to the root of the mandibles, where they unite with the ninth and last ganglion, which is situated under and in contact with the commencement of the esophagus. This ganglion, which is hollow, as perhaps all the others may be, sends off nerves to the maxilla and adjacent parts: and it sends off besides, two large and important branches which ascending on each side of the esophagus unite with two corresponding branches that descend from the brain; which organ is situated immediately in contact with the commencement of the esophagus on its upper surface: so that the esophagus is placed between the ninth ganglion on its lower surface, and the brain on its upper surface, their connecting branches completing the nervous collar which surrounds it at this part.

The brain differs in colour from the ganglions, being of a pale brownish pink, instead of a cream colour, and in size it far exceeds the largest of the ganglions. It consists of two hemispheres, separated by a fissure, from each of which pass out four processes; the first of these processes unites as above described, with a process from the ninth ganglion, to form the nervous collar of the esophagus; the second passes to the root of the antenna; the third, which may be called the optic nerve, passes towards the inner surface of the cornea; and at its extremity swells out into a fringed coronet of an orange red colour; the fourth process, the extremity of which is also of an orange red colour, proceeds to the ocellus or stemma of the corresponding side.

The upper surface of the brain is covered by a mass of soft substance somewhat resembling loose fat.

*The sexual organs of the female.** These organs consist of two ovaries, which occupy a considerable portion of the upper part of the abdomen, and terminate by a narrow duct in a common cavity or uterus, which opens externally under the posterior edge of the last segment but one of the ventral surface of the abdomen. Behind the uterus is an oblong white body, which originating from a cul de sac, and then doubling on itself in the form of a slender tube, terminates in the uterus. The contents of this body resemble a thin white paste. The ovaries are irregularly pear-shaped, and consist of a transparent membrane irregularly convoluted, through which the ova, enveloped in a gelatinous medium, are easily distingushed. In the same ovary the ova are frequently of different sizes and colours; those which are the

* Vide fig. 13.

largest, and which I suppose to be impregnated, are of a brownish yellow colour; they resist a considerable degree of force before they burst, and the contents when pressed out melt as it were into a soft jelly, leaving a tough membrane which enveloped them. The smaller ova are of various sizes and of nearly a white colour, and of a much more slender and compressed form than those which I have supposed to be impregnated. This difference in the degree of maturation corresponds with a fact stated by RÖSEL, that the mole-cricket does not deposit all the eggs of the season at one time. In a few instances I found two or three ova which had entered the narrowest part of the duct and were very near the uterus; and from the appearance of these, which may fairly be supposed to be, if not impregnated, at least in a state fit for impregnation, I have ventured to derive the character of the impregnated ovum.

*The sexual organs of the male.** I had dissected several male gryllotalpæ before I was fortunate enough to meet with the sexual organs fully developed; and while I had as yet met with only one animal bearing the character of full developement, I was not certain whether I judged rightly of the natural state of those parts; or whether their uncommon degree of enlargement were not the effect of disease—the disproportion in size between the state in which they had hitherto occurred, and that to which I now allude is so enormous. However, subsequent dissections presenting the same phenomena, I have no scruple in considering them as indicating full developement.

The testicles of the male are situated similarly to the

* Vide fig. 14.

ovaries of the female, and are not very unlike in general appearance to the ovaries of young females; they differ however in being divided pretty deeply into several unequal lobes, the free extremities of which look towards each other. They send out each a very fine capillary tube or duct; which, descending towards the rectum, is in one part of its passage convoluted on itself so as to resemble the human epididymis partially unravelled.

The excretory duct above described terminates at the bottom of a thick pouch, which is situated between the rectum and the ventral integuments, and in form is not very unlike, though larger than the uterus, opening externally, as the uterus does, under the posterior margin of the last but one of the ventral segments of the abdomen.

The interior mechanism of this pouch is extremely curious; for in the upper part there is contained an apparatus somewhat in the shape of a coronet, of the colour and hardness of tortoise-shell: and at right angles to the centre of this there is fitted a similarly hard and horny substance, (in shape resembling a short flat club,) which descends towards the external opening of the pouch.

Behind the pouch are situated one on each side, two oblong white bodies, which are twisted into three spiral coils, and then terminate by an inflected tube at the upper and back part of the pouch. These bodies evidently answer to the vesiculæ seminales of insects in general: and resemble in their external character, and in their white pulpy contents, that oval body which is placed at the back of the uterus. There is also another pair of vesiculæ seminales, as is frequently the case in insects, situated exteriorly to the former;

more slender in form, also and much more convoluted, which apparently terminate near the points where the ducts of the testicles terminate. In the instances of full developement these bodies are enlarged to six times their usual size. Under the circumstances of full developement there is also found, though scarcely perceptible under imperfect developement, a large spherical mass, resembling a ball of eider down, situated immediately at the anterior edge of the pouch above described, and continued on from its substance.

The examination of the mole-cricket has added, as appears from the description of the parts, another exception in the case of the female as well as the male to the general statement, that in insects the sexual organs pass out by the anus: Cuvier mentions, as the only exceptions to this law, the Iuli and libellulæ.*

Casting of the skin. The following are the only observations I have had an opportunity of making as to this point of the history of the mole-cricket. In the process of moulting, the skin of the abdomen appears to split longitudinally down the middle of the upper part; and the skin of the thorax separates in a similar direction; but the skin of the head only separates partially in that direction, and then splits bebetween the stemmata, in a direction towards each of the antennæ; so that the line of separation somewhat resembles the lambdoïdal suture of the human skull.

The corneæ of the eyes are cast with the rest of the skin, as in the case of the snake; but they lose their transparency, and become of a greyish white colour,

Even the covering of the claws is cast.

* Règne Animale, Tom, iii. p. 137.

The newly exposed surface of the whole body is covered with the same kind of down as that which covered the preceding skin; except in the case of the long bristly hairs of the caudal antennæ, which apparently are produced afterwards. The colour of the body immediately after the casting of the skin is yellowish white, and it remains of that colour for a few hours: it afterwards gradually darkens.

The organ of sound. I have very little doubt that the peculiar sound which is charcteristic of this insect is produced by the wings; for I have observed in several individuals in their perfect state, that, when irritated, they will separate their upper wings by a brisk motion laterally from each other; and that upon their being suddenly brought back to their natural position, a sound is at the same moment produced, resembling that which I have heard the insect spontaneously produce during the season of summer; but I could not fix the power of producing this sound to either sex exclusively.

There is a peculiar organ, forming a part of the common integuments of the abdomen, and situated between the fourth and fifth stigma on each side; the anterior portion of which consists of a tense membrane, like fine parchment, of a semilunar form; this organ from its individual character might be supposed to contribute towards the production of the sound, but it is found in the female as well as in the male; and its supposed use is not justified by the presence of any internal mechanism.

In two or three instances I have perceived the internal and upper surface of the second abdominal segment, answering to what is generally called the third thoracic segment, furnished with two oblong concave laminæ, terminating in free

rounded edges, which are probably elastic; but I feel by no means certain that these are exclusively characteristic of the male, though I certainly found them most distinctly developed in a male individual.

But my acquaintance with the interesting insect, the history of which has formed the subject of this paper, did not commence till towards the close of that period of the summer during which the animal is heard to produce its peculiar sound: and I propose therefore to resume the investigation of this point at a future opportunity.

Oxford, Nov. 13, 1824.

Dimensions of a full grown mole-cricket.

	Inches.
Length of the body from the extremity of the lip to the extremity of the vent - -	2.0
Length of the head - - -	0.165
——— thoracic division - - -	0.5
——— abdominal division - - -	1.33
Breadth of the thorax - - - - -	0.5
——— abdomen - - - -	0.5
Length of the antennæ of the head - -	0.825
——— caudal antennæ - -	0.666
Length of the whole alimentary canal - -	2.0
——— esophagus - - - -	0.5
Length from the crop to the great intestine -	0.5
Length of the great intestine - - -	1.0

EXPLANATION OF PLATE XV.

Fig. 1. Skeleton of the head, viewed from the under side.

Fig. 2. A side view of the animal in its common attitude.

Fig. 3. Skeleton of the thorax.

Fig. 4. Sternum, &c. with the upper part of the thorax adhering.

Fig. 5. Exterior surface of the left fore leg.

Fig. 5[a]. Parasitic insect infesting the roots of the wings; of its natural size, and also enlarged.

Fig. 6. Esophagus, crop, gizzard, cæca, great intestines, hepatic organ, and anal glands.

Fig. 7. Interior view of gizzard.

Fig. 8. Ditto of a portion of ditto.

Fig. 9. Intestinal worm of the mole-cricket; natural size, and enlarged.

Fig. 9[a]. Upper part of great intestine, with four rows of glands, and the orifice of the hepatic duct.

Fig. 10. The stigmata of the left side; with the organ (situated between the fourth and fifth stigmata) described in page 224.

Fig. 11. The nine ganglions.

Fig. 12. The brain, surrounding the esophagus.

Fig. 13. The female sexual organs.

Fig. 14. The male ditto.

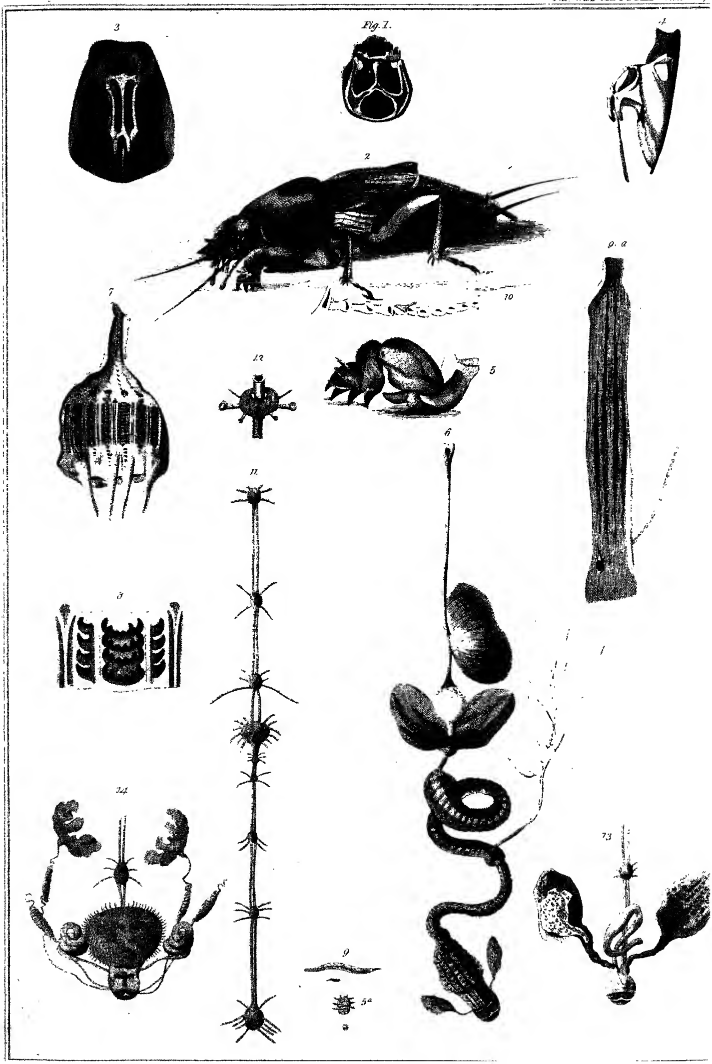
Fig. 1.
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XI. *Further observations on Planariæ.* By J. R. Johnson, *M. D. F. R. S.*

Read March 10, 1825.

About three years since I presented to the notice of the Royal Society a few observations on the genus *planaria*. From that period to the present having had no opportunity of extending my researches to more than two or three species in addition to those formerly described, I was unwilling to trespass upon the time of the Society by any further remarks, until I had ascertained the remaining species of this genus.

A circumstance, however, attending some experiments in which I have been lately engaged, of rather a strange character, forming another interesting feature in the history of these very extraordinary animals, induces me to lay before the Society the present communication.

The circumstance to which I allude, is that of the *P. cornuta* obtaining a second or additional head by an artificial incision, thus constituting a *double headed planaria*.

At the period of my transmitting my former paper to the Society, I was not aware that any English author had written upon the same subject; but was afterwards much surprised on learning that a Gentleman of Edinburgh, Mr. Dalyell, had published an account of these animals in 1814, in a work, having for its title "Observations on some interesting Phœnomena in Animal Physiology, exhibited by several species of *planaria*." Failing to procure this work at the booksellers,

I was at length fortunate in obtaining it through the liberality of its very ingenious author, who obligingly presented me with his only remaining copy.

This Gentleman, after noticing the considerable reproductive powers of the *planariæ* in general, but more particularly conspicuous in that species he terms the *P. felina*, and which from his description I conjecture to be the *P. cornuta*, observes, that having occasionally seen some of these creatures deviate from their natural figure in having two tails, &c. (an event however of so rare an occurrence, that I have in no instance met with such in the many thousands submitted to my inspection) it occurred to him that monstrosities of this kind might be obtained by artificial means, founding the practicabily of this measure on what had passed under his reviews. One of these monstrosities he thus describes, "the planaria in relation to others was of small size, its tail was bifid, and out of the cleft grew a body, separated and distinct from the main trunk of the animal, which by some strange and anomalous proceeding had been surmounted by a head, lively and well defined. In subjecting this planaria to the microscope, numerous black specks, the supposed eyes, appeared surrounding the larger head, and they environed the margin of the smaller head also. In the course of a week or little more the posterior head had separated by spontaneous division, and had disappeared. But soon afterwards a kind of projection occupied its place; and it was not without amazement that I beheld this projection vegetate into a new head, resembling the one which had been lost. About a month having elapsed, it was well shaped and entire. My belief being thus corroborated in the probable effect of experiment,

it was reasonable to conclude, that if separating parts became complete animals, if a mutilated trunk regained the defective portion; and if a head, the most important of all organs, was evolved from every inconsiderable fragment, supernumerary parts might, by some particular operation, be produced; yet it was long before *reiterated trials* were rewarded *with success*, and I had almost determined to abandon the enquiry, conceiving that a certain nicety, of which I was not master, should be practised, and that it had been beyond my ability to detect the secret cause of failure."

Notwithstanding the unpromising commencement of this Gentleman's labours he still persevered, and at length noticed, that one of the planariæ upon which he had made an incision a little below the head, had, to quote his own words "an unnatural prominence, which interrupted the general contour of the side. October 25th, nearly four weeks after the operation, the superfluous reproduction was clearly recognised to be the rudiments of a new head. On the 18th of November the operation of nature was fully accomplished; a new and perfect body crowned by a head had grown out of the side of the parent animal, distant about two thirds of the total length from the extremity of the tail."

The work, from which the above extracts have been copied, being now out of print, I have taken the liberty of transferring the delineation of this *double-headed planaria*, under a magnified form, to the drawing accompanying this paper. Vide Plate XVI. fig. 3.

With the view of ascertaining whether, in my hands, these experiments would prove equally successful, I took the ear-

liest opportunity of putting them in practice, but with evident mistrust as to the result (not however in the slightest degree doubting the accuracy of the above report), conceiving that no circumstance of this nature had yet occurred, in the many and repeated experiments I had performed upon these animals during the past and the preceding summer.

Having a considerable number of the *P. cornutæ* in my possession, I took at least one hundred of the most active, and made an incision on the side of the body in each, but only succeeded *in one solitary instance* in obtaining the wished for result.

Looking over these planariæ after the lapse of nearly a fortnight, I discovered that the incisions had, in by far the greater number healed, so that no evident difference existed between them and perfect unmutilated planariæ. Preternatural excrescenses had taken place in several, and others had separated at the place of incision so as to become two animals, but *only one planaria*, as before noticed, exhibited the very singular and astonishing circumstance of *a double head.* The additional head was in about six weeks equally perfect and well formed with the other, although it had not yet acquired the usual deep colour. In fig. 1. is a delineation of this *double headed planaria*, such as it appeared under the microscope when at rest; fig. 2. as seen when in motion. In about two months after it had acquired this additional head, a fragment separated from the tail (the most usual place of separation) and was in progress towards its entire reproduction, when it was accidentally lost—a second, and ultimately a third fragment was spontaneously separated from the same

animal. A delineation of these as they at present appear, (magnified) is given in fig. 4 and 5. The light portions show the parts renewed.

The *planariæ* submitted to my experiments were, it must be confessed, from their long previous confinement, but ill adapted for the purpose; I think it therefore more than probable, that a different result would have followed, had these planariæ been active or vigorous, or but recently taken from their native abode.

From the number of experiments made both by Mr. DALYELL and myself, and from the very few instances in which they proved successful, it may be reasonably inferred, that the production *in the same animal* of a second or *additional* head, is a circumstance of unusual and extraordinary occurrence, and as such may not be unworthy a record in the pages of the Philosophical Transactions.

In addition to my former remarks on the *P. cornuta* and *P. torva*, I have to observe, that I kept a considerable number of each of these species the whole of the last and the former summer, and not having noticed, during that period, any other mode of perpetuating their kind, than that of their detaching small fragments either from the head or tail, I am of opinion they do not, like the other planariæ—at least those I have examined, propagate by eggs; and this may sufficiently account for the reproductive power being so very conspicuous in these species. The *P. torva*, however, does not possess this principle in so high a degree as the *P. cornuta*. In one instance, I recollect one of the latter species casting off two fragments from the tail, the very same night

it was taken, which were only prevented from becoming perfect animals, by an accidental occurrence.

Having found the *hirudo vulgaris* or common rivulet leech to produce its young in greater number when kept separate, I thought the *planariæ* might be similarly affected. To ascertain this point, I took several of the *P. cornutæ* and placed them singly in separate vessels, and in another vessel, by way of contrast, about an equal number together. During the first fortnight scarcely any fragments were detached from the latter, whilst the former, with but few exceptions, had each gone through this process; some indeed throwing off or detaching more than one fragment. This spontaneous separation occurring so soon in those planariæ kept apart, led me to think it was owing to the necessity *then existing* of continuing their species. Hence it would also appear, that this process is *at all times* under command of the animal, and may be called into action upon any particular emergency. And this I think the more evident, from the circumstance of my having lately placed three lively planariæ in the glass globe where the double headed planaria had been hitherto confined alone—that *spontaneously divided* within the short space of four days, (Dec. 24th) in the manner represented in fig. 6, 7, 8.

In regard to the planariæ placed together, although at first extremely indolent, yet they ultimately threw off as many fragments as in the former case; thus proving, that their being kept together or separate makes no further difference, than that where the demand is strong upon them to perpetuate their kind, this process is sooner brought into operation.

The following is the result of this experiment during the first month.

No. of Planariæ.		No. of Fragments.
15	placed together, threw off	16
10	separately -	13
25	produced - -	29

These 25 planariæ, now placed together, detached in the course of the second month 33 additional fragments, making a total of 62. Supposing therefore this operation to continue in full force eight months in the year, (and I find it unchecked even in the present month of January) we should have in the whole 248 fragments, an average of about 10 to each planaria; but if we allow these creatures to multiply in a double or treble degree when at liberty, and supplied with proper food, we may then form a tolerable estimate of the extent to which their reproductive powers might be carried.

In concluding this history of the *P. cornuta*, I may remark, that the smallest portion detached from the tail, so small indeed as to be scarcely perceptible, is sufficient to constitute the active principle or germ of the future animal; but in this case these animals when perfect are so extremely small, as to lead one at the first glance to believe that the parent animals produced their young *perfect* and in a living state; that they were in fact *viviparous*.

I shall close this paper by a few general observations on the *planaria nigra*, the most common of the British planariæ.

P. nigra.

Planaria oblonga, nigerrima, antice truncata.

Long. 5 *lin.* *Lat.* 2 *lin.*

Body, of a fine glossy velvety black, convex above, with an elevated ridge in the centre; plain beneath, truncated before, slightly pointed behind; two ventral foramina; numerous eyes.

This little animal, of which a front view is given of its natural size in fig. 9. is the most sluggish and inactive of all the planariæ I have yet examined. It is commonly found in ditches, attached to the under part of leaves, stones, &c.; it is often seen traversing the surface of the water in an inverted position like the *glossoporæ.*

This species, like those formerly described, is furnished with a retractile trumpet-shaped *proboscis*, issuing from a circular aperture in the middle of the abdomen, and so capable of extension, when in search of food, as to equal in length the animal itself. A delineation of this curious apparatus, (which I shall in future take as a characteristic type of the genus I am describing) is given in a magnified view of the under part of one of these planariæ in fig. 10.

This singular apparatus by means, of which these animals take their food, is not the least of the many strange features in their history; it is indeed so far removed from the common mode of receiving aliment, that doubts might well be entertained as to its real office, were it not clearly pointed out by MÜLLER, and the ingenious author of the work to which we have recently alluded.

The *P. nigra* is oviparous; each ovum, or more properly

speaking, capsule, producing from 2 to 6 young. The period at which the young are excluded varies with the prevailing temperature; the shortest period as seen by the following tables being 20, the longest 53 days, making a difference in this respect alone of more than a month.

No. of Ova or Capsules.	When deposited.	No. of young.	When evolved.	No. of days.
5	Aug. 5	16	Sept. 25	51
1	7	5	29	53
4	14	9	30	47
5	18	31	10	23
12	Sept. 2	48	22	20
—	—	—	—	—

27 capsules containing 109 young, being an average of four young to each capsule.

The *P. nigra*, if artificially divided in two or more parts, will have the lost portion restored in about a fortnight or three weeks. One of these under a magnified form, with a renewed anterior extremity, is delineated at fig. 11, for the purpose of showing a circular range of black specks, or what are commonly called eyes, surrounding the outer margin of the head. This species does not, as far as I have been able to ascertain, separate like the *P. cornuta* by spontaneous division; although, in common with the genus to which it belongs, it is enabled to repair any mutilation to which it may have been exposed.

J. R. JOHNSON, M. D. F. R. S.

Bristol, Jan. 11, 1825.

EXPLANATION OF PLATE XVI.

Fig. 1 and 2. *P. cornuta* (front view magnified) with an additional head, as seen when at rest and in motion.

Fig. 3. *P. felina* with an additional head and body, (probably the same species as the above.)

Fig. 4. A separated fragment from fig. 2. now become a perfect animal; the lighter portion shows the part recently renewed.

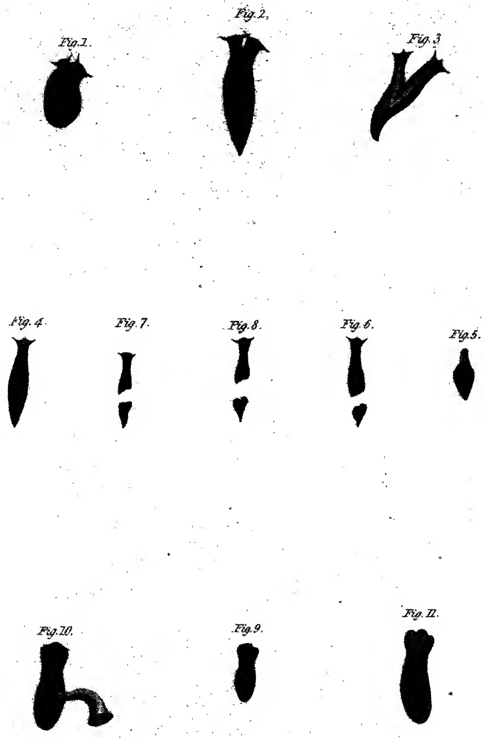
5. Another fragment from the same animal, in its progress towards acquiring a new head.

Fig. 6, 7, 8. Spontaneous divisions of the *P. cornuta.*

Fig. 9. *P. nigra*, front view; natural size.

Fig. 10. Ditto, back view, magnified; with the trumpet-shaped *proboscis* extended as in search of food.

Fig. 11. Ditto, front view magnified; showing a renewed anterior extremity, with a circular range of black specks or dots, supposed to be the eyes.



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XII. *On the influence of nerves and ganglions in producing animal heat. By Sir* Everard Home, *Bart. V. P. R. S. presented by the Society for the Improvement of Animal Chemistry.*

Read March 17, 1825.

In considering this subject, I shall first mention that in the most simple animal structures endowed with life, large enough to admit of dissection, brain and nerves are met with, although many such animals possess no power of preserving a temperature higher than that of the atmosphere by which they are immediately surrounded.

In the oyster and fresh water muscle the whole nervous system consists of two small rounded bodies; of these one is placed upon the œsophagus, one at the opposite end of the body of the animal; they are connected together by two lateral nerves, one on each side.

The internal structure of both these rounded bodies is the same, and resembles that of the brain in other animals, which I have already shown to be composed of small globules, surrounded by a transparent elastic gelatinous liquid: having this structure I shall consider them to represent the brain and the spinal marrow of the animal.

The temperature of the oyster does not exceed that of the surrounding water, since a small thermometer introduced between the shells when kept open by a wedge undergoes no change.

In the garden snail the nervous system resembles that of the muscle, but has also numerous nervous branches going to different parts of the body. The temperature of this species of snail, when its operculum is closed, does not exceed that of the surrounding air: this is proved by making a hole in the shell and introducing a small thermometer, in which the mercury undergoes no change.

It therefore appears that the existence of brain and nerves does not necessarily endow the animal with a power of producing heat.

In the leech, the earth worm, and all the insect tribe, the brain and spinal marrow very closely resemble that of the garden snail; but in all these tribes there is a pair of nerves running down from the spinal marrow the whole length of the body of the animal, which are united together at regular intervals by what are called ganglions, composed of nervous fibres, apparently entangled and agglutinated together; and in all such animals it was proved by Mr. Hunter, in his paper on heat, that their temperature exceeds that of the atmosphere when below 56°, although in very different proportions; the excess in the leech being only one degree, while in a hive of bees it is 26°.

As the only difference between the nervous systems of those animals that have no power of producing heat, and those that have, consists in there being ganglions, I was led to suspect that this power was derived from the ganglions with which the nerves are furnished. Their structure is shown in the splanchnic ganglion.

To ascertain how far there were sufficient grounds for this suspicion, I began to consider, whether any parts of

animals possessed of an unusual temperature were devoid of nerves; the heat of the deer's horn while inclosed in its velvet in June 1824, when only one foot long, I found to be 96°, and on the 12th of July the tip of an antler was $99\frac{1}{2}$; from which it was evident that these horns during their growth have a power of producing heat, independent of the direct influence of the brain or heart; and therefore it was only necessary to ascertain whether there are nerves accompanying their blood vessels, which Mr. BAUER not only ascertained to be the case, but found them equally numerous with the arteries themselves.

This discovery enabled me to institute an experiment, which at once would decide in what degree animal heat was under the influence of ganglionic nerves.

As I might be considered too partial an evidence respecting the different results arising out of such an experiment, I contented myself with superintending it, and made over the operative part to Mr. MAYO, and his associate Mr. CÆSAR HAWKINS, teachers of Anatomy in Berwick-street. The experiment was to consist in dividing all the trunks of the nerves that supplied the velvet of one horn, while those of the other horn were left entire; and see how far under these circumstances the horn would be liable to any diminution of its heat.

The first thing required was to examine into the number of such nervous trunks, and the situations in which they were to be met with. This was done in the head of a deer with antlers, after death.

The experiment was made in Richmond Park on the 21st of July, 1824, about noon, having the dissected nerves be-

fore us to direct the operation. These were found to be the frontal branch of the fifth pair, and the branch of the fifth belonging to the first division which ascends on the outer part of the orbit: this branch in the human body is joined by the trunk of the portio dura of the seventh pair, but in the deer it has no such connection.

Each of these trunks were laid bare by Mr. MAYO in the most satisfactory manner, and a probe passed under the nerve, which was then divided just where they emerge on leaving the great ganglion, which is close to the brain.

That any difference in temperature of the two horns which should occur after the experiment might be registered in the most accurate manner, a hole was bored quite through each of the horns at an equal distance from the tip, just large enough freely to receive the ball of the thermometer.

An hour after the nerves were divided, which was about three o'clock of July the 21st., the temperatures were examined, and so on once a day as long as there was a material difference between them. This will appear by the following diary, only to have continued for five days.

	Atmosphere.	Unnerved Horn.	Uninjured Horn.
July 21,	66°	72°	84°
22	64	69	95
23	64	67	84
24	64	76	84
25	67	87	90

Forty-eight hours after the nerves were divided the temperature of the horn was only 3° higher than that of the atmosphere.

From the time the experiment was made the deer was

kept in a small paddock with two companions. On the 26th of July it had bruised the horn so much, on which the experiment had been made, that the diary could no longer be continued, and that horn was then the hottest of the two.

Upon examination after death no union had taken place between the divided trunk, but it was evident from the recovery of its heat, that some other connection had been formed between the nerves of the horn and these of the head.

This will not appear surprising when I mention that the fallow deer, before they have antlers, shed their horns in June; and immediately after, they again begin to bud, and in the middle of August are completely hardened. Those with antlers mew in April or May, according to their keep, and at the end of August are at their full growth. So that in the space of four months all the nerves that are to supply the deer's horns of a full head have not only begun to form, and arrived at their full growth, but have ceased to exist. This rapidity of growth accounts for their recovering in five days from any check that can be given to their ready communication with one another.

Having gone thus far in my enquiry respecting animal heat, I was determined not to proceed till I had satisfactorily made out whether the placenta is furnished with nerves; and upon that discovery being made by Mr. BAUER's admirable microscopical observations, I found copious new materials to enable me to prosecute the enquiry.

The first step I took was to get my young friend, Mr. CÆSAR HAWKINS, to examine and describe the ganglions

belonging to the nerves of the uterus, those of the nerves of the oviducts in birds and of reptiles, which were found to be more numerous than those of other organs. Mr. HAWKINS's description of them has a place in my paper on the Nerves of the Placenta in the Transactions.

The temperature of the human os tincæ in health was 99°, half a degree lower than the antler of a deer with full-head, in July; but as I knew the nerves belonging to the uterus enlarge during pregnancy, I had no doubt that the temperature of that organ would be increased at that period: in this I was confirmed by finding the oviduct of a frog ready to spawn two degrees hotter than the heart. Upon inquiring among my medical friends who practise midwifery respecting the heat of the pregnant uterus, I was told, that in turning children, they sometimes found the heat of the cavity almost greater than the hand could bear. This information made me most anxious to have its temperature ascertained by a thermometer, as I knew that water heated to 125° degrees is nearly as hot as the hand can well bear.

Upon this occasion I applied to Dr. GRANVILLE, who has upon former occasions assisted me with his knowledge on these subjects, having shown what becomes of the remains of the corpus luteum in ovarial abortions; and ascertained that the two ovaria are equally productive of male and female children, which had been denied; and till Dr. GRANVILLE took up the enquiry, remained without proof. Upon this occasion Dr. GRANVILLE gave me most cordially his assistance, and having been supplied with a proper thermometer sent me the following reports.

First report.

In a natural labour, duration three hours.

The heat of the uterus before delivery -	108°	
after delivery -	105	
Placenta - - - -	104	
The pulsations at the wrist of the mother -	70	beats
in the navel string - -	140	

Second report.

In a labour at 7 months; child alive.

The heat of the uterus before delivery -	100°	
after delivery - -	99	
Placenta - - - -	98	
The pulsations at the wrist of the mother -	60	beats
in the navel string - -	110	

The third report.

In a labour that lasted 38 hours. The child alive, (delivered by forceps).

Six hours before delivery in the intervals of the pains, the heat of the uterus - -	118°	
When the pains strong - - -	120	
After delivery - - -	110	
The placenta - - - -	110	
The pulsations at the wrist of the mother -	100	beats
in the navel string - -	120	

The fourth report.

In a labour that lasted 40 hours; the pelvis deformed.

The heat of the uterus was not accurately ascertained before delivery.	
after delivery - -	115°
When placenta expelled - - - -	118
The placenta itself - - -	112

The instant the child breathes, the pulsations in the chord begin to decrease in frequency till they become the same as at the wrist of the mother, and then cease.

As the balls of some thermometers are so thin, that any pressure made upon them raises the mercury, and renders the instrument inaccurate, it is necessary to remark in this place, that the thermometer employed by Dr. Granville was not capable of having its mercury raised a single degree by the greatest pressure upon the ball that could be made without risk of breaking it.

When the heart of a dog is in action, the heat in the left ventricle is 101, and is the same in the stomach, so that muscular action does not increase animal heat; and the following circumstances, mentioned in Mr. Hunter's paper on this subject, in his work on the Animal Œconomy, proves that its increase or diminution of heat is independent of the action of the arteries. A gentleman while in a state of insensibility from an apoplectic fit, and lying in bed covered up with blankets, had his whole body at one instant become extremely hot, and then suddenly extremely cold, his pulse all the time undergoing no change.

The glow of heat brought into the cheek in the act of

blushing, from whatever cause, has been generally considered to arise from the rush of blood into the smaller vessels; it must however depend on the state of the ganglionic nerves.

Although the nerves when performing their functions in health appear to have no power of producing or keeping up the heat of the animal, there is no doubt that when they are injured or diseased, heat is produced. Of this in the practice of surgery the proofs are without end.

I do not mean at present to go further into this subject, since it would lead me into discussions of some length, respecting the real cause of the increase of temperature excited by inflammation and fever; as however in the first the heat never I believe exceeds the standard heat at the heart; whereas in the second it is raised to 104° or 105°; it is reasonable to believe that the first is from affections of common nerves, the other from affections of ganglionic nerves.

As the torpedo and electrical eel were among the first animals that I ever assisted to dissect, and Mr. HUNTER's account of the structure of the electrical organs, and the wonderful supply of nerves with which they are furnished, was laid before the Royal Society in July 1773; three months after I had enlisted under his banner for the purpose of prosecuting human and comparative anatomy, it will only be considered as natural, that I cannot conclude the present communication, without stating, that the nerves of the torpedo belonging to the electric organs, however numerous, not being ganglionic, do not increase the standard heat of the animal.

As fishes have a lower standard of heat than birds, I wished for some accurate information respecting the ganglions their nerves are furnished with, to determine the proportion they bear to those in birds. I was also desirous of knowing whether there are any ganglions belonging to the nerves that supply the electrical organs of the electrical eel. Mr. HAWKINS's report on both these subjects I shall give in his own words.

"My dear Sir. In the skate I find the following ganglia.

"The olfactory nerve expands into a ganglion of great size, from the lower surface of which many nerves proceed to the membrane of the nose.

"The fifth pair of nerves has a plexiform appearance, chiefly on its inferior or lower root. The lower of the two branches into which the ophthalmic nerve divides has a distinct ganglion upon it.

"The portio dura of the seventh pair of nerves forms a ganglion while passing through the cartilage of the ear.

"The eighth pair of nerves after passing through its foramina enlarges considerably, and that branch which passes along the œsophagus to the stomach forms a considerable plexus on the end of the cardiac portion.

"The spinal nerves originate by two roots, as in quadrupeds, and on the posterior root a ganglion is formed.

"The sympathetic nerve has several ganglia where the branches of the spinal nerves join it; but instead of there being a ganglion at every such junction, as in the quadruped, they are only in the proportion of one to six.

"In examining the preparations of the elecric eel and torpedo in the Hunterian Collection, no ganglia are met with

in the nerves that supply the electric organs; each of these nerves arises separately from the brain, and consists of numerous fasciculi.

yours, &c.

CÆSAR HAWKINS."

From Mr. HAWKINS's examination the ganglions in the skate do not amount to one-sixth part of those in the bird, and the standard heat of this fish is low in proportion; the thermometer in the stomach being only 40°, in the rectum 38°, while the surrounding water was 36°.

EXPLANATION OF PLATE XVII.

In which is exhibited the external and internal appearance of the great splanchnic ganglion.

Fig. 1. The ganglion in situ upon the aorta; natural size.

Fig. 2. The ganglion enclosed in its outer or dura matral covering; magnified two diameters.

Fig. 3. A longitudinal section; magnified in the same degree.

Fig. 4. A small portion from which the outer or dura matral covering has been removed, but is still inclosed in the inner or pia matral coat; magnified six diameters.

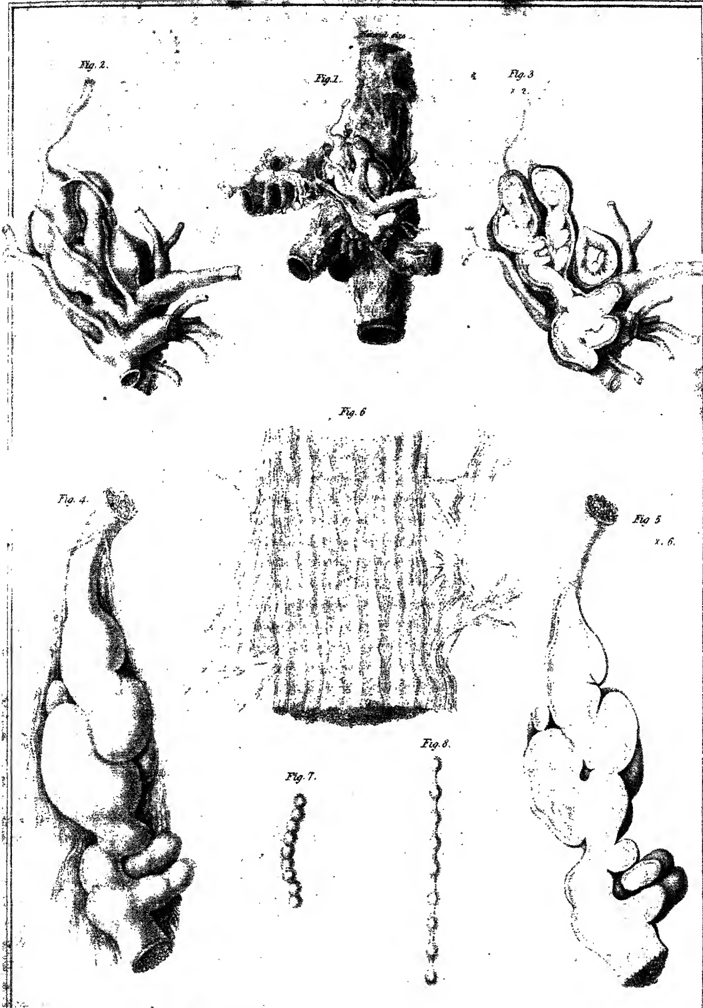
Fig. 5. A longitudinal section; magnified in the same degree.

Fig. 6. A very small portion of the internal substance of the ganglion; magnified twenty diameters, to show that it consists of fasciculi of globular fibres from $\frac{1}{3000}$ to $\frac{1}{4000}$ part of an inch in diameter, similar to those in the brain, connected together by a transparent elastic jelly: this jelly

is so much less readily soluble in distilled water than that met with in the brain, that after eight days maceration in it, the fasciculi are not so readily separated as those of the brain are in two.

Fig. 7. A portion of a single globular fibre in its natural or contracted state, only $\frac{1}{400}$ part of an inch long.

Fig. 8. The same portion of fibre extended by means of the great elasticity of the jelly which connects the globules to more than double its former length.



Franz Bauer del.

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XIII. *An Essay on Egyptian Mummies; with observations on the art of embalming among the ancient Egyptians.* By A. B. Granville, *M.D; F.R.S; F.L.S; F.G.S; M.R.I. one of His Royal Highness the Duke of* Clarence's *Physicians in Ordinary, &c. &c.*

Read April 14, 1825.

In the year 1821, Sir Archibald Edmonstone, whose interesting work on two of the Oäses of Upper Egypt has been so favourably received by the public, presented me with a mummy, which he had purchased at Gournou, on the 24th of March, 1819, from one of the inhabitants of the sepulchral excavations on the side of the mountain, at the back of which are the celebrated tombs of the kings of Thebes. It cost about four dollars. There was no outer case to it; and it is difficult to conceive how the beauty and perfect condition of the surface of the single case in which the mummy was inclosed, could have been so well preserved without any external covering. It appears from Sir Archibald's testimony, confirmed by my own observations, that the mummies which have a second, or an outer case, like the one bought at the same time by Sir Archibald Edmondstone's fellow traveller, Mr. Hoghton, and now lying unopened at his seat near Preston, in Lancashire, have been folded, externally, with greater care than the one about to be described; and that the outward folds are ornamented with variegated stripes of linen. These observations accord with those made by Jomard and Royer.

The first, or inner case, too, of those mummies is covered with a kind of paper, on which the figures and hieroglyphics are painted with much greater brilliancy of colour. Similar remarks apply to the mummy presented to the Hunterian Museum at Glasgow, by Mr. HEYWOOD, a Smyrna merchant, the second or inner case of which is said to be of wonderful beauty and brilliancy.

The single case of the mummy which I am about to describe, appears to be made of sycamore wood, two inches in thickness, consisting of two equal portions (anterior and posterior, as the case is made to stand on its feet) fastened together by pegs of the same material. It is covered, inside and out, with a kind of shell, or coat of plaster, or lime, of considerable thickness. Externally, this coat is painted with symbols and hieroglyphics running in horizontal and longitudinal lines laid on a deep orange ground, the whole being highly varnished. Internally, the surface is divided into horizontal broad stripes, except at the sides, where the stripes run in a perpendicular direction. These stripes are alternately white and yellow, and on both are inscribed hieroglyphic characters an inch in length, constituting, to all appearance, one continued composition; probably a prayer, or invocation for the dead; or the biographical record of the individual contained within the case.

The form of the case is that known to belong to most of the Egyptian mummies brought to Europe, and will be better understood by inspection of Plate XVIII. fig. 1. It measures six feet five-tenths of an inch in its greatest length; and its circumference taken at three different points, the superior or shoulders, the central, and the inferior, immediately

above the feet, is 5 ft. 2 in., 4 ft. 11 $\frac{1}{10}$ in., 3 ft. 8 $\frac{6}{10}$ in. The case is now deposited at my house.

When the mummy came into my possession, it was precisely in the state in which it was found when the case was first opened by Sir ARCHIBALD EDMONDSTONE, covered with cerecloth and bandages most skilfully arranged, and applied with a neatness and precision, that would baffle even the imitative power of the most adroit surgeon of the present day. There is no species of bandage which ancient or modern surgery has devised, described, or employed, that did not appear to have been used in securing the surface of the mummy from external air; and these are repeated so many times, that on weighing the whole mass of them after their removal, they were found to weigh twenty-eight pounds avoirdupois.

In unravelling these complicated envelopes in the presence of two or three medical friends, and Sir ARCHIBALD himself, we could not but be struck with the precision with which the circular, the spiral, the uniting, the retaining, the expellent, and the creeping roller had been applied. The neatness of the turns, and the judicious selection of their size, length, and forms, in order to adapt them to the different parts intended to be protected, and calculated so as to give to the whole an air of smoothness without a wrinkle, or the least appearance of slackness from the varying form of the limbs, were really surprising. We here met with the *couvrechef*, the scapularium, the 18-tailed bandage, the T bandage, as well as the *linteum scissum*, and *capistrum*. Nor were we less pleased to find the many pieces of neatly folded linen, placed like compresses, in all those parts of the body,

which, presenting natural depressions, or hollows, would, unless thus filled up, have proved as many obstacles to the firm and steady application of the bandages. Each limb, nay, each finger and toe, had a separate bandage next to the skin.

These observations respecting the art of bandaging among the ancient inhabitants of Egypt, as displayed in their best class of mummies, have not, as far as I recollect, been made before to the extent here alluded to, and will throw a new light on the history of that branch of practical surgery.

The principal rollers appear to be made of a very compact, yet elastic linen, some of them from four to five yards in length, without any stitch or seam in any part of them. There were also some large square pieces thrown around the head, thorax and abdomen, of a less elastic texture. These pieces were found to alternate with the complete swathing of the whole body. They occurred four distinct times; while the bandaging, with rollers and other fasciæ, was repeated, at least, twenty times.

The numerous bandages by which the mummy was thus enveloped, were themselves wholly covered by a roller three inches and a half wide and eleven yards long, which, after making a few turns around both feet, ascended in graceful spirals to the head, whence descending again as far as the breast, it was fixed there. The termination of this outer roller is remarkable for the loose threads hanging from it in the shape of a fringe, and for certain traces of characters imprinted on it, similar to those described and delineated by Jomard in the *Description de l'Egypte.* One or two of these

characters have corroded the linen, leaving the perforated traces of their form. A fac-simile of this curious fragment will be found in Plate XVIII. fig. 3.

Besides this outer fascia, there was another bandage thrown over the head, brought in front of the chest, crossed there, and carried behind the back, where, being crossed also, it was again brought in front to be once more crossed and returned backward, and ultimately stretched from behind, before, down to the feet, where it crossed a third time in the manner delineated with great precision in Plate XVIII. fig. 2. The shape, form, and position of the limbs lay thus completely concealed, the mummy presenting a homogeneous outline resembling an elongated oval, the superior end of which was twice the width of the inferior.

There was, besides, laid upon the face, above the bandages, a thick mass of linen, by no means neatly folded up, covered by a considerable layer of a black bituminous substance, which became soft on long exposure to moisture, but which, while in that situation, most effectually concealed the features: so that in the present instance, there appears to have existed no desire in the surviving relatives to preserve the lineaments of a cherished friend, as must have been the case with regard to those mummies described by more than one author, in which the bandages applied to the head, had been so skilfully managed as to retain every feature of the face.

The other remaining observations with which I shall trouble the Society on the subject of these bandages, have reference to the materials of which they are made, and the substance with which they seem to be impregnated.

I have satisfied myself that both cotton and linen have been employed in the preparation of our mummy, although HERODOTUS mentions only cotton (*Byssus*) as the material used for the purpose. Most mummies have been described as wholly enveloped in linen cloth, and some persons are disposed to doubt the existence of cotton cloth in any, not excepting in the one now under consideration.

But with respect to the last point, a simple experiment has, I think, set the question at rest. If the surface of old linen, and of old cotton cloth be rubbed briskly and for some minutes with a rounded piece of glass or ivory, after being washed and freed from all extraneous matter, the former will be found to have acquired considerable lustre; while the latter will present no other difference than that of having the threads flattened by the operation. By means of this test I selected several pieces of cotton cloth from among the many bandages of our mummy, which I submitted to the inspection of an experienced manufacturer, who declared them to be of that material.

Having removed, after an operation of upwards of an hour, the various envelopes of the mummy, I directed my attention to its anatomical condition and state of preservation.

It was at once ascertained that the subject was a female, and that no ventral incision, as described by HERODOTUS, had been practised to extract the viscera.

The external parts of generation, on which not a vestige of hair was found, had been brought in close contact, and notwithstanding their shrivelled condition, were readily recognised. The mammæ must have been large during life, for they were found to extend as low down as the 7th

rib, against which they are closely pressed by the arms passing over them. But on lifting the latter, the breasts themselves were raised with little exertion. Of these organs there remain, of course, little more than the integuments, which are of considerable thickness, and exhibit the nipples with their surrounding areolæ in a perfectly distinct manner.

The head is closely shaved; the short hair, which is of a brown colour, can be felt on passing the hand over it; and on close inspection, may be distinctly seen. Externally the cranium appears not to have been disturbed in any way. The eyelids were in close contact. The nose has been flattened down towards the right cheek, by the action of the bandages. The lips, from being retracted, allow the teeth of the upper and lower jaw to be seen, perfectly white and in a sound condition. The arms are crossed over the chest, the fore arms directed obliquely upwards, towards the extremities of the shoulders. The fingers of the left hand alone were bent inwardly, the thumb remaining extended. No papyrus, or other object of interest was found within the grasp of the left hand, but a mere lump of rags which had been previously dipped in the same bituminous substance observed in other portions of the envelopes.

It is well known that papyri, idols, and other objects have been found placed under the arm pits of some of the mummies; but here nothing of the sort was discovered. Only a few glass beads of a blue and green colour, and bugles in all respects similar to those which decorate the dresses of our modern ladies, and made of the same material, dropped from between some of the folds of the bandages, while we unrolled them, as if they had been thrown in gra-

tuitously during the operation, by workmen who had been employing a large quantity of the same ornaments in preparing some more costly mummy, such as is described by JOMARD. It will be recollected, that this gentleman found some mummies in which glass bugles and beads in profusion, disposed in a sort of *trellis-work*, imbedded on bituminous substance, had been fixed here and there, over the surface of the body, in obedience, no doubt, to instructions received to that effect from the opulent surviving relatives. I am the more inclined to adopt the above conjecture with regard to the presence of the few beads and bugles found in my mummy, from the circumstance of my having found, likewise, a portion of reddish clay with characters painted on it, (either a fragment of the wall of the chamber in which the embalmers were at work, or of some case belonging to another mummy) placed in such a manner as to act as a compress on the inside of the left leg in contact with the skin Here it served to fill up a hollow which it accurately fitted; thus keeping the bandage, which passed over it, perfectly tight, but which would otherwise have been slack. This instance of indifference in the choice of materials to produce a particular end, on the part of the embalmers, would, in my opinion, account also for the accidental presence of the beads; and renders it unnecessary to seek for any learned or recondite explanation of their object.

Following up my description of the external appearances of our mummy, I have to remark that the inferior extremities were brought together in close contact at the knees and feet, which latter were kept in that position by a contrivance similar to that which obtains to this very day in most parts

of Europe, of fastening the two great toes by means of a piece of rag or tape.

Numerous and deep wrinkles appeared on the integuments of the abdomen, denoting that before death, this part of the body must have had very considerable dimensions; a conjecture, the correctness of which subsequent inquiries have completely demonstrated.

All these general appearances are well marked in Plate XIX.

The general surface of the body is of a deep brown colour, approaching to black, and is quite dry. In parts where the larger muscles lie, as the thighs for instance, the surface feels quite soft to the touch, and the muscles yield slightly to pressure. The cuticle appears to have been removed throughout, except at the extreme points of the fingers and toes, where it can yet be seen curled up, retaining the nails, of a deep brown colour, in their situation. Some of these, however, quitted their fastening when the slightest attempt was made to detach them.

The dimensions of the mummy appeared to me to deserve the next consideration; and they were taken with great accuracy. Such an opportunity as that before me, of ascertaining the size and proportions of an Egyptian woman, who had probably lived before the building of the pyramids of Memphis, could not be allowed to escape; especially as no admeasurement of a really perfect female mummy has been recorded in modern times. I deemed it, therefore, an object of importance in the study of the natural history of man, to have those admeasurements ascertained with precision. It is well known, that the Egyptian form has been assumed as

the type of a specific variety of the Ethiopian race, particularly by the venerable BLUMENBACH, from certain supposed peculiarities of outward conformation. The consideration of what follows will enable us, as far as a solitary instance can do, to judge of the correctness of such conjectural generalizations.

	Feet	In.
Height of the mummy from the vertex of the head to the inferior surface of the calcaneum	5	$0.\frac{7}{10}$

Thus divided.

Length of the head from the vertex to the first vertebra of the neck - - - -	0	$6.\frac{4}{10}$
Length of the back bone from the first vertebra of the neck, to the articulation of the os sacrum with the os coccygis - - - -	1	10
Length of the thigh from the centre of the head of the femur to the centre of the knee pan -	1	$5.\frac{3}{10}$
Length of the leg from the centre of the knee pan to the inferior surface of the calcaneum	1	$3.\frac{4}{10}$
Total	5	$0.\frac{7}{10}$

The dimensions of the upper extremities and of the foot, are these:

	Feet.	Inch.	
Length of the arm - - -	1	$1.\frac{5}{10}$	} 2 6.
—— of the fore arm - - -	0	$9.\frac{5}{10}$	}
—— of the hand from the tip of the middle finger, to the articulation at the wrist - -	0	7	
Length of the foot - - -	0	$7.\frac{6}{10}$.	

These dimensions will be found accurately marked in Plate XIX.*

Now we find, on comparing the principal of these dimensions, with those of the Venus de Medicis, as given by WINKELMAN, CAMPER, and others, that the difference between them is so slight, as not to deserve notice. Our mummy is that of a person rather taller. The celebrated Medicean statue, which stands as the representative of a perfect beauty, is five feet in height, like our mummy, and the relative admeasurements of the arm, fore-arm, and hand in each, are precisely similar.

But in a female skeleton, it is the pelvis that presents the most striking difference in different races. Nothing, for instance, can be farther removed from the symmetrical form, and from the dimensions of the pelvis in the Caucasian or European race, than the same part in the Negro or Ethiopian race. Of this fact, I shall be able to convince such of the Fellows of this Society, as are not conversant in these matters, by exhibiting the most perfect pelvis of a well grown Negro girl, which I prepared some years ago, in contrast with that of our mummy, which I likewise carefully dissected, and caused to be represented by the same accurate artist in Plate XX. When subjected to this comparative test, the pelvis of our female mummy will be found to come nearer to the *beau idéal* of the Caucasian structure, than does that of women of Europe in general, and to equal in depth, amplitude, and rotundity of outlines, the Circassian form.

In illustration of this remark, I made the following measurements.

	In.
Greatest distance or width of the pelvis from the highest point of the ridge of the ilium on one side, to that of the other side - - - -	$11.\frac{5}{10}$

	In.
Distance between the two anterio-superior spinous processes of the ilia - - - -	10
Distance between the tuberosities of the ischium -	$3.\frac{5}{10}$
Elevation of the branches of the ischium to join the descending branches of the pubis, and form the sub-pubian arch - - - - -	3
Greatest elevation of the os innominatum or haunch bone, from the tubera of the ischium to the highest point of the crest of the ilia - - - -	8

Diameter of the pelvis.

Transverse, or bi-iliac diameter - - - -	$5.\frac{5}{10}$
Anterio-posterior, or sacro-pubian diameter - -	$4.\frac{6}{10}$
Oblique, or sacro-ilio-cotyloid diameter - -	$5.\frac{8}{10}$

Not only are these the most perfect dimensions which a female pelvis can have, but they are precisely in the proportion which the longest diameter bears to the shortest, in the Venus of the Florentine Gallery, according to CAMPER, namely, as 46 to 34; whereas in the Negro or Ethiopian race, the proportion is 39 to $27\frac{1}{2}$, or what amounts to the same thing, the longest diameter of the pelvis of the Negro girl above-mentioned is only $3.\frac{2}{10}$ inches, while the shortest is no more than $3.\frac{6}{10}$ inches. In this respect my admeasurements agree with those given by SOEMMERING.

What has just been observed of the skeleton generally, and of the pelvis in particular, applies with equal force to the form and dimensions of the head. So far from having any trait of Ethiopian character in it, this part of our mummy exhibits a formation in no way differing from the European.

On looking at Plate XXI. which represents with scrupulous accuracy the contour of the head of the natural

size, it is impossible not to be struck with the likeness it bears to the skull of the Georgian female represented in the "*Decas tertia Craniorum*" of BLUMENBACH's very instructive collection. In both we have the facial angle approaching nearly to a right angle: and the configuration of the vertex and occiput in each is such, as must attract attention for its elegance, and the indication of a something more important than mere beauty.

It may be affirmed then, that CUVIER's opinion respecting the Caucasian origin of the Egyptians, founded on his examination of upwards of fifty heads of mummies, is corroborated by the preceding observations; and that the systems which were founded on the Negro form, are destroyed by almost all the recent, and certainly the most accurate investigations of this interesting subject. It is a curious fact, which has been noticed by more than one traveller, that whole families are to be found in Upper Egypt, in whom the general character of the head and face strongly resembles that of the best mummies discovered in the hypogei of Thebes; and not less so, the human figures represented in the ancient monuments of that country.

Having proceeded thus far in my inquiry into the state of preservation of the mummy before me, I determined, perfect, and beautiful as it was, to make it the object of further research by subjecting it to the anatomical knife, and thus to sacrifice a most complete specimen of the Egyptian art of embalming, in hopes of eliciting some new facts illustrative of so curious and interesting a subject; for it is to be observed, that the deficiency of our knowledge on the art of preparing mummies by the ancient Egyptians, both as to the mode of

operating, and of the degree of perfection to which that art was carried among them, has arisen from imperfect and inferior specimens having been generally employed for the purpose of investigation, the best and most perfect mummies (resembling the one I have undertaken to describe) having, invariably, been preserved intact, and, in most cases, uncovered, as valuable objects of curiosity, in private or public Museums. A rapid glance at what has been publicly recorded on this head, will prove the correctness of my assertion.

The Royal Society itself has contributed but little towards the knowledge of this interesting branch of the natural history of man. The subject of Egyptian mummies was brought before it, by two of its members, who from talent and professional avocations, were well calculated to do it justice, had their opportunities been more favourable. The first paper on this subject in the Transactions, is by Dr. HADLEY, who, in 1763, examined a mummy which he had received from the Royal Society, and an account of which he presented in the following year. The paper contains a very clear statement of the successive operations for ascertaining the real condition of the mummy, but seems not to have added much to what was already known, at that time, respecting the mode of preparation.

The mummy retained not the smallest vestige of the soft parts, except some of the tendons of the feet, to the sole of one of which a *bulbous* root, perhaps an onion, was discovered firmly bound by fillets and pitch; reminding us of JUVENAL'S lines:

"O sanctas gentes, quibus hæc nascuntur in hortis
"Numina!"

The bones were all more or less brittle, and some of them separated into splinters in the progress of the examination.

After an interval of thirty years, we find the subject of Egyptian mummies again before the Royal Society, in consequence of a letter from Professor Blumenbach, to Sir Joseph Banks, being read at one of the meetings in 1794; giving an account of three small mummies, and a larger one, opened by the Professor when in London. The latter, as well as one of the former, belonged to the British Museum, and the curators had allowed him to select them from among those deposited in that national collection. In addition to these, Blumenbach re-examined the mummy of a child supposed to have been six years of age, which had been inspected before. The first of these proved to be nothing else than a mass of bandages, strongly impregnated with resinous substance, without the smallest vestige of a human body within them, affording another instance, in addition to those noticed by other writers on the subject, of the impositions practised either by the Egyptian embalmers, or by the modern traffickers in mummies. The second mummy opened by Blumenbach proved to be that of an ibis. In the third supposed mummy, only one or two fragments of a human body were discovered; while in the fourth, the largest, indeed the only real mummy, nothing but naked bones were found within the bandages, a result not far different from that which Blumenbach subsequently obtained from the examination of two other mummies belonging to private individuals, which he had an opportunity of opening before he quitted this country. Such is the sum total of the information to be found in the Transactions of the Royal Society on the subject of

Egyptian mummies, and the extent of its contributions towards the elucidation of this interesting topic, if we except the little that Dr. GREW has said in his printed catalogue of the Museum of the Society in 1681.

Nor had the inquiries of scientific men on the continent been more successful until lately. Thus KESTNER, who described the mummy at Leipzig; HERTZOG who opened the one at Gotha, in which more idols, beetles, frogs and nilometers were found than had ever been met with under similar circumstances; GRYPHIUS, who in the year 1662, gave an account of two mummies in the Dispensary of CRUSIUS at Breslau; and lastly BRÜNNIEL, who dissected the mummy at Copenhagen, found little more than fragments of bones, or whole skeletons in a dry and unsatisfactory state. BRÜCKMAN and STORR, the one at Cassel, the other at Stuttgard, are quoted by BLUMENBACH, as having written on the subject of mummies; but I have not had the means of procuring their descriptions, which, however, to judge from BLUMENBACH'S language, contain no better account of the state of those specimens of Egyptian art, than he himself had been able to give from his own experience.

Long before either Doctor HADLEY or BLUMENBACH had directed their attention to Egyptian mummies, ROUELLE, an eminent French chemist, and CAYLUS, an antiquarian, had treated the same subject with minute precision, although not with better results. Of two papers, which the former had promised, one only was published in the Mémoires of the French Academy of Sciences. In that paper, ROUELLE has given an account of several mummies he had examined, with a view to ascertain the mode in which they had been em-

balmed; and he has described several chemical operations to which he subjected them, in order to discover the nature of the ingredients employed by the Egyptian embalmers. The result of these experiments by no means settled the question they were intended to resolve. With regard to the anatomical state of the mummies examined by him, the information he has given us is very deficient. All he has said reduces itself to a repetition of the common adage "dry as a mummy." Like Dr. HADLEY, BLUMENBACH, and many subsequent writers, he came to the conclusion, that Egyptian mummies are invariably found in a state of aridity, without the least vestige of the soft parts or viscera, and are wholly deprived of humidity, in fact, that they are mere skeletons enveloped in "cerecloth." It will be seen, that such an opinion requires considerable modification.

The next information of importance we possess on the subject of Egyptian mummies, is to be found in the third and fourth volume of the Transactions of the Royal Society of Gottingen. Of two papers on the subject by Professor HEYNE contained in those volumes, the first relates to the antiquity of mummies generally; and the second gives a description of a mummy presented by the King of Denmark, to the Museum of the Royal Society of Gottingen, on which Professor GMELIN instituted various chemical experiments detailed in a separate paper, intended to throw some light on the art of embalming. Numerous as those experiments appear to have been, conducted, moreover, with great care and precision, they nevertheless lead not to more satisfactory conclusions, than the experiments of his predecessor ROUELLE, between whose results and GMELIN's there exists considerable

discrepancy. With respect to the state of integrity of the mummy itself, it is mentioned by Professor HEYNE, that not only had the viscera been removed, but that the muscles also, and every soft part, had been taken away by accurate dissection, made with some sharp instrument; for nothing was found to intervene between the dry substance of the bones and the bandages.

It is needless in this place to advert in a particular manner to the writings of older authors, who have more frequently indulged in conjectures than adhered to facts. They have treated the obscure, yet interesting subject of Egyptian mummies, with more erudition than discrimination, and have not removed the difficulties by which it is surrounded. Much curious information, however, may be collected from their works, especially from those of KIRCHER, PIETRO DELLA VALLE, GREENHILL, POCOCK, BREMOND, MALLET, Dr. MIDDLETON, and others.

The temporary occupation of Egypt by the French army offered a wide field of observation to the antiquarians and the men of science of that nation, the fruits of whose labours have been inserted in a splendid work which must be familiar to the Fellows of this Society. Among the many objects of research to be found in that work, it appears that that of mummies engaged the attention of several very competent individuals, such as DENON, JOMARD, LARREY and ROYER. These gentlemen directed their inquiries to the number of those preparations to be found in the many excavations they visited, to their state of preservation, and to the probable method by which they had been embalmed. The number of mummies discovered by them was prodigious,

and although they state, that the degree of preservation in which the mummies were found varied considerably, they all agree, more or less, in asserting that, as far as they had examined them, there appeared little more than the skeletons remaining. Jomard, indeed, mentions generally, that on removing the bandages, " on observe un corps noir et difforme," and all of them are equally silent on the fact or possibility of the viscera being still in existence in any mummy. Royer, who has taken a more extended view of the subject, and has described with great accuracy, the appearances in two or three distinct classes of mummies, does not mention any of the facts in reference to them, such as I shall presently relate in connection with my own mummy. This omission induces me to believe that the French naturalists never met with a perfect mummy, and that, therefore, the description of a mummy in every respect much better preserved than any that has hitherto been noticed, must be a desirable object to the antiquarian, the learned commentators on ancient historians, and to men of science in general. Baron Larrey's Memoirs are chiefly intended to determine the question of the identity of the present race of Copts with the aboriginal Egyptians, whose descent he traces from the Abyssinians and Ethiopians by a comparative examination of the crania of several mummies he had collected in the desert of Saqquarah, and of those of the modern Copts found in a cemetery near Alexandria. The mummies of Saqquarah, however, are acknowledged to be very inferior to those of Upper Egypt by all travellers; and cannot, therefore, be put in competition with the latter, in an inquiry into the art of embalming among the ancient Egyptians.

Independently of the information thus collected from the writings of different authors, calculated to convince me that chance had put me in possession of a better, and a differently prepared mummy from any that had hitherto been recorded; many curious facts corroborative of that conviction, and capable of illustrating the anatomical history of mummies in general, were communicated to me by the late Dr. BAILLIE, by Sir E. HOME, Mr. BRODIE, Mr. CLIFT, and others. It would appear from their statements, that the inquiry into the condition of these singular preparations had, from time to time, engaged their attention; and that if nothing very new or very interesting was discovered by those eminent anatomists respecting them, the circumstance is to be attributed to the cause already alluded to, namely, the imperfect state of the mummies which fell under their inspection. Both the late Dr. BAILLIE and Mr. WILSON mentioned to me that they were present at the opening of a mummy by Mr. HUNTER, who found it to consist of a mere skeleton, with the skin over it perfectly dry; the whole presenting so confused a mass that no one particular part could be recognised. Mr. BRODIE saw and examined three mummies that belonged to Lord MOUNTNORRIS, and which he found quite dry and uninteresting. Another mummy, brought to England several years ago by Colonel LEAKE, and at the dissection of which Mr. BRODIE was also present in 1807, was not found in better condition. The same observation applies nearly to that which my friend Mr. HAMILTON, the late Under Secretary of State for Foreign Affairs, sent to the College of Surgeons, and which was examined by Sir E. HOME, Mr. BRODIE, and Mr. CLIFT, in the presence of Sir J. BANKS, Mr. HATCHETT, and others. Mr.

Brodie, who took notes of the dissection, and Mr. Hatchett, have stated to me, that there were none of the viscera in the mummy in question; that it was not in a flexible state, and that the muscles could scarcely be distinguished. Sir E. Home himself, on the other hand, cannot tax his memory as to the precise parts discovered, the dissection not having been completed, in consequence of the remains of the mummy being destroyed in some of the souterrains of the College, from the effect of dampness in a newly erected building. Mr. Clift mentioned to me that the external parts of generation were perfect; and Sir Everard recollects that the face was in a high state of preservation. If so, it is to be lamented that a circumstance, over which Sir Everard had no control, should have prevented him from prosecuting an enquiry, which no man could have rendered more instructive; and the publication of which would probably have done away with the necessity of the present communication to the Society.

Sir Everard Home made some observations on another mummy brought from Thebes by the late Captain Kennet, of the Engineers, in 1806; the particulars of which he has kindly communicated to me. The mummy in this instance was that of a male; and, as far as could be judged from external appearances, seemed to be in good condition. No internal examination was permitted. The head had not the appearance of that of an African. The face was entirely exposed, as well as the chest, and the anterior part of the abdomen. The skin was entire in all these places. On the upper part of the head, as also on the chin, the hair was preserved. The teeth were perfect, and the skin was nearly quite black, a circumstance, which Sir Everard thinks,

must be attributed to its having been stained with some gum. The brain had been removed through one of the orbits, into which false eye-balls had been introduced. The eye-lids were entirely removed, probably from accident. Indications of muscles were observed on the abdomen, the scapulæ, the back, and on the nates. The legs were not uncovered; but the toes were all exposed. The arms were placed so that the hands came upon each groin, there being a middle space at the pudendum, of about two inches, between them. The male organs were so enveloped as not to be traced in any degree whatever. Sir EVERARD took notice of the principal dimensions of this mummy, which, as affording the means of comparison between the two sexes, may properly find a place here after those of my female mummy.

	Feet.	Inches,
Length of the body, from the vertex of the head to the bottom of the heel - -	5	2
Breadth across the shoulders -	1	3
Length of the arms, from the top of the shoulder to the end of the fingers -	2	6
Breadth from trochanter to trochanter	1	0
Length of the foot - - - - -	0	9

To those who are familiar with the accounts published by recent travellers in Egypt, it will be needless to repeat that Dr. BRADLEY on the one hand, and Dr. RICHARDSON on the other, acknowledge that the mummies which they had an opportunity of examining appeared to consist of little more than mere dry bones.

My friend, Mr. WALTER DAVIDSON, of the house of HERRIES and FARQUHAR, has also added to my store of

information on the present subject. He purchased a mummy from the excavations near Thebes, at Gournon, in February, 1820, selected out of a dozen which he opened, as the best preserved. It proved to be that of a male. It was quite dry; the hair and teeth were most perfect, the former being very long, in great profusion, and smoothly combed down. The body contained only a large quantity of gum, and there was no flesh, or very little of it, on the bones. Every part was brittle. It was enveloped in cotton bandages to a great extent, and was contained within two cases. His fellow traveller, T. Coates, Esquire, of Newcastle, brought from Egypt another mummy, which was presented to the Literary Society of that town, and of which an account appeared in some of the public papers of last year. This mummy was not opened. Within the last few months a highly preserved mummy, and one which, to judge from the description given in the public papers, I should be inclined to class with my own, has been dissected and exhibited before the Literary Society of Bristol. We are promised a detailed account of the appearances by a competent person; and if these should correspond with what is detailed in this paper, an additional value will be given to my observations, which I could scarcely have hoped they would so soon receive.

The facility which I deemed it my duty to afford to every individual interested in science, of witnessing the demonstrations of my mummy, brought to my house, among others, Mr. Wilmot Horton, Under Secretary of State for the Colonial Department. Pleased with what he there saw, this gentleman was kind enough to place at my disposal, the head and right arm of a male mummy, which, though not

so curious in point of preservation compared to other specimens, are objects of no inconsiderable interest, from the locality in which they are said to have been discovered, namely, near Tripoli, on the coast of Africa. They were forwarded by the British Consul resident in that town; but as no circumstance connected with the discovery is known, it would, perhaps, be premature to come to any conclusion as to the probability of the art of preparing mummies having been exercised among the inhabitants of the north of Africa, as it had been by those of the east.

These remains of a mummy are not altogether devoid of interest, in as much as they supply us with corroborative proofs of the general principles of the art of embalming, having been such as I shall describe in this paper; and as affording additional evidence of its strong power of preservation.

The head, in this case, was covered with a few bandages of coarse linen closely adhering, and, indeed, intimately connected with the integuments and muscles of the face, by a black resinous substance, which must have been applied hot, as it has burnt the soft parts to the very bone, and even some of the teeth. The hair is preserved, but it is with great difficulty that it can be disentangled from the hard and brittle resin. It is about two inches long, of a reddish brown, and in slight curls and tufts. Hair grew down the cheeks and on the chin, about an inch in length. I removed the bandage, and thus denuded the head and face altogether in most parts, carrying away, necessarily, the integuments and muscles. The head is not prepared in the best manner, but according to one of the least expensive processes. The brain was

removed through the nostrils, and in the operation the os unguis of the right side was injured. The eyes were preserved, but in taking away the bandages they came away with them. By immersion in hot water, I was enabled to separate the external coat of the eye ball, which became as soft and globular as in a recent specimen, though discoloured. There is a very remarkable feature in the skull, and that is the extreme depth of the orbits, which amounts to $2\frac{1}{2}$ inches, tapering inwardly, so as to present the appearance of a perfect cone.

Whether this head bears marks of being that of an African, in the full sense of the word, or not, I am not able to decide. The contour of the head, the maxillary bones and jaws, and the appearance of the hair, incline me to that opinion; but the Members of the Society will have an opportunity of judging for themselves, by inspecting the head after the meeting. Certain am I, that it is not the head of a Negro.

The arm, sent with the head from Tripoli, is uncovered. The muscles are preserved, but they are harder than in my other perfect mummy. The hand is stretched. There is only a portion of the humerus, which seems to have been fractured off, not cut regularly, from the appearance of its splintery extremity. The length of what remains is 8 inches. That of the fore arm is $11\frac{1}{4}$ inches, and the hand, from the wrist to the tip of the middle finger, is $7\frac{1}{2}$ inches long. This specimen also will be submitted to the inspection of the Members after the meeting.

Having thus brought within narrow limits the literary history of Egyptian mummies in general, I shall proceed to

the conclusion of my account of the dissection of the one I have described, by which I trust the Society will be enabled to form an opinion of the degree of importance that belongs to the present communication.

An incision having been made into the parietes of the abdomen, just below the ribs, and continued down to the hip bone, on both sides, and carried along the margin of the pubis, the whole of the integuments and muscles were removed, so as to expose that cavity completely to view. The objects which then presented themselves were a portion of the stomach adhering to the diaphragm, the spleen much reduced in size and flattened, attached to the super-renal capsule of the left kidney, and the left kidney itself, imbedded in, but not adhering to the latter, and retaining its ureter, which descended into the bladder. This, as well as the uterus and its appendages, were observed in situ, exhibiting strong marks of having been in a diseased state for some time previously to the death of the individual. Fragments only of the intestinal tube could be found, some of them of considerable dimensions, and among them part of the cœcum, with its vermiform appendix, and portions of the ilium. Several large pieces of the peritoneal membrane were likewise observed. (See Plate XXII. fig. 1, 2, 3.)

There were also several lumps of a particular species of brittle resin, two or three small pieces of myrrh in their simplest and natural state, and a few larger lumps, of an irregular shape, of some compound of a bituminous and resinous nature, mixed up with an argillaceous earth. These seemed to have been forced up to fill the cavity of the abdomen, after the removal of the largest portions of the intestines, and of

as much more of the contents of that cavity as the embalmers could get at, by the very clumsy process which appears to have been employed in this case, for the extraction of those parts through the anus. This orifice was cut in various directions, probably with the intention of enlarging it; but, more likely, in consequence of the forcible introduction of the instrument employed in extracting some of the viscera. No traces of the right kidney could be found, nor of the liver or minor glands of the abdomen; although, among the many fragments of membranes and other soft parts which lay in confusion, and were removed for better inspection, the late Dr. Baillie, who was present at one of the demonstrations, detected the gall-bladder slightly lacerated, but in other respects perfect, retaining a small portion of the peritoneal covering of the liver attached to it, as well as considerable remains of its own ducts.

The cavity of the abdomen being emptied of all its contents, I continued the circular incision back to the spine, which I divided at the first lumbar vertebra. I next sawed off the thighs a few inches from the hip, and dissected carefully all the soft parts from the pelvis, so as to ascertain the condition and dimensions of this important part of the female skeleton. In performing this last operation, which occupied me two hours a day for nearly a week, (some medical or scientific friends being present at each sitting), we could not help being struck with the remarkable degree of preservation of the muscles, such as had never before been noticed in Egyptian mummies, and such as to admit of their being separated from one another, as readily as in the dissection of a recent subject. Nor was the perfect condition of the articulatory

membranes and ligaments less surprising, which allowed us to impart to the great articulation of the thigh with the ilium, its various movements, a circumstance seldom observed, even in modern preparations of the pelvis.

Some of the dissected muscles, as well as the denuded pelvis itself, will be submitted to the inspection of the Fellows after the Meeting of the Society.*

The cavity of the thorax was next examined, and this I effected without disturbing the anterior portions of the ribs or breast bone, by simply detaching the diaphragm all round, and bringing it away. It was found that the pericardium, which adhered partially to the diaphragm, came away with it, and that a laceration had taken place at the same time in that sac.

This circumstance denoting that the heart was present, I introduced my hand to remove it, when it was found suspended, *in situ*, by its large blood vessels, in a very contracted state, attached to the lungs by its natural connections with them. The latter organs adhered throughout their posterior surface to the ribs, and were brought away altogether in as perfect a state as could be effected.

All these various parts are accurately represented in Plate XXIII. fig. 1, 2.†

The last cavity examined was that of the cranium; for this purpose it was sawed in two, horizontally, and when

* Among the detached muscles exhibited in the most distinct manner, there were the triceps femoris, the sartorius, portion of the vastus externus, and the principal abdominal muscles.

† All the parts represented in Plate XXII. and XXIII. were exhibited after the meeting, to the Fellows and Visitors present, on three successive Thursdays.

thus opened, it was ascertained that the brain had been removed through the nostrils; the plates of the inner nasal bones having been destroyed in the operation by the instrument employed, as evidenced by the state of those parts. It is a matter of no little surprise how, under circumstances of so much difficulty, the operators could have contrived to remove every vestige of the membranes investing the brain, one of which is known to adhere firmly in most subjects to the inner surface of the superior cranial bones. There can scarcely be a doubt but that some injection had been thrown into the cavity in question, to clear it out in so perfect a manner; for no instrument could have effected such a purpose. A black resinous substance, but in a small quantity, was found adhering to the inner surface of the occipital bone, which must have been thrown in quite hot, as it had penetrated through, and burnt partially, the superior part of the lambdoidal suture through which the liquid escaped, so as to be now seen extravasated under the scalp. But how this liquid resin was thrown in, and for what purpose, it is not easy to conjecture. It could only have been made to penetrate through the opening which had previously been made in the ethmoid bone, to extract the brain; and if so, it is difficult to conceive in what manner it was made to reach the spot it now occupies without having adhered to any other intermediate portion of the cranium. It was remarked, at the time of opening the head, that its inner surface was studded with small crystals of what appeared to be an animal substance, resembling *steatine*.

The last observation I have to make on the structural condition of this mummy, refers to the state of the eyes,

which appear not to have been disturbed; and to the state of the mouth, which was as carefully examined as circumstances would admit, without destroying the contour and general appearance of the face. The tongue is preserved, and neither above nor below it was there found any coin or piece of metal, as recorded of some of the mummies, but a lump of rags dipped in pitch. The teeth, as I before remarked, are perfectly white and intact; nor did I observe that peculiar cylindrical form of the incisores which has been assumed by some naturalists, as one of the characters of the head in the Ethiopian race.

In order to complete the present essay on Egyptian mummies, I must now trouble the Society with the farther details of my observations on the age of the female under our consideration, and on the disease of which I conceive her to have died, as deduced from the examination of the parts. When we reflect for a moment, that the individual in question, according to the more generally received opinion respecting the antiquity of mummies found in the hypogei of Thebes, had probably lived upwards of three thousand years ago,* it will bespeak a very extraordinary power of preservation in the mode of embalming then practised, in some cases at least, to be able to say, that the female of which we are speaking, died at an age between fifty and fifty-five years; that she had borne children; and that the disease which appears to have destroyed her was ovarian dropsy attended with structural derangement of the uterine system generally.

* Consult Mons. JOMARD's Memoir on the antiquity of the hypogei at Thebes, Mons. ROYER on the art of embalming, and the recent publications of Monsieur CHAMPOLLION.

That such are the facts, I appeal to the state of the bones of the ilium, and of the uterus with its appendages, for proof.

The first exhibit that peculiar degree of thinning in the centre of their osseous plates which has been noticed in women by Professor CHAUSSIER and others, in the course of a great number of observations, as an indication of their having borne children, and of their having passed the fortieth year. This thinning of the bones, in the particular part just mentioned, has never been observed under forty years of age, and becomes gradually greater until fifty-five, when it has reached its maximum, however longer the woman may continue to live. In my mummy it will be seen, on looking at the pelvis, or at Plate XX. that the thinning of the iliac bones seems to have reached its maximum; and as there are no characters of decrepitude in the individual, it appears to me, that from fifty to fifty-five was about the number of years the individual had lived. The thinning of the bones in question has not been observed in women who have not borne children, nor am I aware that it has been noticed in the male sex, except in the shoulder blades of porters, long used to carry heavy weights on their back. In confirmation of this I have to state, that in more than one pelvis in my collection, with the history of which I am perfectly acquainted, I find the above law to hold good. The thinning of the central portions of the ilium in this mummy is so complete, that small fragments have come away in consequence of their being frequently touched by the numerous persons who saw the pelvis at my house, and were incredulous as to its real texture without touching it.

With regard to the disease, the effects of which I detected,

I have to state, in support of my assertion, that the womb is of larger dimensions than it is known to have at the age in question: that the ovarium and broad ligament of the right side are enveloped in a mass of diseased structure, while the Fallopian tube of the same side is perfectly sound and beautifully preserved; and lastly, that the contracted parietes of what (to judge from the dimensions of the remains) must have been a large sac connected with the left ovarium, leave no room to doubt of the correctness of the opinion I have ventured to express. This opinion, I have the satisfaction to add, has not been disputed by a single individual out of the many very competent judges to whom I submitted the parts, among whom I may mention the late Dr. BAILLIE, and Mr. WILSON, Mr. CARPUE, Mr. BROOKS, and others. The whole of the uterine system, as now described, forming the most ancient pathological preparation of its kind, is now in my possession, and will be exhibited to the Members after the meeting, and may be compared with its accurate delineation as given in Plate XXII. fig. 1.*

* Another mark, denoting the previous existence of disease, I detected on the scalp, namely, the remains of that peculiar cutaneous affection of the head, which has been denominated *Porrigo decalvans*, from its effect of destroying the hair as well as of preventing its growth. Was it for this, that the head of this mummy had been shaved, as I have already stated, so as to admit a readier application of remedies to the morbid part, as practised at the present day? or was it for any other particular reason? No mention is made, in any author, of females having their head shaved, though the cutting off of the hair in men is frequently alluded to. Again, with what sort of instrument has the operation been executed? It certainly could not have been performed with scissors, however skilfully constructed, as the hair could not have been cut so close, nor of such uniform length with them. If with any instrument approaching to our razors in structure, of what material was it made? These are highly curious inquiries, which naturally spring from the examination of the condition of this mummy.

The next points of inquiry to which I directed my attention were, First, to discover, if possible, the method by which this perfect specimen of Egyptian mummies had been preserved. Secondly, to ascertain how far the description given by ancient writers of the art of embalming among the Egyptians, applied to the present specimen. And lastly, to determine the nature of the substances employed for the purpose.

In pursuing this investigation, I flattered myself that the Royal Society would consider it as something more than a mere object of useless curiosity.

In order to carry on my inquiry respecting the three points above-mentioned with that precision which alone could lead to a satisfactory conclusion, I proceeded to note down all the principal facts resulting from a close examination of the mummy, as detailed in this paper; next to ask myself how those facts could be explained; and lastly, if explained, whether the facts themselves could be reproduced by following the method which the explanation might point out. I shall leave it to the Society to determine, whether I have been successful in my attempt.

The *first* fact to be noticed, in regard to the preparation of the mummy, is the chestnut brown tint of all the bandages, denoting the presence of some colouring matter in them, the nature of which it was important to ascertain, in order to judge of the intention of those who employed it. For this purpose I made a few experiments with portions of the bandages taken from different parts of the body, when it was found that they had all been steeped in some vegetable solution, which, when treated with gelatine, exhibited the presence of tannin in considerable quantity, a circumstance far-

ther corroborated by the peculiar taste of the infusion. Now, as every particle of the bandages had been equally died with this vegetable solution; and as it appears evident, from other circumstances, that such a process had not been adopted for the sake of giving to the envelopes of the mummy the particular colour in question, may we not infer that the Egyptian embalmers were acquainted with the antiseptic power of astringent and slightly bitter vegetable infusions, a power which modern discoveries have attributed to the presence of the peculiar principle already mentioned?

This inference is confirmed by the *second* fact to be noticed, namely, the appearance and condition of the integuments, which, besides being of a dark brown colour, differ in no respect from prepared leather, particularly those of the abdomen, the thighs, and the mammæ. The Society will have an opportunity of examining several portions of these integuments, and will be struck with the similarity alluded to. Indeed they might be taken for prepared leather at first sight, and the knowledge which I obtained, by a second series of experiments, that a solution of some vegetable astringent, similar to that used for the bandages, but much stronger, had been employed to produce that appearance, must prove conclusive on this point. A question then will naturally arise, was it the bark of the acacia, so plentiful in Egypt, that was employed for the purpose; or did the Egyptians import oak bark from the coast of Syria, where that tree grows in abundance *

* It is not improbable, that a gum, not unlike kino, may have been the substance used for the purpose of tanning the integuments, as I found, among the various lumps of resin contained in the abdomen, several portions of such a substance,

The two preliminary and curious facts just detailed, connected with the art of embalming among the Egyptians, have never been noticed before. Neither HERODOTUS, nor DIODORUS SICULUS, mention them, and all the more modern writers are silent on the subject.

The *next* fact worthy of notice, is the appearance of minute saline crystals, found in great abundance in almost every part of the external, but more particularly of the internal surface of the body. These, at first, had escaped notice; but upon the various portions of the dissected mummy being exposed to the open air, in one of the rooms on the ground floor in my house for some weeks, where a fire was kept, the appearance of the saline particles became strikingly visible. This saline efflorescence I gently swept off the surface with a new brush, and subjected to various analytical experiments, from which it results, that it consists of nitrate of potash, carbonate, sulphate, and muriate of soda, and traces of lime. Now, as as none of these salts have ever been observed to form spontaneously, either within or upon the surface of preserved human bodies, particularly where the contact of external air has been so studiously excluded as in the present case, it follows, that in the preparation of mummies, the embalmers must have had recourse to the immersion of the body into a saline solution of a mixed kind. HERODOTUS, indeed, states that the body was covered with natron for the space of seventy days; but it is more probable, that the water of the celebrated natron lakes, which lay so conveniently at hand, rendered more active by previous evapo-

specimens of which I exhibited to the Society, and which gave to distilled water a deep brown colour, from which a precipitate is obtained by gelatine.

ration, was used for the purpose. The presence of lime may be accounted for by supposing, that in a preliminary operation, the cuticle, which, as I before stated, could not be detected in any part of the body, except the head and the extremity of the toes, and has been found invariably wanting in all other mummies, was removed by means of that alkaline substance. This circumstance again goes far to show that the Egyptian embalmers were acquainted with an important physiological truth, namely, that in order to promote the absorption of liquid substances, particularly of the tanning liquor and saline solution, applied to the external surface of the body, the cuticle must first be removed.

The presence of saline substances in mummies has been noticed by more than one modern writer, especially by Mons. ROYER, already mentioned in the course of this essay; but the conjecture as to the origin of the salts themselves, has not been hinted at before.

A *fourth* fact, deserving of our attention, is the presence of a resino-bituminous substance between some of the folds of the remaining portions of the peritoneal membrane. On collecting this substance, and instituting some experiments upon it, I ascertained that the bitumen was mixed with a greater proportion of wax, so as to have rendered the mixture perfectly plastic. To have penetrated thus far, and to have lodged between closely adhering membraneous folds, this mixture must either have been injected quite warm into the cavity of the abdomen, or the body itself must have been plunged into a vessel containing a liquefied mixture of wax and bitumen, and there kept for some hours or days, over a gentle fire. The latter operation, not noticed by the older historians, has

indeed been surmised by some of the modern writers on the subject; but in none of them have I been able to find a corroborating proof of the correctness of such a surmise. The examination of my mummy has afforded me that proof, in the shape of a *fifth fact*, namely, the thoroughly impregnated state of the bones, membranes, and muscles, in every part of the body, by the same waxy and bituminous substance. The inspection of the bones of the pelvis, of those of the thighs, and of the vertebræ, as well as of some of the muscles, and membranes, to be submitted to the Society, will shew this abundantly. Now such a condition of the parts could not have been produced, but by maceration or immersion, for a length of time, of the whole body, into a liquefied mixture of those two ingredients; accordingly we must conclude that such a process was actually followed by the embalmers; unless we feel disposed to believe that they injected the body through the blood-vessels; an operation of which there is not the most distant evidence in the mummy before us.

The adoption of my view on this point, is farther authorized by the soft and pliant condition of the capsular membranes, of the cellular texture, and above all, of the two coverings of the spinal marrow, than which nothing can be more beautiful or striking; whether we admire their perfect preservation, or reflect on the number of centuries through which these delicate tissues have travelled. I have already noticed to the Society the flexibility of the joints, a circumstance which is entirely due to the process here explained; and now I have to add that this process is made out beyond contradiction, by my having been able to separate the wax by means of

combustion and ebullition, from the soft parts, particularly the muscles, the singularly distinct fibres of which, beautifully arranged and displayed, the Society will not omit remarking.

In examining the dissected parts of the mummy, which I have carefully displayed for public inspection after the meeting, the Members will not fail being struck with the difference that exists between the two *nates* detached from the body. The one has been left in the state in which it was handed down to us by the Egyptian embalmers, dark, tanned, contracted, and impregnated with the mummifying ingredients; the other, on the contrary, has been deprived, in toto, by my process, of those ingredients, (the principal of which is bees wax, as will be seen from the quantity which I collected); so as to appear like the same part in a recent subject, soft, elastic, of a yellowish white, with the cutaneous pores very distinct, and with its muscles, adipose substance, and blood vessels perfectly striking.

The *sixth,* and last fact to be noticed, is the presence of several moderately sized lumps of an earthy matter, mixed with pieces of resin, found loose in the cavity of the abdomen. That these were thrown into that cavity for the double purpose of filling up the space left in it by the abstraction of some of the viscera, and of adding, at the same time, to the antiseptic power of the process employed in embalming, are conjectures that will perhaps be readily admitted. The experiments made to ascertain the nature of the earthy substance in question, tend to prove the latter part of these conjectural propositions. It was found to consist of the same saline compounds, noticed on the surface of the mummy, mixed with

argillaceous earth. Now, if the embalmers used the water from the natron lakes, as I have laid down good grounds for believing, nothing is more probable, than that they also made use of the earthy sediment of that water which contains the salt in question, and which could be procured in abundance at the margin of those lakes, where it has been observed by the naturalists who accompanied the French expedition into Egypt.

As to the nature of the resin and bitumen used as ingredients in the embalming process, it is a question of comparatively little interest. Nor does it matter much, whether aromatic vegetable substances were employed or not. In the mummy before us, two or three small pieces of myrrh in a loose state were found, and evidence is not wanting of both resin and bitumen, though not in their purest form, having been had recourse to. But their presence seems by no means necessary for the completion of that admirable method of embalming, devised and followed by the ancient Egyptians, which my inquiries have been directed to ascertain, and which may be summed up in a few words by saying: that it consisted in impregnating the body with bees wax.

The various circumstances detailed in this essay furnish us with sufficient reasons for believing, that in the most perfect, and, I would call them, the *primitive* specimens of the art of embalming, the progressive stages of the Egyptian method must have been as follows:

A. Immediately after death the body was committed to the care of the embalmers, when, in the majority of cases, the viscera of the abdomen, either wholly, or partially, were forthwith removed; in some cases through an incision on the one

side of the abdomen, as stated by HERODOTUS, and as proved by some of the mummies examined; and in others through the anus, in which latter case, the extremity of the rectum was previously disengaged from its attachments all round by the knife, and the intestines imperfectly extracted. The cavity of the thorax in the most perfect specimens was not disturbed.

B. The head was emptied, in all instances, of its contents, either through the nostrils, by breaking through the superior nasal bones, as in the instance under our consideration, as well as in that of the head from Tripoli, already mentioned, or through one of the orbits, the eyes being previously taken out, and artificial ones substituted in their place, after the operation, as in the instances of the mummies examined by Sir E. HOME and Mr. BRODIE. The cavity of the cranium was repeatedly washed out by injections with some fluid, which had the power of not only bringing away every vestige of the substance of the brain, but even of the enveloping membranes of it. Yet the liquid could not have been of a corrosive nature, else the tentorium, or that membranous floor which supports the brain must have disappeared with the meninges; whereas it is still in existence, and does not appear to have been in the least injured. A small quantity of hot liquid rosin was then injected into the cranium.

C. The next step taken in the embalming process, was to cover the body with quick lime for a few hours, and after to rub the surface of it with a blunt knife, or some such instrument as would most effectually assist in removing the cuticle. The scalp, however, does not appear to have been touched; and care was taken also not to expose the root of the nails

to the action of the alkali, as it was intended that these should remain in all cases. In the mummy I have described, this point has been so much attended to by the embalmers, that the nail of the principal toe of the right foot having been detached, it was replaced and retained in its position by three or four turns of thread passed around it; and in this state it must have continued for the last thirty centuries.

D. The operation of removing the cuticle being accomplished, the body was immersed into a capacious vessel, containing a liquefied mixture of wax and resin, the former predominating; and some sort of bituminous substance being added, not however essential to the process. In this situation the body was suffered to remain a certain number of days over a gentle fire, with the avowed intention of allowing the liquefied mixture to penetrate the innermost and minutest structure; nor can there exist any doubt, but that on this part of the embalming process depended not only its great preservative power, but also its various degrees of perfection. Thus, when the process was properly managed and watched, mummies, such as the one under consideration, would be produced; whereas when neglected or slovenly conducted, the mummy resulting from it, would present those appearances of dryness, blackness, and brittleness, together with the carbonification of the muscles and intimate adherence of the integuments to the bones, which have been noticed by Dr. Hadley, Professor Gmelin, Blumenbach, Hunter, Dr. Baillie, Mr. Brodie, Jomard and others, when they examined imperfect or inferior mummies. The fraudulent subtraction of the allotted quantity of wax required for the principal and important part of the embalming process we are now con-

sidering, or the neglecting to regulate the fire in using the wax and bitumen, would necessarily give rise to the latter results, which the covering bandages were sure to hide from the eye of the surviving relatives to whom the body was to be returned. It is also fair to presume, that inability or unwillingness on the part of friends and relatives to pay for the ingredients or for the labour necessary to carry on the operations just described, have, on many occasions, been the cause of mummies being prepared in that imperfect manner which has been noticed in so many instances.

E. When the body was taken out of the warm liquid mixture, every part of it must have been in a very soft and supple condition, wholly unsusceptible of putrefaction. The next steps therefore to be taken, with a view to convert it into a perfect mummy, must have been those, which, had they been taken before that part of the process that has been just described, would have exposed the body to inevitable putrefaction, in a climate like that of Egypt. I allude to the tanning of the integuments, and the exposing of their surface to the additional influence of those salts, the presence of which, as well as that of tannin, I have most clearly demonstrated.

Whether an infusion of the vegetable astringent employed for tanning the integuments was had recourse to in the first instance, and the immersion of the body into the concentrated water of the natron lakes followed, or whether the tanning liquid was itself made by infusing the vegetable astringents themselves in the water of the natron lakes, and the body then immersed into it, are questions, which it is neither possible, nor important to decide; the body was unquestion-

ably submitted to the operation of both those means, but in what order, it is difficult to ascertain; and when the embalmers judged by the condition of the integuments, that they were sufficiently impregnated with the active principles employed, the body was allowed to dry for a few hours, and then the bandages previously prepared with a solution of *tannin* also, as proved by my experiments, were applied to the different parts, beginning with each separate limb.

While the operation of bandaging took place, the mummy must have been in a very supple state, else the numerous deep longitudinal wrinkles observed in all those parts where the integuments are generally looser, as in the upper part of the thighs and arms, as well as over the abdomen, and at the breasts, could not have existed. These wrinkles, so well marked in Plate XIX. must have been produced by the bandages at the time of their application.

It appears also, that with a view of rendering the bandages more supple in particular places, where such a condition was required, and of obviating the inconvenience of slackness in some of the turns, they were daubed over in a few places with two different substances, the one consisting of wax and resin, the other of resin alone, both applied warm; so that, while the first served to give pliancy to some of the linen employed, the second caused the slack and loose edges of the bandages to adhere together, by which process the whole was rendered compact and firm, without producing hardness.

The lumps of myrrh, resin, and bituminous earth, noticed in the abdomen, were pushed up through the enlarged aperture of the anus, immediately before the application of the bandages, for the purposes already detailed.

The preceding explanatory description of what appears, from the unquestionable facts collected in the course of my inquiry, to have been the best, and, in my opinion, the primitive mode of preparing mummies by the ancient Egyptians, differs from that found in HERODOTUS, as well as from those accounts which we read in other writers who came after him. It does not however appear that the eminent historian just mentioned had ever been present at the embalming of a mummy, or that he ever had an opportunity of examining one of them. He must, therefore, like many other travellers, have noted down what he had collected from hearsay, in which, amidst much that was surmised, there was something approaching to the truth. It is in evidence that the art was kept a profound mystery among those who professed it, so that the different modes of embalming described with such orderly minuteness of details by HERODOTUS, could only have been conjectural. It is a curious fact, that, with the exception of the lateral incision, and immersion into a saline solution mentioned by that historian, we find no confirmatory evidence of the other steps of the supposed processes of embalming detailed by him in any of the various mummies that have hitherto been examined. And in the one now submitted to the inspection of the Society, by far the most perfect that has yet been publicly described, we have none of the characteristic features of the three several modes of embalming which we are told were followed by the ancient Egyptians; while, on the other hand, some of the lesser features of each process are strikingly apparent. We have, in fact, the presence of that which HERODOTUS asserted was invariably removed in the better

prepared mummies, and some of those parts are absent, on the other hand, which he stated never to have been touched in the inferior class of those singular preparations. These facts will be duly valued by the scholar, and the commentators of that historian; and the explanation now given of the real mode of mummifying, will enable the lexicographer to advance with confidence, that the name *mummy* was given to such preparations from the circumstance of *wax* (*mum* in the Cophtic language), being the really preservative ingredient employed in their preparation.

I have had occasion in the course of this paper to observe, that as by carefully taking into consideration the various facts which presented themselves during the examination of our mummy, it was natural to suppose, that the mode in which it had been prepared would be discovered; so would that discovery be confirmed if, by acting on those facts, something resembling a mummy could be produced; and in the specimens which will be submitted to the members after the meeting, the different steps will be seen, by which I was led to what may be considered as an imitation of the Egyptian mummies.*

* There were exhibited after the meeting four different specimens of imitative mummies, each of them illustrative of one or two of the successive stages of the process of embalming detailed in this essay; the last being intended to illustrate all the stages together, and exhibiting a close resemblance to the Egyptian mummy itself. A still born child had been employed for the purpose, and this modern mummy has now been in existence upwards of three years, without bandage or covering of any kind, exposed to all sorts of temperature and rough usage without betraying the slightest vestige of decay or putrefaction. It is rather darker than the Egyptian mummy from the circumstance of a too concentrated solution of tannin having been employed in preparing it.

I purposely omit speaking of the various modes of embalming adopted by different nations, or of those which may have prevailed at different epochs in Egypt; although in the course of my investigation I collected ample materials for entering into such a subject. The art of embalming, with a view to the preservation of the human body, for an indefinite series of years, as strictly illustrated by the mummies of ancient Egypt, does not appear to have been practised with success by any other nation. We find no remains of such high antiquity in any other part of the world; and the mummies of Mexico, those of the Atlantic islanders, the dried bodies found in the catacombs of some of the states bordering on the Mediterranean, are but of yesterday, compared to the age of the mummy which I have had the honour of bringing under the notice of the Society. Indeed the art soon began to decline among the Egyptians themselves, and the mummies found in the hypogei which bear evidence of having been more recently erected, as well as those of the plain of Saqquarah, are, in every respect, inferior to the *primitive* mummies. Whether this arose from the growing ignorance of the real process, the directions respecting which could only have been handed down traditionally; or from carelessness in the operation; or from indifference on the part of the people toward such an object; or from all these causes united, it is not easy now to determine. Certain it is, that the genuine process of embalming, among the Egyptians under the dynasty of the Pharaohs described in this paper, appears to have been progressively disregarded, and forgotten among them, until at last it was lost altogether. Nor does it appear ever to have been known by other nations,

In order to appreciate properly the durability of the bodies prepared by the Egyptian process, it is essential to observe, that the mummy I have described with so much minuteness, after having resisted putrefaction for above three thousand years, covered by bandages, inclosed in a thick wooden case, and placed in recesses, far from the external influence of atmospheric vicissitudes, has since withstood the inclemency and variations of an English climate, without any of those protecting circumstances; nay, exposed purposely, but ineffectually, for four years, to the various causes that are known to favour putrefaction.*

The deep feelings of interest that have of late been excited respecting the Egyptians, have induced me to extend my present inquiry to a greater length, than I should have done under less inviting circumstances. It was impossible not to feel extremely interested in the subject; and when I beheld before me the heart of an Egyptian female, whom imagination, aided by historical records, may fancy to have been cotemporary with the great Sesostris, I could not help experiencing a degree of enthusiasm, a portion of which, methought, I could impart to others.

I recollect with pleasure the sensation which the demon-

* A singular contrast this, with what has since happened to one of the nates alluded to in a previous note. Being divested of the protecting and embalming ingredient, by the process I there alluded to, this part has partially run into putrefaction, and emits the peculiar smell of animal substances, placed under similar circumstances. Nay, in the case of one of the large muscles of the thigh, and a large portion of the integument, which I similarly deprived of their protecting ingredients, such has been the rapidity with which putrefaction has followed, that although well covered, the vessels containing those parts emitted the most insufferable smell, and the parts themselves were found infested with myriads of large maggots.

stration of the various parts of this mummy, at the time it was first opened, excited amongst upwards of an hundred scientific and literary characters, who in the course of six weeks honoured me with their presence at my house to witness the dissection, and by whom I was encouraged to follow up the investigation, and to communicate the result to the public. It is in obedience to their suggestion, and more especially to the recommendation of the President of the Royal Society, that I have taken a comprehensive view of the whole subject, instead of limiting myself to the dry description of a solitary specimen.

Fig. 1.

Fig. 2.

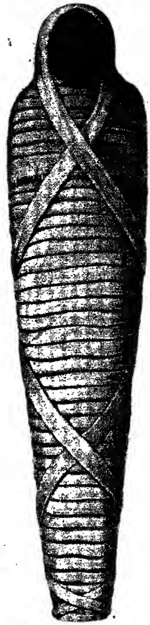
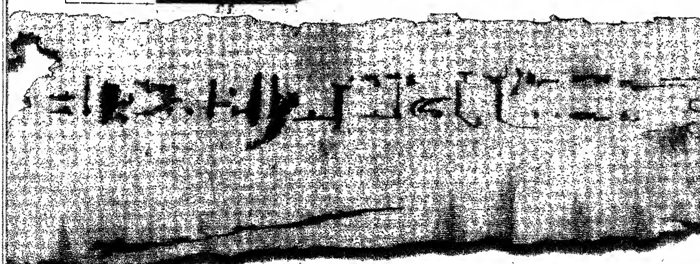
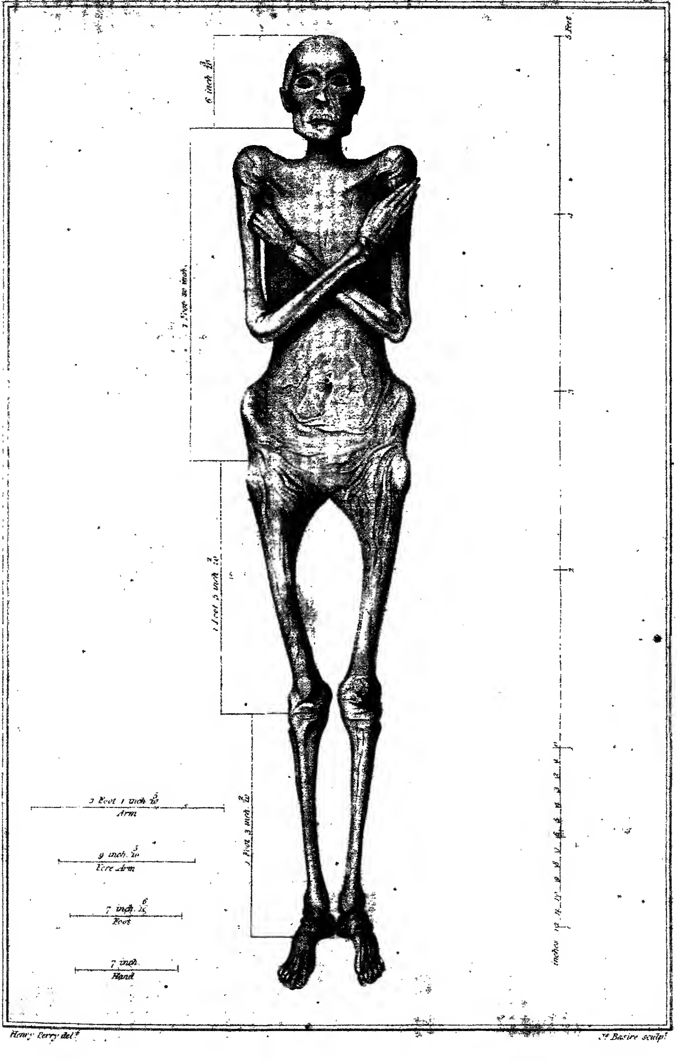


Fig. 3.

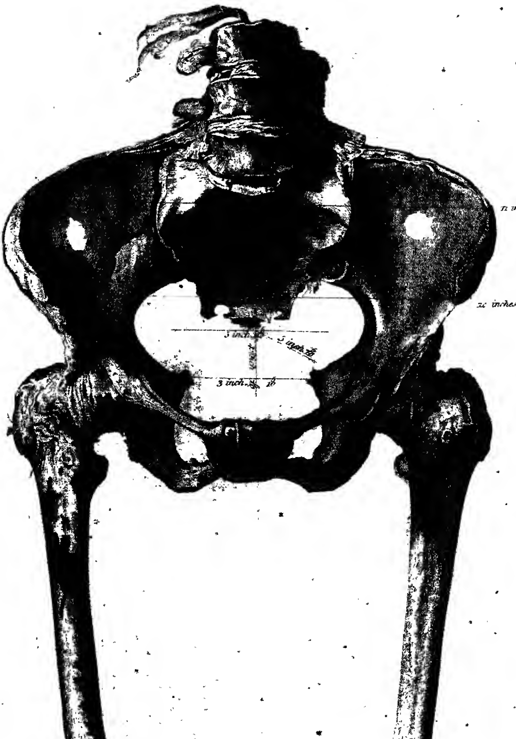


Drawn by Henry Perry, [illegible] Street, Hanover Square.

Engraved by J. Basire.



Henry Perry del.t J.s Basire sculp.t



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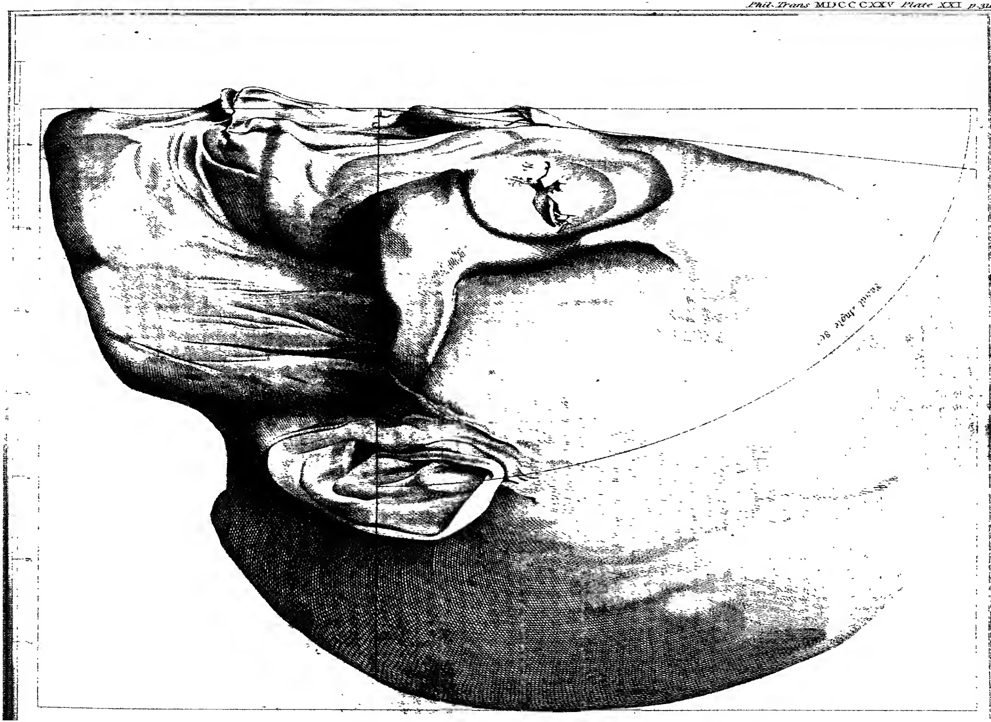

Facial Angle 90°

Fig. 1.

Fig. 2

Fig. 3.

Henry Perry del.t

J. Basire sculp.t

Fig. 1.

Fig. 2.

Fig. 3.

Henry Perry del.t J.s Basire sculp.t

Fig. 1.

Fig. 2.

Henry Perry del.t

J.s Basire sculp.t

XIV. *On the temporary magnetic effect induced in iron bodies by rotation. In a Letter to* J. F. W. Herschel, *Esq. Sec. R. S. by* Peter Barlow, *F. R. S. Communicated April 14th, 1825.*

Read May 5, 1825.

Dear Sir,

It is more than two years since, in a conversation I had with you on subjects connected with magnetism, you enquired what effect I thought might result from giving to an iron ball a rapid rotation? The subject however dropped, and it did not occur to me again, till in some speculative views in which I was lately engaged, as to the cause of the rotation of the earth's magnetic poles, the apparent irregularity of the terrestrial directive powers, &c. I was led to consider that, probably, rotation might have a certain influence. We know that iron is rendered magnetic by various processes, as drilling, hammering, &c. and it was possible also by rotation; your query now occurred to my mind; and knowing at the same time that Mr. Christie had found a permanent change in the magnetic state of an iron plate by a mere change of position on its axis, it seemed highly probable this change, due only to a simple inversion, would be increased by a rapid rotation. In this respect, however, I was deceived; for I found afterwards, that all the effect that was produced was merely temporary; and if any permanent change did take place, it was too small in my cast iron shell to be observed with the small compass I employed in these experiments.

Being however thus urged to the inquiry, as well by my own speculative views as by your query, and encouraged by Mr. Christie's results, I resolved to put the idea to the test of experiment, and to attempt it at once upon a scale that should decide the question in the first instance.

As soon as I had determined upon the experiment, I found an excellent opportunity of making the first trial, through the kindness of Generals Cuppage and Millar, of the Royal Artillery, who gave me permission to have a 13 inch mortar shell fixed to the mandrel of one of the powerful turning lathes worked by the steam engine in the Royal Arsenal. This having been done, and the compass properly placed near the shell, I turned the shell slowly round, in order to ascertain whether in this case, as in Mr. Christie's, there were any effects depending on a change of position; but if there were any, it was so small in the cast iron shell as not to have been rendered sensible with the small compass I employed. The wheel being now put in geer, the shell commenced its revolutions at the rate of 640 per minute, and the needle was deflected out several degrees, at which it remained perfectly stationary while the ball was in motion: but it returned immediately to its original bearing as soon as the motion ceased.

I now inverted the motion of the shell, and the needle was deflected about the same quantity the contrary way, observing a similar steady direction as in the former case; but as before, it returned to its original bearing the moment the motion was discontinued.

These experiments were repeated several times before some scientific officers of the artillery and engineers, always with the same results.

I afterwards found, that the needle being placed in different situations, its motion was reversed, although the direction of motion in the shell was the same; the amount of the whole deflection also differed very considerably according to the situation of the compass, its direction in some cases having been wholly reversed, while in others no perceptible motion was produced, although the rotation of the shell remained the same both in direction and in speed.

I was therefore desirous of undertaking a regular set of experiments, in order to reduce the several apparently anomalous results to some certain law of action; and as the shell in question was rather too heavy for us to feel a perfect security, as to personal safety, when it was in rapid rotation, and moreover, as its effects were larger than seemed necessary for the purpose, I now selected a Shrapnel shell of 8 inches in diameter, which weighed only 30 lbs. and chose another lathe, whose axis was nearly north and south, that in the former instance having been east and west. I had also a table made with a circular hole in it, which I could place at any height above, below, or about the centre of the ball; I could also set my compass on any azimuth on the same, and observe the effects of the direct and reversed motion; but after several days observations, I found the results so complicated, and the needle so much influenced by the iron work of the lathe and other machinery, that it would be useless to proceed, unless I could contrive to produce the rotation out of the way of any disturbing cause of the kind above mentioned.

This also, through the kindness of Colonel Sir Alexander Dickson, and the officers above named, I was enabled to

accomplish; and having got the machine erected on my own premises, I was soon enabled to clear up the difficulties which had hitherto so much embarrassed my proceedings, although even here, in the first instance, I found some results very difficult to explain.

The machine I now employed is shown in the annexed drawing. Plate XXIV.

ABCD is a strong wooden frame, resembling that of a common electrical machine, the shell S being hung in the same manner as the cylinder; the axis is made in two parts of gun metal, and very strong; *s s* are two strong screw bolts and nuts, which were used for fixing the frame firmly to the top of the table, the bolt passing through from below. EGF is a substantial table with its feet sunk into the ground, and the floor of the room cut away where they passed through, in order to prevent any effect of shaking on the stand carrying the compass.

The stand consisted of an upright pedestal filled with sand, to render it steady, and to this was fixed the table ML, with a semicircular hole cut in it, so that it might be placed near the shell. This table might be elevated or depressed at pleasure, and it was divided into the points, quarter points, &c. of the compass.

By means of different holes bored in the top of the table, the machine might be placed N and S, E and W, &c. at pleasure, and the motion of the shell be inverted by turning the handle to the right or left. The large wheel is six times the diameter of the small one; and as it might easily be turned twice in a second, the number of revolutions of the shell were gradually about 720 per minute. The little apparatus

seen above the shell, is a small stand and sliding wire, carrying a common lamp glass, in which a very small dipping needle was suspended by silk; and when the lamp glass was out of the ring, the latter served for setting the horizontal needle on, so as to bring it over any required point of the shell. It should be observed that the pedestal was moveable, and might therefore be placed on either side of the machine. The stand and upright figure 2, is one of two large magnets ultimately employed for neutralizing the needle.

The machine being thus prepared, I screwed it down; first with its axis in the magnetic meridian, and then placed the compass successively at the several points on the table all round, and registered the deviation produced at each, with the motion of the shell direct and reversed. I then removed it, and placed the axis east and west, and again registered in the same manner; but the results were very irregular with respect to quantity. Although I obtained some uniformity *regarding direction only*, viz. in both cases I found four points of change at about 30° from each extremity of the axis, or four points of non action. For example, when the axis was in the meridian from N 30° E to N 30° W, the motion of the needle arising from the rotation was made to the right. From N 30° W to S 30° W to the left. From S 30° W to S 30° E to the right. From S 30° E to N 30° E to the left; the direction of motion in the shell being the same; with the direction of motion reversed, the deviation was reversed also. While at these four points themselves, the needle had no motion. I tried also a variety of other positions, but I could obtain no such results as to lead to a concise expression of

the effect, and for this reason I shall not trouble you with the detail of them.

It at length occurred to me, that the reason of my failure arose from the compound influence under which the needle was placed, viz. that of the iron ball and of the earth; I therefore now neutralized it from the effect of both, by means of magnets properly disposed, adjusting it always before the rotation to a direction tangential to the ball, so that whatever effect was produced at each point, might at least become decided as to its direction. I now immediately arrived at that kind of general law I had been in search of; for I found when things were thus arranged, that whatever might be the direction of the axis of rotation, if the motion of the ball were made towards the needle, the north end of the latter was attracted; and if from the needle, the north end was repelled by the iron, the points immediately in the axis (when of course the motion of the shell was parallel to the needle) being neutral, or those at which the change of direction took place; in other words, if the motion of the shell continue the same, and the compass be successively placed all round the ball, in that semi-circle (from one axis to the other) in which the motion is towards the needle, the north end approaches the ball, and in the other semicircle it recedes, or the south end approaches; the points of non action being in the two extremities of the axis, and those of maximum effect in two opposite points at right angles to the axis; in which two latter the needle, when properly neutralized, points directly to the centre of the ball.

This will be perhaps better understood by reference to fig. 3, where S is the shell, *a b* its axis, and *n s*, *n s*, &c. the

needle in its various positions prior to the motion, and $n' s'$, $n' s'$, &c. its direction as resulting from the motion; the rotation of the shell being from c towards d.. of course with the rotation reversed, the effect will be reversed also.

Now this effect you will, I think, find to be perfectly consistent with the view you have taken of the subject, in your letter of Jan. 13th, where you say in reference to your former query, and to the views I then entertained, "I should rather have expected a diminution of the magnetic polarity, commensurate to the rapidity of rotation and a change in the direction of the magnetic axis of the globe, from parallelism to that of the earth, to a position somewhere intermediate between that and the axis of rotation, but approaching nearer the latter as the velocity increased, &c."

The fact is, that the needle in my experiments being under no influence prior to the rotation from either the iron or the earth, the direction which it takes up in consequence of the motion, enables us to discover the precise direction of the new forces thus impressed upon the shell, and it will be seen immediately to indicate a polarization of the latter in the direction $c\,d$; that is, in a direction perpendicular to the axis of motion, and to the plane passing through that axis and the actual poles of the ball.

You will of course understand that I do not mean that such a polarization actually takes place; I mean merely that the cohesive power of the iron is such, as to resist in a certain degree the inductive powers of the earth, whereby the magnetic forces are changed, as you have suggested, from their original direction, parallel to the magnetic axis of the ball into a position oblique to it, which oblique forces

being resolved into two, the one parallel to the original axis, and the other perpendicular to it, and the former being nearly neutralized by the magnets used for the purpose in the first instance, the perpendicular forces will act upon the needle in the same manner as if the ball were really polarized in the direction above alluded to.

Having got this view of the subject, I soon found that many of my former results, which appeared to have scarcely any conformity among themselves, were perfectly consistent with this hypothesis: of these the experiments given above, before the needle was neutralized, may be mentioned. In these I found the point of change to be at about 30° on each side of the axis, so that the arcs in which similar effects were produced were divided into the unequal portions of 60°, 120°, 60°, and 120°, which appeared to be anomalous; but according to the view now taken of the subject, this is perfectly consistent; it is precisely what ought to happen according to the law tan. dip. = 2 tan. mag. lat. and which actually takes place on the earth. That is in passing from the magnetic equator 30° towards the pole, the dipping needle has actually described a quadrant, as referred to its position at the equator; and it would describe a quadrant, in an opposite direction in going 30° towards the other pole; so that in passing through 60° the needle is actually inverted; but if we start from mag. lat. 30° through the pole, we must pass through an arc of 120° before the direction of the needle is inverted, and the same in the other half of the meridian; and in like manner by referring the motion of my needle as induced by the rotation of the shell to its original magnetic direction, it is obvious that I ought to have found, as I actually

did before I was aware of the cause, a point of change at 30° distance on each side of the meridian passing through the axis; which meridian, as respects the induced power, is actually the equator of the new magnetic sphere.

To render this more obvious, let us refer to fig. 4, in which AB represents the axis of rotation of the shell, the black lines the needle in its natural direction, and the dotted lines the direction the needle has a tendency to assume according to the law above named, in consequence of the magnetism impressed by the rotation in the line *n s*. Beginning at the point A, if we say the motion is from left to right, that is from *n* to *n'*, it will be from right to left at 60°, 75°, 90°, &c. till we arrive again at 30°; at this point as at the former the new power is exerted in the actual direction of the needle, and if it were greater than its natural directive power, it would wholly invert it; in this case it would pass to either hand; but as the new power cannot invert it, it has no tendency to deflect it, and it therefore remains stationary. Thus one of the results which was at first the most perplexing, serves to confirm the law we have established.

On similar principles, if we conceive a circle passing vertically from 90° to 90°, and if the needle be perfectly neutralized at different positions in this circle, and rendered parallel to the axis at each, then in every case the needle will have a tendency to take up a position directly at right angles to the axis of the shell, and it will point in opposite directions at certain parts of this circle: thus, if to fix the idea we conceive the axis to be in the meridian, and the motion of the shell from west to east, then at the east point of the horizon the needle will point to the west, and it will do the

same at all points between the horizon and an altitude of 60°; beyond this, the north end will point to the east till we have passed the zenith 30° on the west side; and then again from this point to the west horizon the north end will again point to the west; and similar changes will take place below the ball. This, which is a necessary consequence of our hypothesis, is completely verified by experiment.

It will of course be understood, that the supposition of the axis being in the meridian is merely to fix the idea; for a similar motion takes place whatever direction the axis of the shell may have.

It is presumed, that what has now been stated is sufficient, without referring to any further experiments, to establish the principal fact adverted to in this letter, viz. that when any iron body is put in rapid rotation on any line not coinciding with its magnetic axis, a temporary derangement takes place in its magnetic powers, which in its effects is equivalent to a new axis of polarization perpendicular to the plane passing through its axis of polarization and rotation.

I have stated in the beginning of this letter the motives which led me to undertake these experiments; but notwithstanding I have certainly found a stronger effect produced by the rotation than I anticipated, yet it does not appear to be of a kind to throw any new light upon the difficult subject of terrestrial magnetism. I think there are strong reasons for assuming, that the magnetism of the earth is of that kind which we call induced magnetism; but at present we have no knowledge of the inductive principle, and are therefore unable to judge, how far the earth's rotation may be influential in producing those discrepancies from the gene-

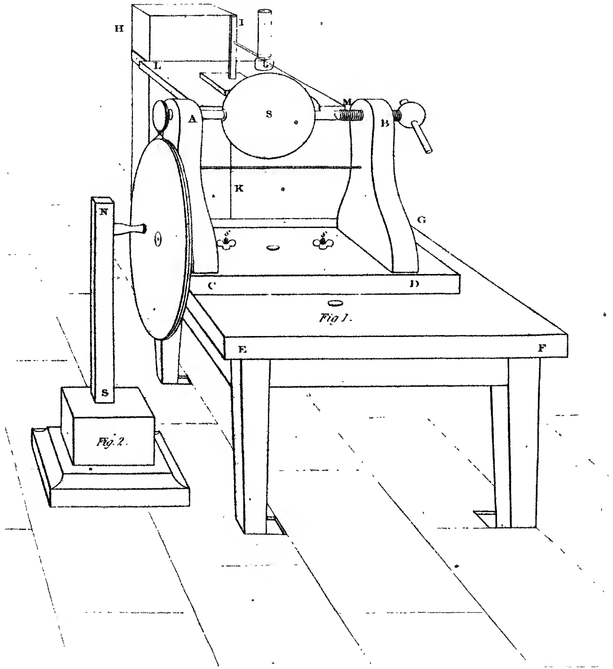


Fig 1.

Fig. 2.

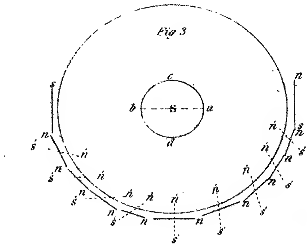


Fig 3

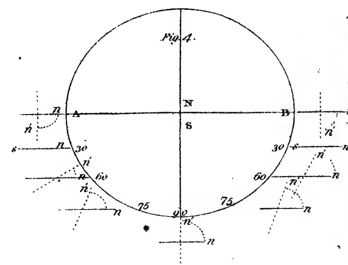


Fig. 4.

ral laws which are known to exist. The formula, which we owe to Mr. BIOT and to Mr. KRAFT, expressing the law of the dips in different latitudes, viz. tan. dip. = 2 tang. mag. lat. certainly agrees with observation in many cases, but the variation computed on the same principles, and which necessarily ought also to coincide with observation, is widely in error. It seems therefore very obvious that some disturbing cause exists, but whether any part of it can be attributed to the rotation of the earth is, notwithstanding the preceding results, very doubtful; at the same time I may perhaps be allowed to observe, that one of the essential conditions for the production of such an effect has place in the earth, viz. that it does not revolve about its polarized axis; and if the inductive principle through which it receives its magnetism be exterior to itself, then it would follow almost of necessity that some such an effect should take place. I beg however to be understood as advancing nothing in this letter, beyond the mere experimental fact above stated, which, even if it should find no useful application, may perhaps be thought sufficiently curious to be recorded in the Transactions of the Royal Society.

PETER BARLOW.

XV. *Further researches on the preservation of metals by electro-chemical means.* By *Sir* HUMPHRY DAVY, *Bart. Pres. R. S.*

Read June 9, 1825.

IN two papers read before the Royal Society, I have described the effects of small quantities of electro-positive metals in preventing the corrosion or chemical changes of copper exposed to sea water, and I have stated that the results appear to be of the same kind, whether the experiments are made upon a minute scale, and in confined portions of water, or on large masses, and in the ocean.

The first and preliminary experiments proved, that the copper sheeting of ships might be preserved by this method; but another and a no less important circumstance was to be attended to, how far the cleanness of the bottom, or its freedom from the adhesion of weeds or shell fish, would be influenced by this preservation.

The use of the copper sheathing on the bottom of ships is two fold: First, to protect the wood from destruction by worms:

And secondly, to prevent the adhesion of weeds, barnacles, and other shell fish. No worms can penetrate the wood as long as the surface of the copper remains perfect; but when copper has been applied to the bottom of a ship for a certain time, a green coating or rust, consisting of oxide, submuriate

and carbonate of copper and carbonate of magnesia forms upon it, to which weeds and shell fish adhere.

As long as the whole surface of the copper changes or corrodes, no such adhesions can occur; but when this green rust has partially formed, the copper below is protected by it, and there is an unequal action produced, the electrical effect of the oxide, submuriate, and carbonate of copper formed, being to produce a more rapid corrosion of the parts still exposed to sea water; so that the sheets are often found perforated with holes in one part, after being used five or six years, and comparatively sound in other parts.

There is nothing in the poisonous nature of the metal which prevents these adhesions. It is *the solution* by which they are *prevented—the wear* of surface. Weeds and shell fish readily adhere to the poisonous salts of lead which form upon the lead protecting the fore part of the keel; and to the copper, in any chemical combination in which it is insoluble.

In general in ships in the navy the first effect of the adhesion of weeds is perceived upon the heads of the mixed metal nails, which consist of copper alloyed by a small quantity of tin. The oxides of tin and copper which form upon the head of the nail and in the space round it, defend the metal from the action of sea water; and being negative with respect to it, a stronger corroding effect is produced in its immediate vicinity, so that the copper is often worn into deep and irregular cavities in these parts.

When copper is unequally worn, likewise in harbours or seas where the water is loaded with mud or mechanical deposits, this mud or these deposits rest in the rough parts or

depressions in the copper, and in the parts where the different sheets join, and afford a soil or bed in which sea weeds can fix their roots, and to which zoophytes and shell fish can adhere.

As far as my experiments have gone, small quantities of other metals, such as iron, tin, zinc, or arsenic, in alloy in copper, have appeared to promote the formation of an insoluble compound on the surface; and consequently there is much reason to believe must be favourable to the adhesion of weeds and insects.

I have referred in my last paper to the circumstance of the carbonate of lime and magnesia forming upon sheets of copper, protected by a quantity of iron above $\frac{1}{120}$ parts, when these sheets were in harbour and at rest.

The various experiments that I have caused to be made at Portsmouth, show all the circumstances of this kind of action, and I have likewise elucidated them by experiments made on a smaller scale, and in limited quantities of water. It appears from these experiments, that sheets of copper at rest in sea water, always increase in weight from the deposition of the alkaline and earthy substances, when defended by a quantity of cast iron under $\frac{1}{150}$ of their surface, and if in a limited or confined quantity of water, when the proportion of the defending metal is under $\frac{1}{4000}$. With quantities below these respectively proportional for the sea, and limited quantities of water, the copper corrodes; at first it slightly increases in weight, and then slowly loses weight. Thus a sheet of copper 4 feet long, 14 inches wide, and weighing 9 lb. 6 oz., protected by $\frac{1}{100}$ of its surface of cast iron, gained in ten weeks and five days, 12 drachms, and was coated

over with carbonate of lime and magnesia: a sheet of copper of the same size protected by $\frac{1}{150}$, gained only 1 drachm in the same time, and a part of it was green from the adhering salts of copper; whilst an unprotected sheet of the same class, both as to size and weight, and exposed for the same time, and as nearly as possible under the same circumstances, had lost 14 drachms; but experiments of this kind, though they agree when carried on under precisely similar circumstances, must of necessity be very irregular in their results, when made in different seas and situations, being influenced by the degree of saltness, and the nature of the impregnations of the water, the strength of tide and of the waves, the temperature, &c.

In examining sheets which had been defended by small quantities of iron in proportions under $\frac{1}{200}$ and above $\frac{1}{1000}$, whether they were exposed alone, or on the sides of boats, there seemed to me no adhesions of confervæ, except in cases where the oxide of iron covered the copper immediately round the protectors; and even in these instances such adhesions were extremely trifling, and might be considered rather as the vegetations caught by the rough surface of the oxide of iron, than as actually growing upon it.

Till the month of July 1824 all the experiments had been tried in harbour, and in comparatively still water; and though it could hardly be doubted, that the same principles would prevail in cases where ships were in motion, and on the ocean; yet still it was desirable to determine this by direct experiment; and I took the opportunity of an expedition intended to ascertain some points of longitude in the north seas, and which afforded me the use of a steam boat, to make these

researches. Sheets of copper carefully weighed, and with different quanties of protecting metal, and some unprotected, were exposed upon canvass so as to be electrically insulated upon the bow of the steam boat; and were weighed and examined at different periods, after being exposed in the north seas to the action of the water during the most rapid motion of the vessel. Very rough weather interfered with some of these experiments, and many of the sheets were lost, and the protectors of others were washed away; but the general results were as satisfactory as if the whole series of the arrangements had been compleat. It was found that undefended sheets of copper of a foot square lost about 6.55 grains in passing at a rate averaging that of eight miles an hour in twelve hours; but a sheet, having the same surface, defended by rather less than $\frac{1}{500}$ lost 5.5 grains; and that like sheets defended by $\frac{1}{70}$ and $\frac{1}{100}$ of malleable iron were similarly worn, and underwent nearly the same loss, that of two grains, in passing through the same space of water. These experiments (the results of which were confirmed by those of others made during the whole of a voyage to and from Heligoland, but in which during the return the protectors were lost) shows that motion does not affect the nature of the limits and quantity of the protecting metal; and likewise prove, that independently of the chemical, there is a mechanical wear of the copper in sailing, and which on the most exposed part of the ship, and in the most rapid course, bears a relation to it of nearly 2 to 4.55.

I used the very delicate balance belonging to the Royal Society in these experiments; the sheets of copper weighed between 7 and 8000 grains; and I was fully enabled to ascer-

tain by means of this balance, a diminution of weight upon so large a quantity, equal to $\frac{1}{100}$ of a grain. It was evident from a very minute inspection of the sheet with the largest quantity of protecting metal, that there was not any adhesion of alkaline or earthy substances to its surface.

Having observed in examining the results of some of the experiments on the effects of single masses of protecting metal on the sheeting of ships, that there was in some cases in which sheets with old fastening had been used, tarnish or corrosion, which seemed to increase with the distance from the protecting metal, it became necessary to investigate this circumstance, and to ascertain the extent of the diminution of electrical action in instances of imperfect or irregular conducting surfaces.

With single sheets or wires of copper, and in small confined quantities of sea water, there seemed to be no indications of diminution of conducting power, or of the preservative effects of zinc or iron, however divided or diffused the surface of the copper, provided there was a perfect metallic connection through the mass. Thus, a small piece of copper containing about 32 square inches, was perfectly protected by a quantity of zinc which was less than $\frac{1}{2000}$ part of the whole surface; and a copper wire of several feet in length was prevented from tarnishing by a piece of zinc wire which was less than $\frac{1}{1200}$ part of its length. In these cases the protecting metal corroded with great rapidity, and in a few hours was entirely destroyed; but when applied in the form of wire and covered, except at its transverse surface, with cement, its protecting influence upon the same minute scale was exhibited for many days. A part of these results depend upon the absorption

of the oxygen dissolved in the water when its quantity is limited, by the oxidable metal, and of course the proportion of this metal must be much larger when the water is constantly changing; but the experiments seem to show that any diminution of protecting effect at a distance, does not depend upon the nature of the metallic, but of the imperfect or fluid conductor.

This indeed is shown by many other results.

A piece of zinc and a piece of copper in the same vessel of sea water, but not in contact, were connected by different lengths of fine silver wire of different thickness. It was found that whatever lengths of wire of $\frac{1}{300}$ of an inch were used, there was no diminution of the protecting effect of the zinc; and the experiment was carried so far as to employ the whole of a quantity of extremely fine wire, amounting to upwards of forty feet in length, and of a diameter equal only to $\frac{100}{98742}$ of an inch, when the results were precisely the same as if the zinc and copper had been in immediate contact.

Pieces of charcoal, which is the worst amongst the more perfect conductors, were connected by being tied together, and made the medium of communication between zinc and copper, upon the same principles, and with the same views as those just described, and with precisely the same consequences.

In my first experiments upon the effects of increasing the length or diminishing the mass of the imperfect or fluid conducting surface in interfering with the preserving effects of metals, I used long narrow tubes; but I found them very inconvenient; and I had recourse to the more simple method of employing cotton or tow for this purpose.

Several feet of copper wire in a spiral form were connected

with a small piece of zinc wire of about half an inch in length. The zinc and a portion of the copper were introduced into one glass, and the coils of copper wire were introduced into other glasses, so as to form a series of six or seven glasses, which were filled with sea water, and made part of the same voltaic arrangement, by being connected with pieces of tow moistened in sea water.

It was found in these experiments, that when the pieces of tow connecting the glasses were half an inch in thickness, the preserving effect of the zinc in the first glass was no where diminished, but extended apparently equally through the whole series.

When the pieces of tow were about the fifth of an inch in thickness, a diminution of the preserving effects of the zinc was perceived in the fourth glass, in which there was a slight solution of copper; in the fifth glass this result was still more distinct, and so on till in the seventh glass there was a considerable corrosion of the copper.

When the tow was only the tenth of an inch in thickness, the preserving effect of the zinc extended only to the third glass; and in each glass more remote, the effect of corrosion was more distinct, till in the seventh glass it was nearly the same as if there had been no protecting metal. All the chemical changes dependent upon negative electricity were successively and elegantly exhibited in this experiment. In the first glass containing the zinc, there was a considerable and hasty deposition of earthy and alkaline matter, and crystals of carbonate of soda adhered to the copper at the surface where it was clean and bright; but in the lower part it was coated with revived metallic zinc. In the second glass the

wire was covered over with fine crystals of carbonate of lime; and the same phænomenon of the separation of carbonate of soda occurred, but in a less degree. In the third glass the wire was clean, but without depositions; and the presence of alkaline matter could only be distinguished by chemical tests. In the fourth glass the copper was bright, evidently in consequence of a slight but general corrosion, but with a scarcely sensible deposit; in the fifth, the deposit was very visible; and in the seventh the wire was covered with green rust.

These results, which showed that a very small quantity only of the imperfect or fluid conductor was sufficient to transmit the electrical power, or to compleat the chain, induced me to try if copper nailed upon wood, and protected merely by zinc or iron on the under surface, or that next the wood, would not be defended from corrosion. For this purpose I covered a piece of wood with small sheets of copper, a nail of zinc of about the $\frac{1}{200}$ part of the surface of the copper being previously driven into the wood: the apparatus was plunged in a large jar of sea water: it remained perfectly bright for many weeks, and when examined, it was found that the zinc had only suffered partial corrosion; that the wood was moist, and that on the interior of the copper there was a considerable portion of revived zinc, so that the negative electricity, by its operation, provided materials for its future and constant excitement. In several trials of the same kind, iron was used with the same results; and in all these experiments there appeared to be this peculiarity in the appearance of the copper, that unless the protecting metal below was in very large mass, there were no depositions of

calcareous or magnesian earths upon the metal; it was clean and bright, but never coated. The copper in these experiments was nailed sometimes upon paper, sometimes upon the mere wood, and sometimes upon linen; and the communication was partially interrupted between the external surface and the internal surface by cement; but even one side or junction of a sheet seemed to allow sufficient communication between the moisture on the under surface and the sea water without, to produce the electrical effect of preservation.

These results upon perfect and imperfect conductors led to another enquiry, important as it relates to the practical application of the principle; namely, as to the extent and nature of the contact or relation between the copper and the preserving metal. I could not produce any protecting action of zinc or iron upon copper through the thinnest stratum of air, or the finest leaf of mica, or of dry paper; but the action of the metals did not seem to be much impaired by the ordinary coating of oxide or rust; nor was it destroyed when the finest bibulous or silver paper, as it is commonly called, was between them, being moistened with sea water. I made an experiment with different folds of this paper. Pieces of copper were covered with one, two, three, four, five and six folds; and over them were placed pieces of zinc, which were fastened closely to them by thread; each piece of copper so protected was exposed in a vessel of sea water, so that the the folds of paper were all moist.

It was found in the case in which a single leaf of paper was between the zinc and the copper, there was no corrosion of the copper; in the case in which there were two leaves, there was a very slight effect; with three, the corrosion was dis-

tinct: and it increased, till with the six folds the protecting power appeared to be lost: and in the case of the single leaf, there was this difference from the result of immediate contact, that there was no deposition of earthy matter. Showing that there was no absolute minute contact of the metals through the moist paper; which was likewise proved by other experiments: for a thin plate of mica, as I have just mentioned, entirely destroyed the protecting effect of zinc: and yet when a hole was made in it, so as to admit a very thin layer of moisture between the zinc and copper, the corrosion of the copper, though not destroyed, was considerably diminished.

The rapid corrosion of iron and zinc, particularly when used to protect metals, only in very small quantities, induced me to try some experiments as to their electro-chemical powers in menstrua out of the contact, or to a certain extent removed from the contact of air, such as might be used for moistening paper under the copper sheathing of ships: the results of these experiments I shall now detail. A small piece of iron was placed in one glass filled with a saturated solution of brine, which contains little or no air; copper, attached by a wire to the iron, was placed in a vessel containing sea water, which was connected with the brine by moistened tow. The copper did not corrode, and yet the iron was scarcely sensibly acted upon, and that only at the surface of the brine; and a much less effect was produced upon it in many weeks than would have been occasioned by sea water in as many days.

With zinc and brine in the same kind of connection there was a similar result: but the solution of the zinc was com-

paratively more rapid than that of the iron, and the copper was rendered more highly negative, as was shown by a slight deposition of earthy matter upon it.

A solution of potassa, or of alkaline substances possessing the electro-positive energy, has nearly the same effect on saline solutions as if they were deprived of air; and when mixed with sea water impedes the action of metals upon them; but if used in quantity in combinations such as these I have just described, in which iron is the protecting metal, it destroys the result, and renders the iron negative. Thus, if iron and copper in contact, or fastened to each other by wires, be in two vessels of sea water connected by moist cotton or asbestos, all the various circumstances of protection of the two metals by each other may be exhibited by means of solution of potassa. By adding a few drops of solution of potassa to the water in the glass containing the iron, the negative powers of the copper in the other glass are diminished; so that the deposition of the calcareous and magnesian earths upon it is considerably lessened; by a little more solution of potassa the deposition is destroyed, but still the copper remains clean. The corrosion of the iron, which before was rapid, is now almost at an end; and a few drops more of the solution of potassa produces a perfect equilibrium: so that neither of the metals undergoes any change, and the whole system is in a state of perfect repose. By making the fluid in the glass containing the iron still more alkaline, it no longer corrodes; and the green tint of the sea water shows that the copper is now the positively electrified metal; and when the solution in the glass containing the iron is strongly alkaline, the copper in the other glass corrodes with

great rapidity, and the iron remains in the electro-negative and indestructible state.

I began this paper by some observations upon the nature of the processes by which copper sheeting is destroyed by sea water, and on the causes by which it is preserved clean, or rendered foul by adhesions of marine vegetables or animals; I shall conclude it by some further remarks on the same subject, and with some practical inferences and some theoretical elucidations, which naturally arise from the results detailed in the foregoing pages.

The very first experiment that I made on harbour-boats at Portsmouth, proved that a single mass of iron protected fully and entirely many sheets of copper, whether in waves, tides, or currents, so as to make them negatively electrical, and in such a degree as to occasion the deposition of earthy matter upon them; but observations on the effects of the single contact of iron upon a number of sheets of copper, where the junctions and nails were covered with rust, and that had been in a ship for some years, showed that the action was weakened in the case of imperfect connexions by distance, and that the sheets near the protector were more defended than those remote from it. Upon this idea I proposed, that when ships, of which the copper sheathing was old and worn, were to be protected, a greater proportion of iron should be used, and that if possible it should be more distributed. The first experiment of this kind was tried on the Sammarang, of 28 guns, in March, 1824, and which had been coppered three years before in India. Cast iron, equal in surface to about $\frac{1}{80}$ of that of the copper was applied in four masses, two near the stern, two on the bows. She

made a voyage to Nova Scotia, and returned in January 1825. A false and entirely unfounded statement respecting this vessel was published in most of the newspapers, that the bottom was covered with weeds and barnacles. I was present at Portsmouth soon after she was brought into dock: there was not the smallest weed or shell-fish upon the whole of the bottom from a few feet round the stern protectors to the lead on her bow. Round the stern protectors there was a slight adhesion of rust of iron, and upon this there were some zoophytes of the capillary kind, of an inch and a half or two inches in length, and a number of minute barnacles, both Lepas anatifera and Balanus tintinnabulum. For a considerable space round the protectors, both on the stern and bow, the copper was bright; but the colour became green towards the central parts of the ship; yet even here the rust or verdigrease was a light powder, and only small in quantity, and did not adhere, or come off in scales, and there had been evidently little copper lost in the voyage. That the protectors had not been the cause of the trifling and perfectly insignificant adhesions by any electrical effect, or by occasioning any deposition of earthy matter upon the copper, was evident from this—that the lead on the bow, the part of the ship most exposed to the friction of the water, contained these adhesions in a much more accumulated state than that in which they existed near the stern; and there were none at all on the clean copper round the protectors in the bow; and the slight coating of oxide of iron seems to have been the cause of their appearance.

I had seen this ship come into dock in the spring of 1824, before she was protected, covered with thick green carbonate

and submuriate of copper, and with a number of long weeds, principally fuci, and a quantity of zoophytes, adhering to different parts of the bottom; so that this first experiment was highly satisfactory, though made under very unfavourable circumstances.

The only two instances of vessels which have been recently coppered, and which have made voyages furnished with protectors, that I have had an opportunity of examining, are the Elizabeth yacht, belonging to the Earl of DARNLEY, and the Carnebrea Castle, an Indiaman, belonging to Messrs. WIGRAM. The yacht was protected by about $\frac{1}{125}$ part of malleable iron placed in two masses in the stern. She had been occasionally employed in sailing, and had been sometimes in harbour, during six months. When I saw her in November she was perfectly clean, and the copper apparently untouched. Lord DARNLEY informed me that there never had been the slightest adhesion of either weed or shell-fish to her copper, but that a few small barnacles had once appeared on the loose oxide of iron in the neighbourhood of the protectors, which however were immediately and easily washed off. The Carnebrea Castle, a large vessel of upwards of 650 tons, was furnished with four protectors, two on the stern, and two on the bow, equal together to about $\frac{1}{104}$ of the surface of the copper. She had been protected more than twelve months, and had made the voyage to Calcutta and back. She came into the river perfectly bright; and when examined in the dry dock was found entirely free from any adhesion, and offered a beautiful and almost polished surface; and there seemed to be no greater wear of copper than could be accounted for from mechanical causes.

Had these vessels been at rest, I have no doubt there would have been adhesions, at least in Portsmouth or Sheerness harbours, where the water is constantly muddy, and where the smallest irregularity or roughness of surface, from either wear, or the deposition of calcareous matter, or the formation of oxides or carbonates, enable the solid matter floating in the water to rest. There is a ship, the Howe, one of the largest in the Navy, now lying at Sheerness, which was protected by a quantity of cast iron judged sufficient to save all her copper, nearly fifteen months ago. She has not been examined; but I expect and hope that the bottom will be covered with adhesions, which must be the case if her copper is not corroded; but notwithstanding this, whenever she is wanted for sea, it will only be necessary to put her into dock for a day or two, scrape her copper, and wash it with a small quantity of acidulous water, and she will be in the same state as if newly coppered.

At Liverpool, as I am informed, several ships have been protected, and have returned after voyages to the West Indies, and even to the East Indies. The proportion of protecting metal in all of them has been beyond what I have recommended, $\frac{1}{90}$ to $\frac{1}{70}$; yet two of them have been found perfectly clean, and with the copper untouched after voyages to Demarara; and another nearly in the same state, after two voyages to the same place. Two others have had their bottoms more or less covered with barnacles; but the preservation of the copper has been in all cases judged complete. The iron has been placed along the keel on both sides; and the barnacles, in cases where they have existed, have been generally upon the flat of the bottom; from which it may

be concluded, that they adhered either to the oxide of iron, or the calcareous deposits occasioned by the excess of negative electricity.

In the navy the proportion adopted has been only $\frac{1}{250}$ of cast iron, at least for vessels in actual service, and when the object is more cleanness than the preservation of the copper.

It is very difficult to point out the circumstances which have rendered results, such as these mentioned with respect to Liverpool traders, so different under apparently the same circumstances, i. e. why ships should exhibit no adhesions or barnacles after two voyages, whilst on another ship, with the same quantity of protection, they should be found after a single voyage.* This may probably depend upon one ship having remained at rest in harbour longer than another, or having been becalmed for a short time in shallow seas, where ova of shell fish, or young shell fish existed; or upon oxide of iron being formed, and not washed off, in consequence of calm weather, and which consolidating, was not afterwards separated in the voyage. From what I can learn, however, the chance of a certain degree of foulness, in consequence of the application of the full proportion of protecting metal, will not prevent ship owners from employing this proportion, as the saving of copper is a very great object; and as long as the copper is sound, no danger is to be apprehended from worms.

It ought to be kept in mind that the larger a ship, the more the experiment is influenced by the imperfect conducting power of the sea water, and consequently the proportion of protecting metal may be larger without being in excess.

* The quality of the copper may be another cause.

I have mentioned these circumstances because they apply to ships already coppered, and because I have heard that a Liverpool ship, of which it was doubtful whether the copper was in a state such as would enable her to make another voyage to India with security, has, by the application of protectors of $\frac{1}{70}$, made this voyage,* without apparently any wear of her sheeting; and that she is now preparing with the same protectors to make another voyage.

In cases when ships are to be newly sheathed, the experiments which have been detailed in the preceding pages render it likely, that the most advantageous way of applying protection will be under, and not over the copper: the electrical circuit being made in the sea water passing through the places of junction in the sheets; and in this way every sheet of copper may be provided with nails of iron or zinc, for protecting them to any extent required. By driving the nail into the wood through paper wetted with brine *above* the tarred paper, or felt, or any other substance that may be employed, the incipient action will be diminished; and there is this great advantage, that a considerable part of the metal will, if the protectors are placed in the centre of the sheet, be deposited and re-dissolved: so there is reason to believe that small masses of metal will act for a great length of time. Zinc, in consequence of its forming little or no insoluble compound in brine or sea water, will be preferable to iron for this purpose; and whether this metal or iron be used, the waste will be much less than if the metal was exposed on the outside: and all difficulties with respect to a proper situation in this last case are avoided.

* The Dorothy.

The copper used for sheathing should be the purest that can be obtained; and in being applied to the ship, its surface should be preserved as smooth and equable as possible: and the nails used for fastening should likewise be of pure copper; and a little difference in their thickness and shape will easily compensate for their want of hardness.

In vessels employed for steam navigation the protecting metal can scarcely be in excess;* as the rapid motion of these ships prevents the chance of any adhesions; and the wear of the copper by proper protection is diminished more than two-thirds.

* I have mentioned in the two last communications on this subject some application of the principle; many others will occur. In submarine constructions—to protect wood, as in piles, from the action of worms, sheathing of copper defended by iron in excess may be used; when the calcareous matter deposited will gradually form a coating of the character and firmness of hard stone.

XVI. *On the Magnetism of Iron arising from its rotation.* By SAMUEL HUNTER CHRISTIE, *Esq. M. A. of Trinity College, Cambridge; Fellow of the Cambridge Philosophical Society; of the Royal Military Academy. Communicated April* 20, 1825, *by* J. F. W. HERSCHEL, *Esq. Sec. R. S.*

Read May 12, 1825.

As the principles on which phænomena depend can only be discovered by a careful investigation of the circumstances attending every new fact which presents itself, its importance must not, in the first instance, be estimated by the magnitude of the effects produced, but by their peculiarity. However minute may be the effects, an inquiry into the laws which govern them, if unattended by any other, will have this advantage, that these laws will serve as an additional test of the correctness of the principles advanced for the explanation of the more striking phænomena, firmly establishing their truth, if the consequences of those principles, or being incompatible with them, pointing out their fallacy. Thus the severest test that the principle of gravitation has been subjected to, is the explanation of the minute irregularities in the planetary motions; and the coincidence of the observed irregularities with those deduced from the application of this principle would have established its truth beyond dispute, had any doubt previously remained. In the experiments which I am about to detail, the effects produced are of this minute character; but as they point out a species of

action not hitherto observed, they will not, I trust, be considered unimportant.

It has been stated that different effects will be produced on iron, as regards its polarity, when struck, twisted, filed, or scoured in different positions, with respect to the magnetic axis or line of the dip; but I am not aware that it has ever been suspected that the simple rotation of iron, in different directions, would have any effect on the manner in which the iron influenced a magnetic needle. This I have discovered to be the case; and that the laws which govern this peculiar action on the needle are so general and uniform, that I have no doubt their causes are as steady in their operation, as those to which the more striking phænomena of magnetism owe their origin. On observing these magnetical phænomena arising purely from rotation, it appeared to me that they might possibly indicate the cause of the earth's magnetism; and this was a further inducement to me thoroughly to investigate the circumstances connected with them. Before giving the particulars of these phænomena, it is necessary that I should mention how I was first led to observe them.

For some time previous I had been engaged in making several series of experiments, with a view to discover the precise manner in which unmagnetised iron acts upon a magnetic needle. For this purpose I had made use of an iron ball 13 inches in diameter, and likewise of a shell 18 inches in diameter, and observed their effects on the needle in various positions, as referred to certain planes passing through its centre. The shell and the needle were placed in the relative positions which I wished to give them, by determining a radius and an angle on an horizontal plane, and a vertical

ordinate. The requisite computations being necessarily tedious, when I wished to pursue the subject further, I found them, from their number, so laborious, that I resolved, if possible, to supersede the necessity of them by the construction of an instrument, by which I could adjust the iron and the needle in their proper relative positions, without any previous computation. In this I succeeded; but as the iron was to be supported on an arm of brass, it became necessary to make use of a plate of iron instead of the heavy shell of nearly 500 lbs. weight; and in consequence of this, when I expected that I had overcome the principal difficulties, I found they had only commenced. It is well known that almost every mass of iron, but especially sheet iron, possesses polarity in a slight degree, and of a very variable nature in some parts of it, whatever care may have been taken in its manufacture; and I soon found, to my no small vexation, that the effects apparently produced by it in that which I made use of were so various, that they would for a long time baffle me in my investigations, if they did not ultimately frustrate all my attempts at drawing any conclusions from the experiments.

The instrument which I have mentioned is represented in Plate XXV. fig. 1. The principal part consists of two strong limbs of brass: one, SQN, a semicircle, 18 inches in diameter, 2.15 inches broad and .3 inch thick: the other consists of two semicircles joined together; S Æ N, 1.2 inches broad and .22 thick, and its outer diameter 18 inches; *s æ n* .9 inch broad, .22 thick, and its inner diameter 9.2 inches. S Æ N *n æ s* and SQN are attached to each other by strong brass pins passing from S to *s* and N to *n*; so that *s* Æ *n* will

revolve about the axis S *s n* N, while S Q N is fixed. S Æ N and S Q N are graduated from Æ and Q towards S and N, as is likewise *s æ n* from *æ* towards *s* and *n*. The semicircle S Q N passes freely through an opening in the support G I, but may be clamped firmly in any position by means of two strong screws, working into the parts G, G′ from the back of the instrument. On the chamfered edge of the opening *g*, in the face of G G′, is an index showing the inclination of the axis S N to the horizon; and on the part K *k* at the foot of the pillar, and attached to it, is an index pointing out on the graduated circle L *l*, fixed on the table T *t*, the situation of the fixed limb S Q N with respect to the magnetic meridian. R *r* is another graduated circle, fixed to the moveable limb S Æ N; which, by the index at *x* on the fixed limb S Q N, shows the angle described by S Æ N from the plane of S Q N. A very strong brass pin, soldered to the foot of the pillar, passes through the table T *t* and a thick circle of wood, to which the legs are attached, and has below a clamping screw, to fix the whole firmly together in any position. The compass box N′ S′ is fitted on to a stand fixed to the support F *f*, which consists of two parts; *f* fitted to G, and F sliding on a tube attached to *f*; so that the compass may be elevated or depressed. An arm A B, to carry the circular plate of iron C *c*, is connected with the moveable limb S *æ* N. The part A *a* consists of two flat pieces, having the limb of the instrument between them, so that the arm may be moved into any position on it, and be fixed in that situation by means of a strong screw working into the face A *a*. On the cylindrical part B *b*, a short hollow cylinder slides freely, having a circular rim raised .6 inch from it, to support the iron plate

C*c* at right angles to the axis of the cylinder. Over the plate of iron is a wooden washer D *d*, which is pressed on it by the screw *h* working on the short cylinder. The cylinder with the plate is fixed in any position on the arm by the clamp M *m*. In the part A*a* of the arm are two openings *o*, *o'*, on the chamfered edges of which are indexes in a line with the axis of the cylinder B*b*, so that when each points to the same arc on the semicircles S Æ N, *s æ n*, the axis of the cylinder B *b* is directed towards their centre, and every point in the edge of the plate is at the same distance from that centre. As the weight of the plate was a considerable strain on the instrument, a scale to contain a counter-weight, was suspended from the ceiling of the room, and the line from it passed through a moveable pulley, attached to the arm B *b*, so that the weight might easily be adjusted to relieve nearly altogether the strain of the plate on the arm in any position. The arm was also occasionally supported, and kept steady in its position, by a sliding rod resting on the table T *t*. The compass consists of a circular box, containing a circle 6 inches in diameter, very accurately divided into degrees, and again into thirds of a degree; and a very light needle, having an agate in its centre, and its point of suspension only .07 inch above the surface of the needle. The extremities of the needle are brought to very fine points; so that by a little practice, with the assistance of a convex lens, I could read off the deviations very correctly to two minutes, being the tenth of the divisions on the circle. To this compass I have another needle, which has a vernier at each end; but this being much larger and heavier, and consequently not so sensible, I greatly prefer the other for all delicate experiments. In the

experiments which I had previously made, and in those which I proposed making with this apparatus, I conceived a sphere to be described about the centre of the needle, referring the situation of the iron to a plane, in which, according to the hypothesis I had adopted, it should equally affect the north and south ends of the needle. The line in which the needle would place itself, if freely suspended by its centre of gravity, I considered as the magnetic axis; the points where this axis cuts the sphere, the poles, the upper being the south, and the lower the north pole; and the great circle at right angles to the axis, the equator, being the plane above mentioned. The position of the iron was thus determined by its latitude and longitude; the longitude being always measured from the eastern intersection of the equator with the horizon. The angle which the axis makes with the horizon I considered to be, according to the most accurate observations, very nearly 70° 30′.*

As I shall have frequently to refer to different adjustments

* In 1818 Captains KATER and SABINE found the dip to be 70° 34 in the Regent's Park; and in 1819, in the same place, Captain SABINE found it to be 70° 33′.27. Since making the greater part of these experiments, I have had opportunities of observing the dip at this place. With a very good instrument, by T. JONES of Charing Cross, having a 7-inch needle, consisting of two circular arcs, on Captain KATER's construction, the mean of 40 observations, 10 with the face of the instrument east, 10 with the face west, and the same with the poles reversed, gave the dip 70° 15.25 on the 23d December, 1821, between the hours of 1 and 4 P. M. the observations being made in my garden. With another instrument, also by T. JONES, having an 8-inch rectangular needle, the mean of 40 observations made in my garden, (about a mile from the former place of observation) near noon on the 5th and 6th May, 1824, gave 70° 06.5 for the dip. With the same instrument, but using a needle on MEYER's construction, the mean of 40 observations near noon on the 8th May, 1824, gave the dip on the same spot 70° 10′.5.

of the instrument, I will here briefly notice their effects. The angle which the axis SN, fig. 1. makes with the horizon HO, being 70° 30′, if SQN is in the magnetic meridian, and the compass is adjusted, so as to have its centre in the centre of the instrument, SN will be the magnetic axis, and the centre of the plate, as there represented, would, by the rotation of the limb SÆN, describe a parallel of latitude, its longitude being indicated on the circle R*r*: and if the indexes at *o*, *o*′ be brought to coincide with Æ, *æ*, the centre of the plate would then describe the equator. If the limb SQN be slided through GG′ until the index there coincide with 19° 30′, from Q towards S, and the indexes *o*, *o*′ be made to coincide with Æ *æ*, as represented in Fig. 2, the centre of the plate, by the revolution of the limb, would describe a secondary both to the meridian and equator, and its latitude would be indicated on the circle R*r*. If the point Q, Fig. 1, or zero on the limb SQN, be brought to coincide with the index at *g*, and the instrument make a quarter of a revolution about GI, so that the index at K may point to 90°, the centre of the plate would describe the meridian when the indexes at *o*, *o*′ coincide with Æ, *æ*; and the latitude would be determined from the degrees indicated on R*r*. This is represented Fig. 3, where the contrary side of the instrument to that seen in Fig. 1, 2, is placed in front, in order to show the situations of the screws, which clamp the arm AB and the limb SQN in their respective situations. Thus, by a proper adjustment of the indexes at K, *g*, *o*, *o*′, the centre of the plate may be made to describe any circle of the sphere.

After making a very few sets of experiments with this instrument, I found that it was necessary to attend very par-

ticularly to the situation of certain points on the iron plate with respect to the limb, since, with one point coinciding with it, the deviation of the needle, when the centre of the plate was on the meridian, would be easterly, and with another point coinciding, westerly; whereas had the iron possessed no partial magnetism, which was the case I wished to investigate, there would have been no deviation when its centre was on the meridian. My first object was to find what points on the plate must coincide with the limb, in order that the plate, when its centre was on the meridian, should cause no deviation in the needle; and it was in my attempts to effect this, which at first sight appears sufficiently easy, that I discovered the leading feature in all the phænomena which I am about to describe.

General description of the phænomena arising from the rotation of an iron plate.

In order to find the points which I have mentioned, I adjusted the instrument so that the plane of the fixed limb was exactly in the magnetic meridian, and then brought the other limb into the same plane: the centre of the plate was then on the magnetic meridian, and its plane perpendicular to that plane, as represented in Fig. 1. I now made the plate revolve in its own plane about the axis B *b*, and noted very carefully its effect on the needle. In doing this I found that if I placed the plate on the arm, so that a certain point, *c* for instance, coincided with the plane of the limb, the deviation was different when the same point, by the revolution of the plate, coincided with the limb again. As it appeared by this that the revolution of the plate had an effect

Fig. 1.

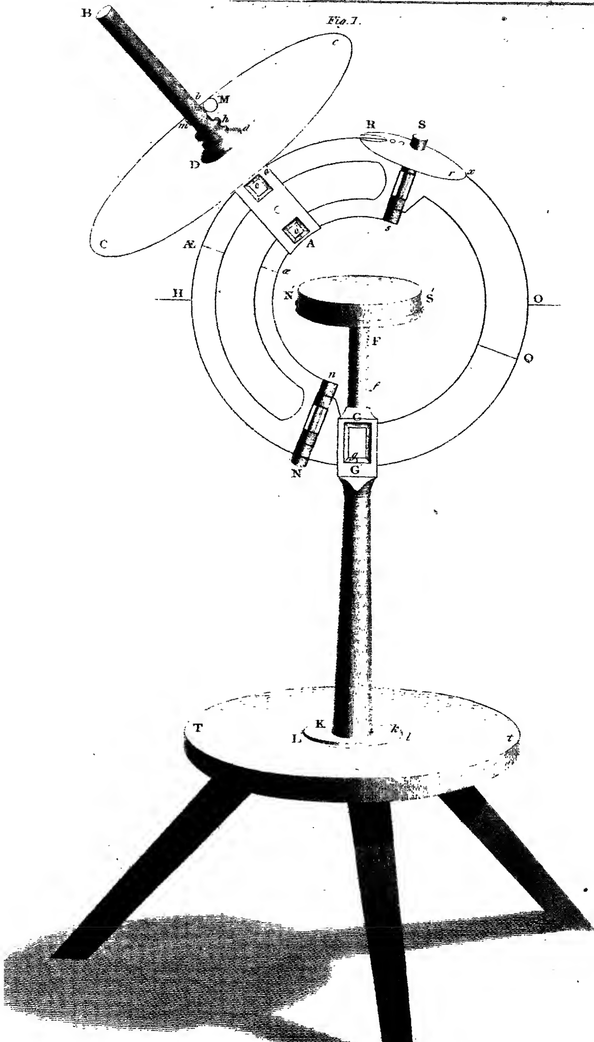


Fig. 2.

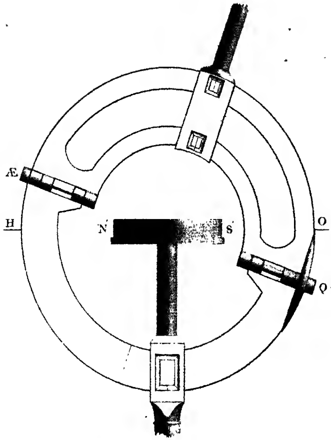


Fig. 3.

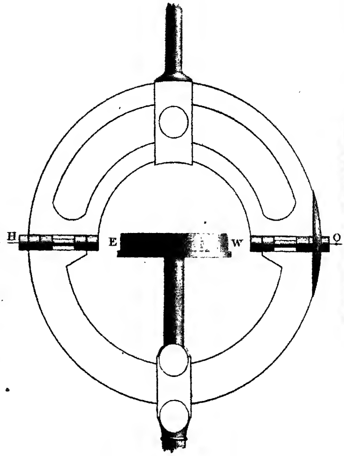


Fig. 4

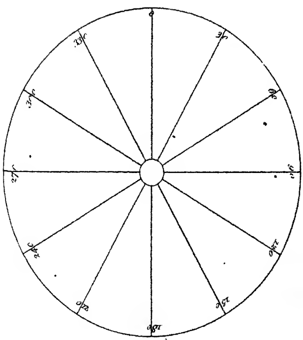


Fig. 5.

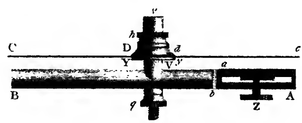
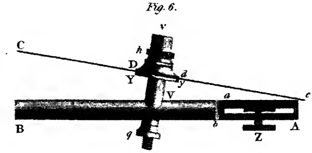


Fig. 6.



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upon the needle, independent of the partial magnetism of particular points, I considered that if the plate were made to revolve the contrary way, the deviation ought to be on the opposite side, and this I found to be the case. I will illustrate this by the observations made when I first noticed the effect. The plate was divided at every 30° of its circumference (Fig. 4.) by lines drawn through the centre, and being placed on the arm, so that 0° coincided with the upper part of the limb, the north end of the needle pointed 10′ east; but when this point again coincided with the limb, by the upper edge of the plate revolving from *west* to *east*, the needle pointed 30′ east: making the plate revolve the contrary way, that is, its upper edge from *east* to *west*, when 0° coincided with the limb, the north end of the needle pointed 28′ west: so that there was a difference of 58′, when every point of the plate had the same position with respect to the needle, according as the plate was brought into that position by revolving from *west* to *east*, or from *east* to *west*. As this appeared extraordinary, I made repeated observations at the time, to ascertain that the effect was independent of any accidental circumstances, and found that the results always accorded with the first, the difference caused by the rotation of the plate being however greater or less according to the position of the plate.

Having fully satisfied myself that, in whatever manner the rotation of the plate might cause this difference, such was really the effect, I next endeavoured to ascertain the nature and degree of the difference, according to the different situations of the centre of the plate. For this purpose I made a great variety of experiments, of which I shall not however

here give the details, as I afterwards repeated them in a more convenient manner, and with greater precision; but shall merely point out the nature of them in general, and the conclusions which I at the time drew from them. The instrument being adjusted, and the arm fixed so that the centre of the plate was in the position which I required, I made the plate revolve so that its upper edge moved from *west* to *east*, and noted the greatest and least deviation of the north end of the needle; I then made the corresponding observations when the plate revolved in the contrary direction: a mean of the differences between the two greatest and between the two least I considered as the effect produced on the needle by the rotation of the plate in opposite directions. Repeating these in a variety of positions, I found that when the centre of the plate was in the magnetic meridian, its plane being always a tangent to the sphere circumscribed about the centre of the needle, the deviation of the needle caused by the rotation of the plate in its plane was the greatest when the centre of the plate was in the equator, and that it decreased from there towards the poles, where it was nothing;* that when its centre was on the equator, this deviation was the greatest when the centre of the plate was on the meridian, or in longitude 90°, and decreased to nothing in the east and west points, or when the longitude of the plate was 0° or 180°; and that when the centre of the plate was in the

* I should here mention, that, from the nature of my instrument, I could not make observations at the *north* pole; but as the results, as far as I could observe, were of the same nature on this side of the equator as on the south side, I think I am warranted in concluding, that *at* the *north* pole the results would likewise be of the same nature as at the south pole.

secondary both to the equator and meridian, the rotation of the plate, whatever might be its latitude, caused no deviation of the needle. In these experiments, the plate which I made use of was a circular one 17.88 inches in diameter, and .099 inch in thickness, weighing 112 oz. The further I had pursued this inquiry, the more I was disposed to attribute the effects I have mentioned to a general magnetic action, arising in a peculiar manner from the *rotation* of the iron; and my next experiments were with the view of ascertaining how far this idea was correct. As similar results might not be obtained with any other plate, I next made use of a plate 12.13 inches in diameter and .075 inch in thickness, weighing 38.75 oz., and with it obtained results precisely of the same nature, though considerably less in quantity. Another objection which occurred to me was this—that the iron being evidently slightly polarised in particular points, the effect might be supposed to arise from an impulse given to the needle by the motion of these points in a particular direction, and that the directive power of the needle not immediately overcoming the slight friction on the pivot, a deviation might thus arise from the rotation of the plate. Had this, however, been the cause of the deviations, I should have expected that, when the centre of the plate was in the meridian, the greatest effect would be produced with the plate parallel to the horizon, and its centre vertical to that of the needle; but I had seen that the greatest deviation took place when the centre of the plate was in the equator, its plane being perpendicular to it; and the deviation arising from the *rotation*, when the plate was parallel to the horizon, was not a fifth of the deviation when the plate was perpendicular to that

plane. Besides it was manifest that if this were the cause, any other impulse would have a similar effect. I therefore made the needle revolve first in one direction and then in that opposite, by means of a small bar magnet, and invariably found that it settled at the same point, in whichever direction the impulse was first given, and the results obtained by the rotation of the plate were in these cases of the same nature as before. It was also evident, that if the deviations I have mentioned arose from this circumstance, the needle being agitated after any particular point of the plate was brought to the limb of the instrument, it ought to settle in the same direction, whether that point were brought into this position by revolving from *east* to *west* or from *west* to *east;* but this, except in the cases I have mentioned, where the rotation produced no deviation, was not found to take place. In order wholly to obviate this objection, in all my future experiments, after any point had been brought to the limb of the instrument, I agitated the needle, and let it settle before I noted the deviation.

Description of particular experiments.

As I had found in my first experiments that I could obtain the nature of the deviation caused by the *rotation* by noting the greatest and least deviations when the plate was made to revolve in contrary directions, but that the quantity of that deviation could not by this means be determined with any degree of precision, I resolved to make my future observations differently. The method I adopted, when the change in the deviation from one point of the plate to another was considerable, was this: the plate being placed in any

required position, I made it revolve once, for example, the upper edge from *east* to *west*, without noting the deviations, bringing the point marked 0° to coincide with the line indicating the position for observation; from hence I continued the revolution of the plate until the point marked 30° coincided with the same line, and, after slightly agitating the needle, noted the deviation; and in the same manner were the points 60°, 90°, 120°, 150°, 180°, 210°, 240°, 270°, 300°, 330°, 360° or 0° brought successively to coincide, and the deviations noted. I now made the plate revolve once from *west* to *east*, without noting the deviations, bringing 0° or 360° to coincide with the same line, and then brought in succession 330°, 300°, 270°, 240°, 210°, 180°, 150°, 120°, 90°, 60°, 30°, 0° to coincide, noting the deviations as before. The sum of the first set divided by 12, I considered as the mean deviation, when the plate revolved from *east* to *west;* and the sum of the others divided by 12, as the mean deviation, when the plate revolved from *west* to *east*: their difference was the mean effect of the *rotation* in contrary directions. This I call the *Deviation due to Rotation;* and to distinguish it from the deviation caused simply by the position of the iron, I call this last the *Absolute Deviation.* When the change in the deviation from one point of the plate to another was not so considerable, I made the observations only for the points 0°, 90°, 180°, 270° on the plate.

I now proceed to the detail of the experiments, and the conclusions I draw from them. In those which I shall first describe, the centre of the plate was always in the magnetic meridian; its plane was perpendicular to the meridian, and a tangent to the sphere, whose centre was the centre of the

needle; and the plate revolved, as in all other cases, in its own plane: they are a repetition of those by which I first discovered several of the facts I have mentioned, but made for the purpose of determining more precisely the deviation caused by the rotation. In making these, the instrument was adjusted so that the index at *g*, fig. 1, pointed to 0°, that at K to 90°, and those at o, o′ to Zero; so that S N was horizontal and pointed east and west, as represented in fig. 3.

In the following table, the numbers in the first column indicate the points of the plate which coincided with the plane of the meridian nearest the south, or upper pole of the sphere, when the several directions of the north end of the needle in the same lines with them were observed; the latitudes and longitudes are those of the centre of the plate as referred to the centre of the needle, the longitudes being measured from *east through north;* the letters at the tops of the columns indicate the direction in which the edge of the plate, nearest the south pole of the sphere, moved; the mean deviation of the needle, when the plate revolved in this direction, is placed in the line below the other deviations; the direction in which the *deviation due to rotation* took place, in the following line; and the whole deviation, arising from making the plate revolve in opposite directions, below this: the deviations observed always refer to the north end of the needle. The distance of the centre of the plate from that of the needle was 9.75 inches; the diameter of the plate, 17.88 inches; thickness, .099 inch; weight, 112 oz.: so that its specific gravity appeared to be 7826. This plate I call No. I.

I. *Table of the deviations of a magnetic needle, caused by the rotation of a circular plate of iron, when its centre was in the magnetic meridian, and its plane a tangent to the sphere.* *Plate No. I.*

Points on the Plate.	Long. 90°. Lat. 0°.		Long. 90°. Lat. 19° 30′ S.		Long. 270°.* Lat. 19° 30′ S.		Long. 90°. Lat. 45° S.		Long. 270° Lat. 45° S.		Long. 90°.† Lat. 70° 30′ S.		Lat. 90° S.‡	
	E to W	W to E	E to W	W to E	W to E	E to W	E to W	W to E	W to E	E to W	E thro' S to W	W thro' S to E	S thro' S to W	W thro' S to E
0	0° 02′ E	1° 44′ E	0° 46′ W	0° 44′ E	0° 34′ E	1° 00′ W	0° 08′ E	0° 56′ E	0° 30′ W	1° 18′ W	0° 10′ E	0° 26′ E	0° 14′ E	0° 14′ E
90	2 10 W	0 34 W	2 06 W	0 32 W	1 34 E	0 04 E	1 54 W	1 04 W	1 12 E	0 22 E	1 22 W	1 06 W	1 08 W	1 06 W
180	1 40 W	0 04 E	0 40 W	0 54 E	0 16 E	1 16 W	1 14 W	0 24 W	0 48 E	0 08 W	1 02 W	0 44 W	0 54 W	0 54 W
270	0 20 W	1 20 E	0 06 W	1 30 E	0 12 W	1 42 W	0 10 E	0 58 E	0 26 W	1 16 W	0 16 E	0 36 E	0 28 E	0 26 E
Mean deviation	1 02 W	0 38 E	0 56 ⅓ W	0 39 E	0 33 E	0 58 ½ W	0 42 ½ W	0 06 ½ E	0 16 E	0 35 W	0 29 ½ W	0 12 W	0 20 W	0 20 W
Direction of deviation	N to W	N to E	N to W	N to E	N to E	N to W	N to W	N to E	N to E	N to W	N to W	N to E	Stationary.	
Deviation due to rotation	1° 40′⅓		1° 33′½		1° 31′⅓		0° 49′		0° 51′		0° 17′½		0° 0′	

	Long. 270°. Lat. 0°.		Long. 90°.* Lat. 19° 30′ N.		Long. 270°. Lat. 19° 30′ N.		Long. 90° Lat. 45° N.		Long. 90°.* Lat. 39° 00′ N.		Long. 270°. Lat. 70° 30′ S.		Long. 90°. Lat. 31° 30′ S.	
	W to E	E to W	E to W	W to E	W to E	E to W	E to W	W to E	E to W	W to E	W to E	E to W	E to W	W to E
0	0° 12′ W	2° 00′ W	0° 54′ W	0° 34′ E	0° 44′ E	0° 50′ W	0° 14′ E	1° 00′ E	0° 44′ W	0° 14′ E	0° 42′ W	1° 00′ W	0° 22′ E	1° 38′ E
90	1 26 E	0 22 W	1 48 W	0 22 W	1 54 E	0 16 E	1 46 W	0 58 W	1 40 W	0 42 W	0 34 E	0 18 E	1 32 W	0 12 W
180	1 24 E	0 28 W	0 38 W	0 50 E	0 30 E	1 08 W	1 16 W	0 26 W	0 16 W	0 36 E	0 18 E	0 02 E	1 42 W	0 24 W
270	0 36 E	1 14 W	0 16 W	1 10 E	0 00	1 36 W	0 20 E	1 10 E	0 12 E	1 08 E	0 44 W	1 00 W	0 34 W	0 44 E
Mean deviation	0 48 ½ E	1 01 W	0 54 W	0 33 E	0 47 E	0 49 ½ W	0 37 W	0 09 ½ E	0 37 W	0 19 E	0 08 ½ W	0 25 W	0 51 ½ W	0 26 ½ E
Direction of deviation	N to E	N to W	N to W	N to E	N to E	N to W	N to W	N to E	N to W	N to E	N to E	N to W	N to W	N to E
Deviation due to rotation	1° 49′½		1° 27′		1° 36′½		0° 46′¼		0° 56′		0° 16′½		1° 18′	

* In these positions the plane of the plate was vertical, and its centre in the same horizontal line as that of the needle.

† In this position the plane of the plate was horizontal, and its centre vertical to that of the needle.

‡ Here the deviations are those corresponding to the coincidence of the points on the plate with the southern meridian.

From these observations it appears, that when the centre of the plate was in the pole of the magnetic sphere, its plane being parallel to the equator, the position of the needle, for any situation of the several points of the plate, was the same whether they were brought into that situation by the plate revolving from *east* through *south* to *west*, or from *west* through *south* to *east*; that is, that the *deviation due to rotation* was nothing:

That the *deviation due to rotation* increased from this point towards the equator, where it was the greatest:

And that the horizontal needle was affected by the rotation of the plate, not according to the situation of the centre of the plate as regarded the poles and equator of the horizontal needle, but as regarded the poles and equator of an imaginary dipping needle passing through the centre of the horizontal needle.

This last is not so evident, from the circumstance of the deviation being nothing when the centre of the plate was in the pole of the dipping needle, and a maximum when in the equator, as from its being very nearly equal at equal distances on each side of the pole, and also of the equator, that is, at very unequal distances from the axis of the horizontal needle; and from the deviations at equal distances from the axis of the horizontal needle being very unequal. For if we compare the *deviation due to rotation* in lat. 70° 30′ S, long. 90°, with that in lat. 70° 30′ S, long. 270°, the difference is only 1′; in the first case, the centre of the plate was at the distance of 90° from the axis of the horizontal needle, and its plane parallel to it; and in the other at the distance of 51°, and its plane making an angle of 39° with this axis. Again,

in the four corresponding situations of lat. 19° 30′, the mean *deviation due to rotation* is 1° 32′, and none of the deviations differ from this by more than 5′, although in two cases the centre of the plate was in the axis of the horizontal needle, and its plane perpendicular to it, and in the two others the centre of the plate was at the distance of 39° from this axis, and its plane made an angle of 51° with it. The mean of the *deviations due to rotation* in the three* corresponding situations of lat. 45° is 49′, from which none of the deviations differ by 3′, notwithstanding the difference in the situations of the centre and plane of the plate, in these cases, with respect to the axis of the horizontal needle. In long. 90° lat. 45° S, the centre of the plate was 64° 30′ above the horizontal axis, and its plane made an angle of 25° 30′ with it; in long. 90° lat. 45° N, it made an angle of 64° 30′ at 25° 30′ below it; and in long. 270° lat. 45° S, it was in a position above it similar to the last. Any doubt, however, on the subject will be removed, if we compare the deviation in long. 90° lat. 39° N with that in long. 270° lat. 0; the one deviation being nearly double of the other, although the centre of the plate was at the distance of 19° 30′ from the axis of the horizontal needle, and its plane made an angle of 70° 30′ with it in both cases. The difference is even more striking, if we compare the deviation in lat. 70° 30′ S, long. 270°, with that in lat. 31° 30′ S, long. 90°, the centre of the plate being in each case at the distance of 51° from the axis of the horizontal needle, and its plane making an angle of 39° with it.

* The nature of the instrument would not admit of observations being made so near to the north pole in long. 270° as lat. 45°, or so near as lat. 70° 30′ on the other side of the support G I.

The differences which we have noticed in the deviations observed at the same distance from the equator, is not more than I have found to arise from a slight change in the adjustment of the centre of the needle to the centre of the instrument, the plate remaining in the same position. These errors of adjustment I found it almost impossible to avoid, owing probably in a great measure to the magnetic centre of the needle not being in the centre of suspension; and it was to counteract their effects, that I generally made observations on contrary sides of the centre.

With respect to the direction in which the *deviation due to rotation* took place, it appears, that the rotation of the plate always caused the *north* end of the needle to move in the same direction as the edge of the plate nearest the *south* pole of the magnetic sphere: so that the deviation of the *north* end of the needle was in the direction in which the *south* edge of the plate moved, and that of the *south* end of the needle in the direction in which the *north* edge moved, referring the edges to the poles of the sphere.

Having ascertained, that when the centre of the plate was in the pole, and its plane *parallel to* the equator, the *deviation due to rotation* was nothing; and some of the first experiments which I had made having indicated that this was also the case when the centre of the plate was in the secondary to the equator and meridian, and its plane, as before, a tangent to the sphere, I wished to ascertain whether such were really the fact. The experiments, the results of which are given in the following table, left no doubt in my mind on the subject. In making them, the instrument was adjusted, so that the index at K (Fig. 1) pointed to zero, that at G to 19° 30′

from Q towards S, and those at o, o′ to zero on the limb S *æ* N, as in Fig. 2. The deviations for the several points of the plate are those observed when these points coincided with the southern or upper part of the secondary to the equator and meridian; and the direction of rotation is, as before, that of the edge of the plate nearest to the south pole of the sphere.

II. *Table of the deviations of a magnetic needle caused by the* rotation *of a circular plate of iron, when its centre was in the secondary to the equator and meridian, and its plane a tangent to the sphere: the distance as before* 9.75 *inches. Plate No. I.*

Points on the Plate.	Long. 0°. Lat. 0°.		Long. 180°. Lat. 0°.		Long. 0°. Lat. 45° S.		Long. 180°. Lat. 45° S.		Long. 0°. Lat. 45° N.		Long. 180°. Lat. 45° N.	
	S to N	N to S	N to S	S to N	S to N	N to S	N to S	S to N	S to N	N to S	N to S	S to N
0	0° 06′ E	0° 06′ E	0° 26′ W	0° 24′ W	7° 24′ W	7° 22′ W	6° 48′ E	6° 48′ E	7° 08′ E	7° 10′ E	7° 32′ W	7° 32′ W
90	0 14 E	0 14 E	0 40 W	0 40 W	7 40	7 40	6 58	6 56	7 18	7 20	8 00	7 58
180	0 04 E	0 04 E	0 32 W	0 32 W	6 52	6 54	5 58	6 00	6 26	6 26	6 52	6 52
270	0 04 W	0 04 W	0 20 W	0 20 W	6 28	6 28	5 48	5 48	6 08	6 08	6 30	6 28
Mean Deviations	0 5 E	0 05 E	0 29½ W	0 29 W	7 06 W	7 06 W	6 23 E	6 23 E	6 45 E	6 46 E	7 13½ W	7 12½ W
Deviation due to rotation	0° 00′		0° 00′ ½		0° 00′		0° 00′		0° 01′		0° 01′	

From these observations, combined with the preceding, we may infer, that if the centre of the plate were made to describe any parallel of latitude, the *deviation due to rotation* would be nothing when the longitude was 0° or 180°, and a maximum when the longitude was 90° or 270°, which is precisely the reverse of the *absolute deviations* that would be produced by the plate describing the parallel of latitude.

The next experiments which I made, were with the view of determining whether the *rotation* of the plate would produce any deviation, when its plane *coincided with the equator.* For this purpose an axis was fixed perpendicularly on the arm of the instrument in such a manner, that, when the plate revolved on it, its plane was parallel to the limb. This is represented in fig. 5: AB is the arm, on the cylindrical part of which, B *b*, is fixed perpendicularly to it the axis V *v*, on which the plate of iron, C *c*, here seen edgewise, revolves. A, *a*, are the two flat pieces, having an opening between them for the limb of the instrument; Z is the clamping screw, and Y *y* the circular rim to support the iron plate, which are not seen in fig. 1.

In order to make these observations, it was necessary to adjust the whole instrument twice; since the deviations for the longitudes 90° and 270° could not be observed with the same adjustment as those for the longitudes 0° and 180°. For the longitudes 90° and 270°, the axis of the instrument was horizontal and pointed east and west, as in fig. 3, and the moveable limb E AW revolved on the axis until its plane, and therefore also that of the iron plate, made an angle of 90° 30′ with the horizon, rising towards the north; so that the compass being elevated until the centre of the needle was in the plane of the plate, the plate was then in the equator. For the other longitudes, the axis of the instrument was inclined to the horizon at an angle of 19° 30′, and in the plane of the meridian, as in fig. 2, and the moveable limb adjusted at right angles to the fixed one: the compass was then elevated to coincide with the plane of the plate.

In these experiments the distance of the centre of the iron

from the centre of the needle was 13.2 inches; but as its edge was only 4.26 inches distant, the differences between the deviations corresponding to the several points on the plate were greatly increased; and therefore, to obviate any inaccuracies that might arise, from the points not being brought into precisely the same situation when the plate revolved in the opposite directions, I increased the number of observations, making twenty-four for each position, namely, twelve points on the plate, as I have before described, the deviation for any point being observed when that point coincided with the line joining the centre of the plate and needle. The letters at the tops of the columns indicate the direction of rotation of the inner edge of the plate, or that nearest the centre of the needle.

III. *Table of the deviations of a magnetic needle, caused by the rotation of a circular plate of iron, when its centre was in the equator, and its plane in the plane of the equator. Plate No. I.*

Points on the Plate.	Long. 0°		Long. 45°		Long. 90°		Long. 135°		Long. 180°		Long. 225°		Long. 270°		Long. 315°	
	N to S	S to N	NW to SE	SE to NW	W to E	E to W	SW to NE	NE to SW	S to N	N to S	SE to NW	NW to SE	E to W	W to E	NE to SW	SW to NE
0	3° 00′ E	3° 00′ E	2° 40′ E	2° 40′ E	0° 30′ E	0° 30′ E	2° 00′ W	2° 00′ W	1° 28′ W	1° 28′ W	3° 06′ W	3° 00′ W	0° 26′ W	0° 32′ W	1° 22′ E	1° 22′ E
30	2 20 E	2 20 E	1 02 E	1 00 E	1 00 E	0 58 E	1 06 W	1 02 W	0 48 W	0 46 W	1 44 W	1 48 W	0 30 W	0 40 W	0 18 E	0 10 E
60	1 14 E	1 14 E	0 18 W	0 12 W	0 40 E	0 34 E	0 30 W	0 30 W	0 00	0 00	1 14 W	1 10 W	0 02 E	0 00	0 12 W	0 16 W
90	0 06 W	0 06 W	1 36 W	1 42 W	1 44 E	1 42 E	0 14 E	0 12 E	1 00 E	0 56 E	0 30 E	0 26 E	0 42 W	0 50 W	0 40 W	0 46 W
120	1 30 W	1 30 W	2 30 W	2 20 W	0 32 E	0 34 E	1 00 E	1 00 E	1 16 E	1 16 E	1 50 E	1 50 E	0 10 E	0 04 E	1 10 W	1 02 W
150	2 20 W	2 26 W	2 24 W	2 12 W	0 32 E	0 40 E	1 02 E	1 02 E	1 16 E	1 16 E	2 10 E	2 10 E	0 26 W	0 30 W	1 00 W	1 00 W
180	2 50 W	2 44 W	1 56 W	1 50 W	0 12 W	0 14 W	1 30 E	1 34 E	1 16 E	1 16 E	2 20 E	2 26 E	0 08 W	0 12 W	1 10 W	1 06 W
210	2 50 W	2 44 W	1 04 W	1 10 W	0 32 W	0 40 W	1 30 E	1 30 E	1 16 E	1 16 E	2 32 E	2 30 E	0 04 W	0 06 W	1 10 W	1 06 W
240	1 56 W	1 56 W	0 16 W	0 10 W	1 18 W	1 18 W	1 00 E	1 00 E	0 56 E	0 56 E	2 24 E	2 18 E	0 30 E	0 26 E	0 40 W	1 40 W
270	0 08 W	0 08 W	1 10 E	1 04 E	1 30 W	1 34 W	0 36 W	0 34 W	0 12 W	0 16 W	0 20 E	0 14 E	1 10 E	1 06 E	0 40 E	0 38 E
300	1 42 E	1 40 E	2 30 E	2 30 E	1 22 W	1 22 W	1 44 W	1 50 W	1 18 W	1 18 W	1 40 W	1 46 W	1 10 E	1 10 E	1 16 E	1 20 E
330	2 50 E	2 56 E	2 48 E	2 50 E	0 16 W	0 20 W	2 16 W	2 16 W	1 50 W	1 50 W	2 44 W	2 48 W	0 14 E	0 16 E	1 42 E	1 40 E
Mean Deviations.	0 02⅚ W	0 02 W	0 00½ E	0 02⅙ E	0 01 W	0 02½ W	0 09⅔ W	0 09½ W	0 07⅔ E	0 06⅓ E	0 08⅔ E	0 06⅚ E	0 05 E	0 01 E	0 03⅓ W	0 03⅚ W
Deviation due to rotation.	0° 00⅚′		0° 01¾′		0° 01½′		0° 00⅙′		0° 01⅙′		0° 01⅔′		0° 04′		0° 00⅙′	

These observations show very clearly, that when the centre of the plate is in the equator, and its plane also coincides with the plane of the equator, the *deviation due to rotation* is always nothing, since the small differences to be observed here in the revolutions in opposite directions are only such as may justly be attributed to slight errors in the adjustments of the centre of the needle or of the plane of the plate, which are almost unavoidable. With regard to the several deviations in the different columns, I should notice, that they are not those actually observed, but derived from them by subtracting the same number from all the deviations observed in two corresponding columns, so that they indicate the same difference of deviations in the two revolutions as those actually observed, and therefore give the same *deviation due to rotation*. The necessity of this reduction arose from the circumstance of my having to adjust the compass to the proper height, so that its centre might be in the plane of the plate, while it was under the influence of the partial magnetism of particular points in the plate; and having done this, when zero of the compass was brought to coincide with the point of the needle it was not necessarily in the magnetic meridian, since the needle was under the influence of this partial magnetism; and as I wished the deviations to be those from the meridian, I reduced the observed deviations as I have mentioned.

Being convinced that the *rotation* of the plate *in* the plane of the equator caused no deviation of the needle, I proceeded to determine the effects produced by its rotation in other planes. In the first set of observations which I made, the centre of the plate was in the meridian, and its plane perpendicular to the plane of the meridian and passing through the

centre of the needle. Before however making these, to avoid the necessity of moving the compass as in the last, I made a slight alteration in the instrument. Instead of having the axis on which the plate revolved perpendicular to the arm, and the plate consequently parallel to the limb, this axis was inclined in such a manner that the plane of the plate passed through the axis of the instrument, as represented fig. 6; so that the axis of the instrument being horizontal, and passing through the centre of the needle perpendicularly to the meridian, as in fig. 3, when the arm of the instrument was adjusted to zero on the limb, the revolution of the limb caused the centre of the plate to describe the magnetic meridian, and at the same time the plane of the plate always passed through the centre of the needle. The distance between the centre of the plate and that of the needle was as in the last 13.2 inches. The observations are given in the following table, where the letters above the columns indicate the direction of rotation of the plate's inner edge.

IV. *Table of the deviation of a magnetic needle, caused by the* rotation *of a circular plate of iron, when the centre of the plate was in the meridian, and its plane in the plane of a secondary to the meridian. Plate No. I.*

Points on the Plate.	Long. 90°. Lat. 0°.		Long. 90°. Lat. 19° 30′ S.		Long. 270°. Lat. 19° 30′ S.*		Long. 90°. Lat. 45° S.		Long. 270°. Lat. 45° S.		Long. 90°. Lat. 70° 30′ S.†		Long. 270°. Lat. 70° 30′ S.		Lat. 90° S.	
	W to E	E to W	W so E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W
0	0° 14′W	0° 14′W	0° 12′W	0° 06′E	0° 12′W	0° 12′E	0° 54′W	0° 00′	0° 48′W	0° 02′E	0° 58′W	0° 20′E	1° 10′W	0° 08′E	1° 10′W	0° 14′E
30	0 12 E	0 08 E	0 02W	0 14 E	0 24W	0 00	0 56W	0 00	1 14W	0 18W	1 02W	0 20 E	1 10W	0 00	1 12W	0 08 E
60	0 20 E	0 18 E	0 06 E	0 24 E	0 24W	0 00	0 32W	0 24 E	1 22W	0 24W	0 58W	0 32 E	1 10W	0 08 E	1 08W	0 20 E
90	1 04 E	1 04 E	0 48 E	1 10E	0 50E	1 14 E	0 22 E	1 18 E	0 20W	0 38 E	0 10W	1 18 E	0 20W	1 00 E	0 22W	1 04 E
120	0 40 E	0 38 E	0 26 E	0 48 E	0 22 E	0 48 E	0 18 E	1 14 E	0 32W	0 28 E	0 04 E	1 26 E	0 20W	1 02 E	0 16W	1 14 E
150	1 16 E	1 14 E	0 56 E	1 22 E	1 00 E	1 26 E	0 50 E	1 44 E	0 10 E	1 06 E	0 20 E	1 40E	0 10 E	1 30 E	0 16 E	1 40 E
180	0 36 E	0 38 E	0 [illegible]2 E	0 58 E	0 44 E	1 08 E	0 36 E	1 32 E	0 10 E	1 08 E	0 20 E	1 36 E	0 12 E	1 24 E	0 16 E	1 40 E
210	0 14 E	0 16 E	0 02 E	0 26 E	0 28 E	0 50 E	0 04W	0 52 E	0 00	0 56 E	0 00	1 12 E	0 12W	1 02 E	0 04W	1 14 E
240	0 38W	0 40W	1 00W	0 38W	0 08W	0 12 E	1 c8W	0 10W	0 30W	0 28 E	0 50W	0 22 E	0 52W	0 28 E	0 50W	0 30 E
270	1 30W	1 28W	1 40W	1 18W	0 46W	0 24W	1 46W	0 48W	0 56W	0 00	1 20W	0 00	1 12W	0 00	1 20W	0 04 E
300	1 04W	1 08W	1 20W	1 00W	0 40W	0 20W	1 54W	0 52W	0 56W	0 02 E	1 22W	0 00	1 20W	0 04W	1 22W	0 00
330	0 30W	0 26W	0 46W	0 26W	0 44W	0 18W	1 30W	0 36W	1 02W	0 06W	1 30W	0 00	1 26W	0 10W	1 30W	1 02W
Mean deviation - -	0 02 $\frac{5}{6}$ E	0 01 $\frac{2}{3}$ E	0 10 $\frac{5}{6}$ W	0 10 $\frac{1}{2}$ E	0 00 $\frac{1}{2}$ E	0 24 E	0 33 $\frac{1}{6}$ W	0 23 $\frac{1}{6}$ E	0 36 $\frac{2}{3}$ W	0 20 E	0 37 $\frac{1}{6}$ W	0 43 $\frac{5}{6}$ E	0 44 $\frac{1}{6}$ W	0 32 $\frac{1}{3}$ E	0 43 $\frac{1}{2}$ W	0 40 $\frac{1}{2}$ E
Direction of deviation	Stationary.		N to W	N to E	N to W	N to E	N to W	N to E	N to W	N to E	N to W	N to E	N to W	N to E	N to W	N to E
Deviation due to rotation	0° 00′ $\frac{1}{2}$		0° 21′ $\frac{1}{3}$		0° 23′ $\frac{1}{2}$		0° 56′ $\frac{1}{3}$		0° 56′ $\frac{2}{3}$		1° 21′		1° 16′ $\frac{1}{2}$		1° 24′	

	Long. 270°. Lat. 0°.		Long. 90°. Lat. 19° 30′ N.*		Long. 270°. Lat. 19° 30′ N.		Long. 90°. Lat. 45° N.		Long. 270°. Lat. 45° N.		Long. 90°. Lat. 70° 30′ N.		Long. 270°. Lat. 70° 30′ N.	Lat. 90° N.	
	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W		W to E	E to W
0	0° 58′W	0° 58′W	0° 08′E	0° 14′W	0° 54′W	1° 20′W	0° 14′E	0° 40′W	0° 44′E	0° 12′W	0° 42′E	0° 38′W	The nature of the instrument does not admit of observations being made in this position of the Plate.	0° 42′E	0° 42′W
30	0 52W	0 52W	0 38 E	0 16 E	0 44W	1 06W	0 50 E	0 00	0 54 E	0 02 E	1 20 E	0 00		1 22 E	0 06W
60	0 46W	0 46W	0 40 E	0 16 E	0 28W	0 48W	1 02 E	0 14 E	0 52 E	0 02W	1 16 E	0 04W		1 22 E	0 00
90	0 46 E	0 46 E	1 06 E	0 42 E	0 36 E	0 14 E	1 18 E	0 20 E	0 54 E	0 02W	1 26 E	0 06 E		1 24 E	0 00
120	0 40 E	0 38 E	0 46 E	0 22 E	0 42 E	0 18 E	1 00 E	0 12 E	0 52 E	0 04W	1 08 E	0 14W		1 18 E	0 10W
150	1 36 E	1 38 E	1 08 E	0 44 E	1 16 E	0 54 E	1 18 E	0 20 E	1 02 E	0 06W	1 16 E	0 04W		1 26 E	0 04W
180	1 14 E	1 16 E	0 24 E	0 00	0 58 E	0 34 E	0 44 E	0 10W	0 40 E	0 18W	0 38 E	0 48W		0 52 E	0 38W
210	1 08 E	1 08 E	0 20 E	0 02W	1 00 E	0 36 E	0 42 E	0 12W	0 22 E	0 38W	0 14 E	1 10W		0 28 E	0 56W
240	0 24 E	0 26 E	0 22W	0 48W	0 18 E	0 04W	0 08 E	0 50W	0 12W	1 16W	0 14W	1 36W		0 02W	1 24W
270	0 32W	0 32W	1 06W	1 30W	0 54W	1 16W	0 38W	1 30W	0 42W	1 36W	0 42W	2 02W		0 32W	2 00W
300	0 46W	0 44W	0 40W	1 02W	1 06W	1 30W	0 28W	1 12W	0 12W	1 06W	0 16W	1 40W		0 16W	1 44W
330	1 06W	1 02W	0 08 E	0 12W	0 56W	1 20W	0 10 E	0 46W	0 28 E	0 24W	0 40 E	0 46W		0 32 E	1 00W
Mean deviation - -	0 04 E	0 04 $\frac{5}{6}$ E	0 15 $\frac{5}{6}$ E	0 07 $\frac{1}{3}$ W	0 01W	0 24W	0 31 $\frac{2}{3}$ E	0 21 $\frac{1}{6}$ W	0 28 $\frac{1}{2}$ E	0 27 $\frac{1}{2}$ W	0 37 $\frac{1}{3}$ E	0 44 $\frac{2}{3}$ W		0 43 E	0 43 $\frac{2}{3}$ W
Direction of deviation	Stationary.		N to E	N to W	N to E	N to W	N to E	N to W	N to E	N to W	N to E	N to W		N to E	N to W
Deviation due to rotation	0° 00′ $\frac{5}{6}$		0° 23′ $\frac{1}{6}$		0° 23′		0° 52′ $\frac{5}{6}$		0° 56′		1° 22′			1° 26′ $\frac{2}{3}$	

* The plane of the plate was here horizontal.

† Here the plane of the plate was vertical.

Here we find, directly contrary to what took place when the plane of the plate was a tangent to the sphere, that the *deviation due to rotation* increases from the equator to the pole where it is a maximum. In this case, however, as in the other, the deviations are very nearly equal at equal distances on each side of the equator; so that, as before, it appears that the horizontal needle was affected by the rotation of the plate, not according to the situation of the centre of the plate with respect to the poles and equator of the horizontal needle, but with respect to the poles and equator of an imaginary dipping needle passing through the centre of the horizontal needle.

With regard to the direction of the *deviation due to rotation*, it appears, that when the centre of the plate had *north latitude*, the *north end* of the needle deviated *in the direction* of the motion of the plate's *inner edge;* and when it had *south latitude*, the *north end* deviated in a *contrary direction* to that of the *inner edge* of the plate, and therefore the *south end* deviated *in the direction* of the *inner edge*: so that, *the end of the needle of the same name as the latitude, always deviated in the direction of the motion of the plate's inner edge.*

Let us compare this with the inference we have drawn from the observations in Table I. viz. that when the centre of the plate is in the meridian, and its plane a tangent to the sphere, the north end of the needle, by the rotation of the plate, deviates in the direction of the motion of the south edge, and the south end in the direction of the north edge of the plate; that is, either end of the needle deviates in a direction contrary to that of the motion of the edge of the plate nearest to the pole of the sphere of the same name as that

end. Now, if from the position which the plate had in the last experiments, namely, its plane passing through the centre of the needle, it be conceived to revolve about its diameter, which is perpendicular to the plane of the meridian, until its plane be a tangent to the sphere, the direction of the revolution about this diameter being of the inner edge towards the pole of the same name as the latitude of the plate's centre, the inner edge will become the edge of the same name as the end of the needle, which, in its first position, according to our inference from the last observations, deviated in the direction of its rotation; but according to the inference drawn from Table I. the end of the needle of the same name as this edge will, in the new position, deviate in a direction contrary to that of its rotation; so that the rotation of the plate being in the same direction in both positions, the deviations by rotation will be in contrary directions in the two cases: and consequently, between the two positions, the plane of the plate must have passed through one in which the rotation would produce no deviation. If we conceive the plate to come into the position of the tangent plane by revolving about its diameter in the opposite direction, that is, by the inner edge moving towards the pole of a contrary name to the latitude, the inner edge will become the edge of the contrary name to the end of the needle, which in the first position, deviated in the direction of its rotation; and therefore that end of the needle will still continue to deviate in the same direction; that is, the direction of the rotation being the same in the two positions, the deviation by rotation will be in the same direction in both cases; and consequently, between the two positions, either there is no position of the plane of the plate

in which the rotation will produce no deviation, or there are two, or some even number of such positions.

I have not been able to determine in all cases experimentally the situation of the plane in which the *deviation due to rotation* vanishes, or whether there may be more than one plane in which this takes place; but all the observations which I have made, confirm me in the opinion which I formed on comparing the preceding results, that when the centre of the plate is in the meridian, there is only one plane between the tangent plane and the plane passing through the centre of the needle in which the deviation due to rotation vanishes, and that that plane is parallel to the equator.

Another conclusion which we may draw from these experiments compared with those in Table I. is this, that when the centre of the plate is in the meridian, and its plane perpendicular both to the meridian and equator, then, supposing the plate always to revolve in the same direction, the deviation will always be in one direction, in whatever point of the meridian the centre of the plate may be; for when the centre of the plate is in longitude 90°, latitude 0, Table I. the plane of the plate has this position, and also when in latitude 90° S. and 90° N. Table IV. and with the same direction of rotation, the deviation will be in one direction in these two cases.

As I had already found, that, when the centre of the plate was in the secondary to the equator and meridian, and its plane a tangent to the sphere, the rotation caused no deviation of the horizontal needle; it appeared to me, that there ought to be no *deviation due to rotation* when the plane of the plate was in any other plane perpendicular to this secondary.

To ascertain how far my views were correct, or otherwise, I adjusted the plate on the arm as in fig. 6. the same as in the last experiments, and the instrument as in fig. 2: so that the axis ÆQ being in the plane of the meridian and inclined to the horizon at an angle of 19° 30′, the centre and plane of the plate were, during the revolution of the limb, always in the position I required. The distance between the centres of the needle and plate was as before 13.2 inches. The following Table exhibits the observations which I made; the letters at the tops of the columns indicate the direction of rotation of the plate's inner edge; and the numbers in the first column, the points on the plate which coincided with the plane of the secondary, when the several directions of the north end of the needle in the same lines with them were observed. The observations were made at every 10° of latitude, as in some cases there was an indication of *deviation due to rotation*.

To ascertain how far my views were correct, or ...

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V. *Table of the deviations of a magnetic needle caused by the rotation of a circular plate of iron when its centre was in the secondary to the equator and meridian, and its plane perpendicular to this secondary, and passing through the centre of the needle. Plate No.* I.

	Long. 0° Lat. 0°		Long. 0° Lat. 10° S.		Long. 0° Lat. 20° S.		Long. 0° Lat. 30° S		Long. 0° Lat. 40° S		Long. 0° Lat. 45° S		Long. 0° Lat. 50° S		Long. 0° Lat. 60° S.		Long. 0° Lat. 70° S.		Long. 0° Lat. 80° S		Lat. 90° S	
	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S
0	1° 16′ E	1° 16′ E	6° 58′ W	6° 56′ W	11° 10′ W	11° 10′ W	16° 12′ W	16° 14′ W	17° 36′ W	17° 30′ W	17° 50′ W	17° 50′ W	16° 56′ W	16° 58′ W	13° 40′ W	13° 38′ W	10° 08′ W	10° 02′ W	5° 08′ W	5° 08′ W	0° 22′ E	0° 22′ E
30	1 24 E	1 22 E	5 30	5 30	10 24	10 24	15 16	15 10	16 52	16 50	17 02	17 00	16 20	16 20	13 32	13 26	10 00	9 54	5 06	5 04	0 20	0 20
60	1 18 E	1 20 E	3 44	3 42	9 22	9 24	13 32	13 32	15 42	15 38	15 56	15 52	15 24	15 22	12 58	12 52	9 32	9 26	4 50	4 50	0 24	0 22
90	0 54 E	0 52 E	3 00	2 58	9 26	9 24	13 14	13 10	15 40	15 40	15 44	15 40	15 14	15 08	12 52	12 48	9 24	9 20	4 50	4 50	0 20	0 22
120	0 20 E	0 18 E	2 56	2 52	9 32	9 32	13 18	13 20	15 48	15 48	16 00	15 58	15 20	15 20	12 56	12 54	9 32	9 30	4 50	4 48	0 18	0 22
150	0 18 W	0 22 W	3 38	3 40	10 22	10 24	14 10	14 16	16 18	16 20	26 38	16 38	15 52	15 50	13 04	13 02	9 46	9 42	4 50	4 48	0 20	0 24
180	1 00 W	1 04 W	4 56	4 52	11 26	11 22	15 30	15 30	17 18	17 22	17 36	17 40	16 40	16 40	13 24	13 24	10 08	10 10	4 52	4 54	0 24	0 28
210	1 44 W	1 42 W	6 24	6 24	12 30	12 30	17 10	17 10	18 18	18 20	18 46	18 42	17 36	17 36	14 00	13 56	10 32	10 30	5 02	5 04	0 22	0 26
240	1 54 W	1 54 W	7 44	7 42	13 04	13 00	18 08	18 06	18 56	18 54	19 24	19 20	18 16	18 12	14 18	14 12	10 46	10 40	5 08	5 08	0 26	0 24
270	1 04 W	1 02 W	8 04	8 04	12 40	12 34	18 00	17 58	18 38	18 36	19 16	19 12	18 02	18 20	14 06	14 00	10 36	10 36	5 10	5 10	0 24	0 24
300	0 04 W	0 02 W	8 06	8 06	12 00	11 56	17 40	17 38	18 24	18 20	19 02	19 02	17 52	17 52	14 00	13 58	10 32	10 30	5 10	5 10	0 22	0 22
330	0 38 E	0 42 E	7 44	7 44	11 34	11 32	16 52	16 52	17 50	17 48	18 26	18 22	17 20	17 16	13 42	13 42	10 18	10 16	5 08	5 08	0 20	0 20
Mean Deviations.	0 01 $\frac{1}{6}$ W	0 01 $\frac{1}{3}$ W	5 43 $\frac{2}{3}$	5 42 $\frac{1}{2}$	11 07 $\frac{1}{2}$	11 06	15 45 $\frac{1}{6}$	15 44 $\frac{2}{3}$	17 17	17 15 $\frac{1}{2}$	17 38 $\frac{1}{3}$	17 36	16 44 $\frac{1}{3}$	16 42 $\frac{5}{6}$	13 32 $\frac{2}{3}$	13 29 $\frac{1}{3}$	10 06 $\frac{1}{6}$	10 03	5 00 $\frac{1}{3}$	5 00 $\frac{1}{6}$	0 21 $\frac{5}{6}$	0 23
Deviations due to rotation.	0 00 $\frac{1}{6}$′		0° 01 $\frac{1}{6}$′		0° 01 $\frac{1}{2}$′		0° 00 $\frac{1}{2}$′		0° 01 $\frac{1}{2}$′		0° 01 $\frac{1}{2}$′		0° 01 $\frac{1}{2}$′		0° 03 $\frac{1}{3}$′		0° 03 $\frac{1}{6}$′		0° 00 $\frac{1}{6}$′		0° 01 $\frac{1}{6}$′	

Although the *deviations due to rotation* are here in some cases greater than might perhaps on a first view be expected, if in the position in which I have supposed the plate, its rotation would really produce no deviation, yet the differences are not in any case more than may, I consider, be fairly attributed to errors in the adjustments. That the deviations, when the plate revolved from south to north, had a tendency most generally to be greater than when it revolved in a contrary direction, as is evident by referring to the Table, appears at first sight more unfavourable to my opinion than the magnitude of the difference; but on further consideration, I think that this will be allowed rather to point out the source of the errors in the results, than the incorrectness of my views, and that these errors arose from the plane of the plate not being in those cases perpendicular to the plane of the secondary to the equator and meridian. The proximity of the edge of the iron to the ends of the needle, varying from 5.16 inches to 4.27 inches at the south end, and from 5.16 inches to 5.92 inches at the north end, I considered to be another source of error; the inequalities arising from the effects of particular points near the edges of the iron on the ends of the needle being the more sensible when the distances are small. All my observations were made as near to the centre of the needle as the instrument would admit, in order that the effects of the rotation, since they were in many cases extremely small, might be the more sensible; and by this means I discovered the nature of the effects produced on the needle by the rotation of the plate; but I am fully convinced, that for the purpose of comparing the results of observation with the conclusions from theory,

it is always desirable, that the observations should be made when the iron is at such a distance from the centre of the needle, that the effects of particular points, near its edges, on the ends of the needle are nearly insensible. Taking these circumstances into consideration, I was quite satisfied from these experiments, that, if the centre of the plate be in the secondary to the equator and meridian, and its plane perpendicular to the plane of that circle, the rotation of the plate will produce no effect on the absolute deviations caused by the mass.

In order to determine what effects would be produced by the rotation of the plate when its centre was in the secondary to the equator and meridian, and its plane in the plane of this circle, the instrument was adjusted as in fig. 1. the index at *g* pointing to 70° 30′; the limb SÆN was then placed at right angles to SQN, and the arm AB attached to it with the iron plate on the axis as in fig. 5; and that the centre of the needle might be in the plane of the plate, the compass box was moved in the direction of the meridian.

Some of my first observations were made with the centre of the plate in the equator, and I immediately found, that the *deviation due to rotation*, instead of being o, as in the cases when the plate revolved in the planes at right angles to its present position, was here considerable; and also that, that of the south end of the needle was in the direction of the upper, or south edge of the plate, contrary to what had been observed in the same plane at the pole (Table IV. lat. 90°). This indicated that there must be, at least, one point in this circle on each side of the pole, where the *deviation due to rotation* was o; and to determine nearly the latitude of this point, I

made observations at every 10° degrees of latitude on each side of the south pole. Before, however, giving these observations, it is necessary that I should state the kind of reliance I place on them as forming a complete set. In order to make the observations near the pole, it was necessary to adjust the instrument as in fig. 3. and after having made the complete set, I suspected that, in the change from the one adjustment to the other, the centre of the plate had been nearer to that of the needle in making the observations near the equator, than those near the pole; and that consequently, the *deviations due to rotation* in the former case, were proportionally too great. I was confirmed in this suspicion on comparing these observations with those which I had, in the first instance, made in lat 0° and in lat. 90°; and still further on comparing them with others, which I subsequently made at the several distances 15, 17, 19, 20 inches; in the corresponding situations. For example, in my first observations, the *deviations due to rotation* in lat. 0°, long. 0°, and in lat. 0° long. 180° were 3° 10′, and 3° 14′, giving a mean 3° 12′ in lat 0; and in lat. 90 S, 1° 31′; when the centres of the plate and needle had been carefully adjusted to the same distance 13.2 inches, in the two cases; whereas the corresponding deviations in the table are 3° 43′ and 1° 29½′; and, by subsequent observations, I found the sum of the deviations at the distances 15, 17, 19 and 20 inches to be in these two cases, 7° 20′ and 3° 32′, to which 3° 12′ and 1° 31′ are very nearly proportional. As however these differences do not in the least affect the conclusions which I at the time drew from this set of observation, and they were all made immediately follow-each other, I prefer giving them as a complete set for the

purpose of illustration; they are contained in the following Table. The numbers in the first column indicate the points on the plate which coincided with the line joining the centres of the plate and needle, when the several observations of the directions of the north end of the needle were made. Of the letters at the *tops* of the columns, the *upper* ones indicate the direction of rotation of the *south*, or *upper edge* of the plate, with respect to the points in the horizon; and the *lower* ones, the direction of the *inner edge*, or that nearest the axis, with regard to the poles of the sphere; the letters at the bottoms of the columns indicate the direction of the *deviation* of the south end of the needle *due to rotation*.

VI. *Table of the deviations of a magnetic needle, caused by the rotation of a circular plate of iron, when its centre was in the secondary to the equator and meridian, and its plane in the plane of this secondary. Plate No. I.*

	Long. 0°. Lat. 0°.		Long. 0°. Lat. 10° S.		Long. 0°. Lat. 20° S.		Long. 0°. Lat. 30° S.		Long. 0°. Lat. 40° S.		Long. 0°. Lat. 45° S.		Long. 0°. Lat. 50° S.		Long. 0°. Lat. 60° S.		Long. 0°. Lat. 70° S.		Long. 0°. Lat. 80° S.		Lat. 90° S.	
Upper Edge.	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E
Inner Edge.	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S		
0	2° 10′ E	1° 24′ W	5° 24′ W	8° 46′ W	13° 00′ W	16° 10′ W	19° 16′ W	21° 30′ W	21° 50′ W	23° 08′ W	21° 44′ W	22° 30′ W	21° 52′ W	22° 16′ W	18° 18′ W	18° 00′ W	13° 22′ W	12° 22′ W	8° 20′ W	6° 58′ W	1° 46′ W	0° 18′ W
30	1 54	1 50	5 40	9 28	13 10	16 44	19 22	21 50	21 54	23 20	21 50	22 40	21 50	22 10	18 30	18 12	13 20	12 20	8 18	6 58	1 48	0 24
60	1 46	1 58	6 34	9 44	13 16	16 50	19 22	21 54	21 56	23 20	21 48	22 40	21 30	21 50	18 24	18 04	13 20	12 18	8 14	6 54	1 46	0 1S
90	0 54	2 38	7 20	11 06	14 14	17 24	20 40	23 06	23 18	24 38	23 20	23 58	22 40	22 56	19 16	18 58	14 00	12 50	9 00	7 36	2 18	0 48
120	0 00	3 40	8 30	12 20	15 12	18 32	21 20	23 38	23 40	24 56	23 38	24 16	22 44	22 58	19 14	18 50	14 00	12 50	8 50	7 26	1 56	0 22
150	0 30 W	4 20	9 34	13 08	15 52	19 10	21 54	24 22	24 16	25 26	23 58	24 36	23 20	23 40	19 22	18 58	14 06	12 52	8 56	7 32	1 58	0 28
180	0 30	4 40	9 36	13 34	15 50	19 30	21 46	24 30	24 04	25 24	24 00	24 38	23 18	23 38	19 18	18 56	14 00	12 52	8 52	7 36	1 42	0 20
210	0 08	4 30	9 10	13 20	15 32	19 02	21 36	24 30	23 38	25 08	23 36	24 20	22 56	23 16	18 46	18 30	13 42	12 46	8 24	7 08	1 24	0 00
240	0 30 E	3 38	8 00	11 38	14 06	17 44	20 04	22 28	22 44	24 04	22 20	23 10	22 08	22 30	18 00	17 42	13 10	12 18	7 50	6 38	0 54	0 38 E
270	1 46	1 54	6 16	9 48	12 26	16 04	18 20	20 46	20 58	22 24	20 56	21 46	21 20	21 42	17 10	16 50	12 56	11 48	7 36	6 10	0 50	0 44
300	2 44	1 20	4 46	8 28	11 48	15 18	17 50	20 20	20 22	21 40	20 36	21 20	21 06	21 26	16 54	16 38	12 56	11 46	7 38	6 12	0 50	0 40
330	2 38	1 20	4 50	8 28	12 06	15 20	18 16	20 26	20 58	22 08	20 48	21 26	21 10	21 26	17 18	16 52	13 10	11 50	7 44	6 18	1 28	0 18
Mean Deviations.	0 56 1/6 E	2 46 W	7 08 1/3	10 49*	13 52 2/3	17 19	19 58 5/6	22 26 2/3	22 28 1/6	23 48	22 22 5/6	23 06 2/3	22 09 1/2	22 19	18 22 1/2	18 02 1/3	13 30 1/6	12 24 1/6	8 18 1/2	6 57 1/6	1 32 2/3 W	0 03 1/6 W
Direction of Deviation by rotation.	S to W	S to E	S to W	S to E	S to W	S to E	S to W	S to E	S to W	S to E	S to W	S to E	S to W	S to E	S to E	S to W	S to E	S to W	S to E	S to W	S to E	S to W
Deviation due to rotation.	3° 42′ 1/6		3° 40′ 2/3		3° 26′ 1/3		2° 27′ 5/6		1° 19′ 5/6		0° 43′ 5/6		0° 19′ 1/2		—0° 20′		—1° 06′		—1° 21 1/4		—1° 29′ 1/2	

	Long. 180°. Lat. 0°.		Long. 180°. Lat. 10° S.		Long. 180°. Lat. 20° S.		Long. 180°. Lat. 30° S.		Long. 180°. Lat. 40′ S.		Long. 180°. Lat. 45° S.		Long. 180°. Lat. 50° S.		Long. 180°. Lat. 60° S.		Long. 180°. Lat. 70° S.		Long. 180°. Lat. 80° S.			
Upper Edge.	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E		
Inner Edge.	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N	N to S	S to N		
0	1° 30′ W	5° 04′ W	6° 48′ E	3° 36′ E	14° 06′ E	10° 50′ E	19° 50′ E	17° 24′ E	21° 44′ E	20° 10′ E	21° 36′ E	20° 50′ E	20° 58′ E	20° 38′ E	17° 26′ E	17° 48′ E	11° 42′ E	12° 48′ E	5° 34′ E	6° 52′ E		
30	0 32	4 18	7 38	4 30	14 52	11 50	20 42	18 18	22 10	20 40	21 46	21 00	21 00	20 40	17 26	17 46	11 42	12 48	5 36	6 50		
60	0 38 E	2 52	8 42	5 20	15 44	12 42	20 00	19 34	22 38	21 30	22 10	21 22	21 10	20 52	17 36	18 00	12 00	12 56	5 38	6 56		
90	2 14	1 20	9 46	6 18	16 48	13 38	23 00	20 22	22 42	21 40	22 12	21 30	21 00	20 42	17 22	17 48	11 32	12 34	5 08	5 28		
120	3 16	0 22	10 48	7 32	18 00	15 00	24 20	21 44	24 02	22 58	23 40	23 00	22 18	22 02	18 16	18 40	12 08	13 10	5 34	6 58		
150	3 12	0 26	10 40	7 12	18 00	15 00	24 10	21 34	24 08	22 50	23 54	23 10	22 24	22 08	18 16	18 40	12 04	13 08	5 30	6 50		
180	3 12	0 26	10 46	7 16	17 58	14 58	24 20	21 36	24 34	23 16	24 30	23 48	22 50	22 32	18 38	19 04	12 22	13 20	3 44	7 00		
210	3 40	0 12	11 30	7 58	19 20	15 54	25 00	22 14	25 24	23 56	25 26	24 34	23 40	23 20	19 12	19 36	12 50	13 58	6 08	7 20		
240	3 38	0 10	11 48	8 16	19 22	15 58	24 50	22 08	25 54	24 38	25 52	25 10	24 12	23 52	19 46	20 08	13 20	14 22	6 40	8 00		
270	2 24	1 12	10 40	7 06	17 48	14 40	23 00	20 28	25 08	23 38	25 04	24 20	23 42	23 22	19 22	19 50	13 12	14 18	6 38	8 00		
300	0 42	2 52	9 10	5 40	16 52	13 26	21 42	19 34	24 06	22 36	24 00	23 18	23 10	22 48	19 00	19 26	12 50	14 06	6 30	7 52		
330	0 50 W	4 28 W	7 40	4 14	15 18	11 40	20 22	18 02	22 56	21 18	22 40	21 56	22 00	21 38	18 18	18 40	12 28	13 36	6 14	7 34		
Mean Deviations.	1 45 1/3 E	1 58 1/6 W	9 39 2/3	6 14 5/6	17 00 2/3	13 48	22 46 1/3	20 14 5/6	23 47 1/6	22 25 5/6	23 34 1/6	22 49 5/6	22 22	22 02 1/6	18 23 1/6	18 47 1/6	12 20 5/6	13 25 1/3	5 54 1/2	7 08 1/3*		
Direction of Deviation by rotation.	S to W	S to E	S to W	S to E	S to W	S to E	S to W	S to E	S to W	S to E	S to W	S to E	S to W	S to E	S to E	S to W	S to E	S to W	S to E	S to W		
Deviation due to rotation.	3° 43′ 5/6		3° 24′ 5/6		3° 12′ 2/3		2° 31′ 1/2		1° 21′ 1/3		0° 44′ 1/3		0° 19′ 1/6		—0° 24′		—1° 04 1/2		—1° 13 5/6			

* In adjusting the compass for the observations in long. 0°, lat. 10° and lat. 0°, the north end of the needle pointed 50′ W of zero in the box, and consequently 50′ should be subtracted from the westerly, and added to the easterly deviations. A similar error of 30′ W was made in the adjustment for lat. 80°, long. 180° and long. 0°, and lat. 90°. These will not affect the *deviations due to rotation*; but the *absolute deviations* must be increased when east, and diminished when west.

It appears, from these observations, that, when the plate revolves in the plane of a secondary to the equator and meridian,

1st. The deviation due to rotation is a maximum when the centre of the plate is in the equator.

2d. It decreases as the plate approaches the pole, and is o between the latitudes 50° and 60°, apparently very nearly at 55°; and from this point it increases till it attains a maximum in a contrary direction at the pole.

3d. At the south pole and on each side down to the latitude 55°, the deviation of the *south* end of the needle, due to rotation, is in the direction of the *north*, or *lower* edge of the plate: or, from the *south pole* down to the latitude 55°, the *south* end of the needle moves *towards* the plate, when the *inner* edge of the plate moves *from* the *south* pole, and *from* the plate when the *inner* edge moves towards the *south* pole.

4th. From the equator towards either pole as far nearly as the latitude 55°, the *south* end of the needle moves in the direction of the *south* edge of the plate; that is, it moves *towards* the plate when the *inner* edge of the plate moves *towards* the *south* pole, and *from* the plate, when that edge moves *from* the *south* pole; also the *north* end of the needle moves *towards* the plate, when the *inner* edge moves *towards* the *north* pole, and *from* the plate, when that edge moves *from* the *north* pole. Consequently towards whichever pole the *inner* edge moves, the corresponding end of the needle will move *towards* the plate from the equator to the latitude of 55° nearly, and the contrary will take place from the latitude 55° to the pole.

The observations which I made with the plate on the north

side of the equator, though not so multiplied as those on the south, were sufficient to show, that the deviations due to rotation observed the same laws on that side of the equator as I had noticed on the south side.

The *deviation due to the rotation of the plate*, when its centre is in the secondary to the equator and meridian, having a peculiar character, namely, two greater maxima when the centre is in the equator, two less maxima, in a contrary direction, when the centre is in either pole, and four points where it vanishes, I consider to be particularly well adapted for forming an estimate of the correctness of any theory which may be adopted for the explanation of the phænomena in general; since the theory must be perfectly compatible with these peculiarities, before it can be applied to the explanation of the less marked phænomena.

As it appeared from these observations, that the point where the deviation due to rotation vanishes, is not far from lat. 55°, the complement of which, 35°, is nearly half the angle of the dip, I wished to ascertain whether the deviation were really 0 in latitude 54° 45′, which I considered to be correctly the complement of half the dip 70° 30′, although I could not see how the angle which the plane makes with the horizon could have an influence on an angle in the plane itself. The following observations show, that in this instance the deviation due to rotation vanishes, or nearly so, when the polar distance of the centre of the plate is equal to half the angle which the dipping needle makes with the horizon. Whether this coincidence is purely accidental, or is a necessary consequence of the manner in which the effect is produced, must remain doubtful, until it can be shown how the action takes

place; it, however, led me to ascertain precisely the point at which the deviation due to rotation vanishes

VII. *Table of the deviations of a magnetic needle caused by the rotation of a circular plate of iron when the centre and plane of the plate were in the secondary to the meridian and equator, and its centre in latitude* 54° 45′.

	Lat. 54° 45′ S.				Lat. 54° 45′ N.			
	Long. 180°.		Long. 0°		Long. 180°.		Long. 0°.	
Upper Edge.	E to W	W to E	W to E	E to W	W to E	E to W	E to W	W to E
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	20 42 E	20 42 E	20 44 W	20 44 W	21 36 W	21 40 W	19 56 E	19 56 E
30	20 30	20 32	19 50	19 50	21 10	21 10	20 18	20 20
60	19 54	19 56	19 80	19 28	20 50	20 50	20 44	20 44
90	19 18	19 20	20 26	20 24	20 16	20 18	20 56	20 58
120	19 50	19 52	20 50	20 46	20 24	20 22	20 48	20 50
150	19 38	19 38	21 42	21 46	19 52	19 50	20 34	20 36
180	20 16	20 16	22 22	22 24	19 46	19 44	19 50	19 50
210	21 30	21 26	22 22	22 20	19 24	19 20	19 40	19 40
240	22 32	22 32	21 34	21 34	19 28	19 28	19 09	19 08
270	22 26	22 28	20 42	20 42	20 28	20 30	19 58	19 00
300	22 02	22 04	20 30	20 30	21 10	21 10	19 38	19 38
330	21 26	21 26	20 36	20 36	21 12	21 18	20 00	20 02
Mean Deviations.	20 50 ½	20 51	20 55 ⅔	20 55 ⅓	20 28	20 28 ⅓	20 02 ⅓	20 03 ½
Deviation due to rotation.	— 0° 00⅔′		+ 0° 00′⅓		— 0° 00⅓′		— 0° 01′	

General law of the deviation due to rotation *deduced from the experiments.*

Having now ascertained the nature of the effects produced on the horizontal needle by the rotation of the plate in different planes, I endeavoured to discover some general law, according to which the direction of the deviation depended on the direction of the rotation of the plate; so that the situation of the centre of the plate, the plane in which it revolved, and

the direction of rotation being given, we might point out immediately the direction in which the deviation would take place.

On comparing together all the facts which I have detailed, I found that this might be effected in the following manner. I refer the deviations of the horizontal needle to the deviations of magnetic particles in the direction of the dip, or to those of a dipping needle passing through its centre; so that, in whatever direction this imaginary dipping needle would deviate by the action of the iron, the horizontal needle would deviate in such a manner as to be in the same vertical plane with it: thus, when the north end of the horizontal needle deviates towards the west, and consequently the south end towards the east, I consider that it has obeyed the deviation of the axis of the imaginary dipping needle, whose northern extremity has deviated towards the west and its southern towards the east; so that the western side of the equator of this dipping needle has deviated towards the south pole of the sphere, and its eastern side towards the north pole. It would follow from this, that if the north and south sides of the equator of the dipping needle (referring to these points in the horizon) deviated towards the poles, no corresponding deviation would be observed in the horizontal needle; the effect, in this case, taking place in the meridian, would only be observable in the angle which the dipping needle made with the horizon. As it is not my intention at present to advance any hypothesis on the subject, I wish this to be considered only as a method of connecting all the phænomena under one general view. Assuming it then for this purpose, it will be found that the *deviations of the horizontal needle due*

to rotation are always such as would be produced *by the sides of the equator of this imaginary dipping needle deviating in directions contrary to the directions in which the edges of the plate move,* that edge of the plate nearest to either edge of the equator producing the greatest effect on it. By referring to the particular laws which I deduced at the time of making the experiments in different planes, it will be seen that they are all comprised under this general law; but this will be rendered more evident by taking an instance.

When the centre of the plate is in the meridian, and its plane a tangent to the sphere, the eastern side of the equator of the imaginary dipping needle, according to the above law, will deviate in a direction contrary to that of the motion of the eastern edge of the plate, and consequently the northern extremity of the axis will deviate in a contrary direction to that of the motion of the plate's northern edge, or it will deviate in the direction in which the southern edge of the plate moves. Hence the horizontal needle obeying the deviations of this dipping needle, the deviations of its north end due to the rotation of the plate will be in the direction in which the south edge of the plate moves, which is the law deduced from the experiments, Table I.

Experiments with the dipping needle.

Having found, in all the experiments which I have described, that the effects produced on the horizontal needle depended on the situation of the plate with respect to the axis and equator of an imaginary dipping needle passing through the centre of the horizontal needle, my next experiments were undertaken with the view of ascertaining whether the effects produced by the rotation of the plate on the dipping

needle itself corresponded with those which I had observed on the horizontal needle. In making these it was necessary to adjust the dipping needle on a stand detached from the instrument, on the arm of which the iron plate revolved, on account of the diameter of the case of the dipping needle being greater than the distance *s n* (fig. 1). It was therefore only in particular positions that I could observe the deviation caused by the rotation of the plate. This however was of the less importance, since, as I expected that the deviations of the dipping needle would be less than those of the horizontal needle nearly in the ratio of sin. 19° 30′ to 1, it was only in the cases in which they were the greatest that I was likely to have been able to observe them.

As the dipping needle, when in the position of the dip, could only vibrate in the plane of the meridian, no effect corresponding to the deviations of the horizontal needle could be observed, either when the centre of the plate was in the intersection of the meridian and equator, and its plane perpendicular to the planes of these circles, or when the centre of the plate was in the secondary to the meridian and equator, and its plane in the plane of this secondary. In order therefore to ascertain the deviations of a needle suspended freely by its centre of gravity, corresponding to those of an horizontal needle, when the plate had those positions, and which I considered to be the principal points to be determined, it was necessary to observe the effect produced on the dipping needle when the centre of the plate was in the equator and exactly east or west of the centre of the needle, and its plane parallel to the plane of vibration of the needle; and also when its centre and plane were in the plane of vibration.

In making these observations, the instrument was adjusted

as in fig. 1, the compass being however removed; the indexes at o, o′ were brought to Æ, æ, on the moveable limb, and that limb was placed at right angles to the fixed limb, so that the plane of the plate was parallel to the magnetic meridian. The dipping needle was then placed as nearly as possible in the required position, and the levels being carefully adjusted, the needle was made to vibrate freely and left to settle. After the plate had been made to revolve several times in the same direction, the point marked o was brought to coincide with the upper part of a line parallel to the magnetic axis, and passing through the centre of the plate. The needle was then slightly agitated, or made to vibrate through a small arc; and when it settled, the dip was noted both at the upper and lower extremity, or the south and north end of the needle. This was repeated for the points marked 60, 120, 180, 240, 300. The plate was now made to revolve in the contrary direction, and similar observations made of the dip of the needle when the several points 300, 240, 180, 120, 60, o, coincided with the upper part of the line parallel to the magnetic axis. Continuing the revolution of the plate in this direction, a second set of observations of the dip were made for the several points from 300 to o. After this, the plate was again made to revolve in its first direction, and a second set of observations made of the dip for the points from o to 300. I considered the mean of all the observations in the two sets, when the plate revolved from o to 300, as the mean dip when the plate revolved in this direction; the mean of all the observations in the two sets, when the plate revolved from 300 to o, as the mean dip when the plate revolved in

this direction; and the difference between these mean dips as the *deviation due to the rotation* of the plate.

As I had experienced that the dipping needle, even when of the best construction, was an instrument from which accurate results could only be obtained by taking a mean of a great number of observations, I was aware that, by making only two for each point of the plate, I was liable to an error in the observations for each point taken separately, but this I considered would be counteracted in taking the mean for all the points; so that the mean results could not err far from the truth. The dipping needle which I made use of was a very good instrument, by JONES, of Charing Cross: the needle, made according to Captain KATER's construction, consisted of two arcs of a circle; its length was 7 inches. The plate was the same I had used in the experiments with the horizontal needle.

For the better distinguishing of the edges of the plate and the direction of its rotation, I conceive two planes at right angles to each other to pass through its centre; one, the plane of the equator or a plane parallel to it, which I call the equatorial plane; the other, the plane of the secondary to the equator and meridian, or a plane parallel to this secondary, which I call the plane of or parallel to the axis. The intersections of the first plane with the edges of the plate, I call the equatorial north and south edges; and the intersections of the second, the polar north and south edges.

In the following table, the numbers in the first column indicate the point on the plate which was in the polar south; the inclinations of the needle corresponding to these positions

of the plate are in the following columns, which are in pairs, the one showing the inclination indicated by the southern extremity of the needle, the other, that by the northern. Above the pairs of columns, is indicated the direction in which the upper or polar south edge of the plate revolved, with reference to the points in the equator, and also the direction in which the equatorial south edge of the plate revolved with reference to the polar points in the plane of the axis. Under the columns, are the mean inclinations of the needle when the plate revolved in opposite directions, and below these, the mean deviation due to rotation.

VIII. *Table of the inclinations of the dipping needle when the centre of the plate was in longitude* 0°, *latitude* 0°, *and its plane parallel to the meridian; so that the axis of rotation of the plate was the same as the axis of vibration of the needle: the distance of the centre of the plate from that of the needle being* 9.5 *inches. Plate No.* I.

Points on the Plate coinciding with the Polar South.	Polar South edge of the Plate to Equatorial North, or Equatorial South edge to Polar South.				Polar South edge of the Plate to Equatorial South, or Equatorial South edge to Polar North.			
	First Set.		Second Set.		First Set.		Second Set.	
	S. end.	N. end.	S. end.	N. end.	S. end.	N. end.	S. end.	N. end.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	70 30	70 45	70 30	70 20	71 40	71 35	71 25	71 20
60	70 10	70 25	70 30	70 20	71 45	71 40	71 05	71 00
120	70 45	71 00	70 25	70 15	71 25	71 20	71 05	71 00
180	70 40	70 55	70 25	70 20	71 00	70 55	72 00	70 55
240	70 55	71 10	70 40	70 30	71 30	71 20	71 20	71 15
300	71 05	71 20	70 45	70 35	71 30	72 00	71 05	71 00
Mean dip.	78° 38′				71° 18′			
Mean deviation due to rotation.	0° 40′							

From these observations it appears that, in this position of the plate, the *deviation* of the *upper*, or *south* end of the needle,

due to rotation, was in the direction in which the *north* or *lower* edge of the plate revolved, and the deviation of the *north* or *lower* end of the needle, in the direction of the rotation of the *upper* or *south* edge of the plate. It would follow from this, that if a needle could be suspended freely by its centre of gravity, and the centre of the plate were in longitude 90°, latitude 0°, and its plane at right angles to the meridian; then also, the *deviation* of the *south* end of the needle *due to rotation*, would be in the direction of the *north* or *lower* edge of the plate, and the deviation of the *north* end, in the direction in which the *south* or *upper* edge revolved; which are precisely the directions of the deviations of the horizontal needle in this position of the plate. (See Table I.)

The law which I have shown to obtain in all the experiments on the horizontal needle, viz. that the sides of the equator of the imaginary dipping needle always deviated in directions contrary to those in which the corresponding edges of the plate moved, I had derived previously to having an opportunity of making any experiments with the dipping needle: a comparison of the above results with this law will more fully illustrate its nature, and at the same time show their perfect accordance. In making this comparison, it is necessary to notice that, an *increase* of the dip of the needle, corresponds to a deviation of the *southern* edge of its equator towards the *south* pole, and of the *northern* edge towards the *north* pole; and on the contrary, a *diminution* of the dip corresponds to a deviation of the *southern* edge of the equator towards the *north* pole, and of the *northern* edge towards the *south* pole. Now, when the equatorial *south* edge of the plate revolved towards the polar *south*, and consequently the

equatorial *north* edge towards the *polar* north, the inclination of the needle was *diminished* by the rotation; that is, the *south* edge of its equator deviated towards the *north* pole, and the *north* edge of its equator towards the *south* pole; or *the edges of the equator, by the rotation of the plate, deviated in directions contrary to those in which the edges of the plate moved.* The same conclusion evidently follows from the observations when the equatorial south edge of the plate revolved towards the polar north, the dip being here increased by the rotation of the plate.

The next observations which I made, were of the inclinations of the dipping needle, when the plane of the plate was in the plane of the meridian or plane of vibration of the dipping needle.

IX. *Table of the inclinations of the dipping needle, when the centre of the plate was in longitude* 90°, *latitude* 0°, *and its plane in the plane of the meridian, or plane of vibration of the dipping needle; the distance of the centre of the plate from that of the needle being* 13.3 *inches. Plate No. I.*

Points on the Plate coinciding with the Polar South.	Polar South edge of the Plate to Equatorial North, or Equatorial South edge to Polar South.				Polar South edge of the Plate to equatorial South, or Equatorial South edge to Polar North.			
	First Set.		Second Set.		First Set.		Second Set.	
	S. end.	N. end.	S. end.	N. end.	S. end.	N. end.	S. end.	N. end.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	70 25	70 35	70 45	70 30	69 40	69 35	69 35	69 15
60	69 50	70 05	70 15	70 05	69 20	69 15	69 30	69 20
120	69 50	70 00	69 55	69 50	68 55	68 55	69 00	68 45
180	70 45	70 55	70 35	70 25	69 35	69 40	69 35	69 25
240	70 55	71 05	70 50	70 40	69 30	69 35	69 20	69 10
300	70 40	70 50	70 30	70 15	69 35	69 40	69 30	69 20
Mean dip	70° 26′ ¼				69° 22′ ½			
Mean deviation due to rotation.	1° 04′							

From these observations it appears that, the plane of the plate being in the plane of vibration of the needle, and its centre in the equator, the *deviation* of the *upper* or *south* end of the needle, *due to rotation*, was in the direction of the rotation of the *upper* or *south* edge of the plate, and of the north end in that of the *north* edge; and we may therefore conclude, that if a needle could be freely suspended by its centre of gravity, and the centre of the plate were in the equator, and its plane in that of the secondary to the meridian and equator, the *deviation* of the *south* end, *due to rotation*, would be in the direction in which the *south* edge of the plate revolved, and of the *north* end, in that in which the *north* edge revolved: which, again are precisely the directions in which we have seen, that the horizontal needle deviated by the rotation of the plate in this position.

X. *Table of the inclinations of the dipping needle, when the centre of the plate was in latitude* 90° *South, and its plane in the plane of the meridian, or plane of vibration of the dipping needle; the distance of the centre of the plate from that of the needle being* 13.3 *inches. Plate No. I.*

Points on the Plate coinciding with the Polar South.	Polar South edge of the Plate to Equatorial North, or Equatorial South edge to Polar South.				Polar South edge of the Plate to Equatorial South, or Equatorial South edge to Polar North.			
	First Set.		Second Set.		First Set.		Second Set.	
	S. end.	N. end.	S. end.	N. end.	S. end.	N. end.	S. end.	N. end.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	71 05	71 05	71 35	71 30	71 40	71 40	71 50	71 40
60	71 00	71 10	71 10	71 05	71 35	71 15	71 35	71 25
120	71 10	71 15	71 25	71 25	71 40	71 30	72 15	72 05
180	71 40	71 45	71 35	71 40	72 00	71 50	72 30	72 20
240	71 40	71 45	71 30	71 35	72 05	72 05	72 15	72 05
300	71 05	71 10	71 25	71 35	71 40	71 55	72 00	71 55
Mean dip.	$71^{\circ}\ 23'\ \frac{1}{3}$*				$71^{\circ}\ 52'\ \frac{1}{12}$			
Mean deviation due to rotation.	$0^{\circ}\ 29'$							

* In these observations the edge of the iron plate was not an inch from the south end of the needle; so that a very small error in the position of the plate's centre will account for the dip in *both* directions of rotation being *greater* than $70^{\circ}\ 15'$, the true dip.

Here, contrary to what took place when the centre of the plate was in the equator, the deviation of the *south* end of the needle is in the direction in which the *lower* or *north* edge of the plate revolved; and we may therefore infer, that the same would be the case if a needle were suspended freely by its centre of gravity, and the plane of the plate were in the plane of the secondary to the meridian and equator, its centre being in latitude 90° S: which also agrees exactly with the directions of the deviation of the horizontal needle, due to rotation, in this position of the plate.

It is evident from these different experiments with the dipping needle, that whatever may be the peculiar effects produced on the iron by its rotation, the *deviations* of the horizontal needle, *due to the rotation*, are of the same nature as those that would arise by referring the deviations of the dipping needle to the horizontal plane.

Further observations with the horizontal needle.

Although, in order to point out the particular laws according to which the rotation of the iron causes the needle to deviate in particular situations of the plate, and to deduce a general law by which the direction of the deviation might in all cases be determined from the direction of rotation, I have been under the necessity of entering into such a detail of the experiments, as has already extended this paper beyond the limits to which I wished to confine it, I yet think it may not be uninteresting to enquire, how far the adoption of particular hypotheses may enable us to account for the several phænomena which I have observed.

I have already stated, that I considered that the deviations

arising from the rotation of the plate, when its centre and plane are in the secondary to the equator and meridian, are those best adapted for forming a comparison with the results obtained from theory. In Table VI. I have given a series of such observed deviations; but as I was not quite satisfied that in making these observations there had not been some small inaccuracies in the different adjustments, when the centre of the plate was near the equator and when near the pole, I should not on this ground have considered a comparison with them as altogether conclusive with respect to the correctness of any theory. In repeating these experiments, I increased the distance of the centre of the plate from that of the needle, as, in order to simplify the calculations, it would be necessary to neglect certain terms, which would be the greater the less was this distance, and consequently if it were increased, the neglecting these terms would the less affect the results of the calculation as compared with the observations. The following Table contains a series of observations similar to those in Table VI, but having the centre of the plate removed to the distance 16 inches from the centre of the needle. In making them, the most scrupulous attention was paid to the different adjustments, so that I can place entire confidence in the results.

Table of the deviations of a magnetic needle, caused by a circular plate of iron, whose centre was in the secondary to the equator and meridian, and plane in the plane of this secondary, the plate having revolved in opposite directions; the distance of the centre of the plate from the centre of the needle being 16 inches. Plate No. I.

Latitude and Longitude of the plate's centre.		Lat. 0°.		Lat. 10° S.		Lat. 20° S.		Lat. 30° S.		Lat. 40° S.		Lat. 50° S.		Lat. 54° 45′ S.		Lat. 60° S.		Lat. 70° S.		Lat. 80° S.		Lat. 90° S.	
		Long. 0°		Long. 0°.		Long. 0°.		Long. 0°.		Long. 0°.		Long. 0°.		Long. 0°.		Long. 0°.		Long. 0°.		Long. 0°.			
Direction of rotation of plate's upper edge.		W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W
Points on the plate coinciding with the line joining the centres of the needle and the plate.	0	1° 02′ W	0° 32′ E	4° 50′ W	3° 20′ W	7° 46′ W	6° 30′ W	9° 38′ W	8° 42′ W	10° 28′ W	9° 54′ W	10° 38′ W	10° 28′ W	10° 08′ W	10° 08′ W	8° 48′ W	9° 00′ W	6° 52′ W	7° 18′ W	3° 54′ W	4° 34′ W	0° 40′ W	1° 24′ W
	30	1 02	0 36	4 24	2 52	7 10	5 52	9 04	8 04	10 00	9 22	9 42	9 32	9 18	9 18	7 56	8 06	6 54	7 20	3 48	4 26	0 24 W	1 08 W
	60	0 58	0 40	4 04	2 32	6 38	5 22	8 38	7 38	9 38	9 00	8 56	8 46	8 46	8 46	7 24	7 32	6 50	7 16	3 44	4 18	0 18 W	0 56 W
	90	0 44	0 54	3 36	2 04	6 10	4 52	8 12	7 10	9 16	8 36	8 18	8 08	8 20	8 20	6 52	7 02	6 36	7 02	3 20	4 02	0 08 E	0 36 W
	120	0 46	0 52	3 32	2 02	6 12	4 56	8 18	7 18	9 22	8 42	8 18	8 06	8 26	8 24	6 58	7 12	6 20	6 48	3 02	3 48	0 26 E	0 24 W
	150	1 06	0 30	3 50	2 22	6 34	5 20	8 42	7 44	9 44	9 08	8 36	8 30	8 40	8 42	7 22	7 36	5 56	6 28	2 40	3 24	0 46 E	0 02 W
	180	1 14	0 26	4 06	2 34	6 58	5 40	9 04	8 02	10 04	9 30	9 02	8 52	9 00	9 02	7 48	8 00	5 26	5 56	2 18	2 58	1 08 E	0 22 E
	210	0 52	0 50	4 04	2 30	7 02	5 44	9 08	8 08	10 12	9 34	9 28	9 18	9 18	9 16	8 10	8 24	5 00	5 26	2 00	2 38	1 16 E	0 28 E
	240	0 34	1 04	4 02	2 30	7 10	5 52	9 16	8 14	10 16	9 38	9 58	9 52	9 42	9 42	8 36	8 48	4 48	5 18	2 02	2 40	1 08 E	0 22 E
	270	0 22	1 12	4 10	2 38	7 18	6 00	9 24	8 24	10 26	9 50	10 34	10 26	10 14	10 14	9 06	9 18	5 00	5 28	2 20	3 00	0 42 E	0 06 W
	300	0 26	1 06	4 26	2 56	7 34	6 20	9 40	8 38	10 36	10 00	11 04	10 56	10 36	10 40	9 28	9 38	5 32	6 02	2 54	3 34	0 06 E	0 38 W
	330	0 46	0 42	4 42	3 14	7 48	6 34	9 46	8 50	10 38	10 06	11 04	10 56	10 34	10 36	9 16	9 30	6 14	6 42	3 28	4 06	0 24 W	1 04 W
Mean deviations.		0 49⅓	0 47	4 08⅚	2 37⅚	7 01⅔	5 45⅙	9 04⅙	8 04⅓	10 03⅓	9 26⅔	9 38⅙	9 29⅙	9 25⅙	9 25⅔	8 08⅔	8 20½	5 57⅓	6 25⅓	2 57½	3 37⅓	0 19½ E	0 25½ W
Deviations due to rotation.		1° 36′⅓		1° 31′		1° 16′½		0° 59′⅚		0° 36′⅔		0° 09′		−0° 00′½		−0° 11′⅚		−0° 28′		−0° 39′⅚		−0 45	

		Long. 180°.		Long. 180°.		Long. 180°.		Long. 180°.		Long. 180°.		Long. 180°.		Long. 180°.		Long. 180°.		Long. 180°.		Long. 180°.		Lat. 90° N.	
Direction of rotation of plate's upper edge.		E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E
Points on the plate coinciding with the line joining the centres of the needle and the plate.	0	1° 20′ E	0° 14′ W	4° 08′ E	2° 38′ E	7° 16′ E	5° 58′ E	9° 30′ E	8° 30′ E	10° 16′ E	9° 38′ E	10° 42′ E	10° 32′ E	10° 06′ E	10° 06′ E	9° 06′ E	9° 18′ E	4° 46′ E	5° 12′ E	1° 44′ E	2° 24′ E	0° 16′ W	0° 26′ E
	30	1 10	0 26	3 52	2 20	7 00	5 42	9 16	8 14	10 10	9 32	10 40	10 30	10 06	10 06	9 14	9 24	5 16	5 40	2 08	2 44	0 14	0 30
	60	0 58	0 40	3 38	2 06	6 46	5 26	9 00	8 00	9 54	9 16	10 12	10 02	10 40	9 40	8 52	9 04	5 42	6 10	2 28	3 06	0 10	0 34
	90	0 28	1 10	3 16	1 44	6 22	5 06	8 32	7 36	9 28	8 56	9 32	9 22	9 02	9 02	8 20	8 32	6 08	6 38	2 50	3 24	0 16	0 30
	120	0 12	1 22	3 06	1 36	6 10	4 56	8 18	7 20	9 14	8 40	8 46	8 38	8 20	8 20	7 38	7 50	6 24	6 56	3 04	3 50	0 06	0 42
	150	0 08	1 22	3 14	1 44	6 18	5 00	8 26	7 26	9 16	8 40	8 24	8 14	8 00	7 58	7 14	7 24	6 36	7 06	3 24	4 06	0 14	0 32
	180	0 18	1 18	3 28	1 56	6 30	5 14	8 44	7 42	9 32	8 52	8 20	8 08	7 56	7 54	7 08	7 16	6 40	7 08	3 38	4 18	0 32	0 14
	210	0 32	1 06	3 46	2 16	6 56	5 36	9 00	8 00	9 44	9 04	8 26	8 12	8 02	7 58	7 10	7 18	6 32	6 58	3 38	4 16	0 34	0 10
	240	0 52	0 46	4 02	2 30	7 10	5 52	9 18	8 18	9 56	9 18	8 42	8 30	8 12	8 12	7 20	7 28	6 04	6 32	3 18	3 58	0 32	0 10
	270	1 00	0 36	4 08	2 38	7 14	5 56	9 22	8 24	9 56	9 20	9 02	8 52	8 28	8 28	7 32	7 42	5 26	5 56	2 46	3 26	0 34	0 12
	300	1 18	0 18	4 18	2 50	7 24	6 06	9 32	8 32	10 08	9 32	9 44	8 32	9 04	9 04	8 04	8 14	4 56	5 24	2 14	2 54	0 26	0 18
	330	1 18	0 10	4 14	2 48	7 22	6 06	9 34	8 36	10 18	9 40	10 24	10 12	9 44	9 42	8 44	8 54	4 40	5 08	1 52	2 32	0 16	0 28
Mean deviations.		0 47⅚	0 47⅓	3 45⅚	2 15½	6 52⅓	5 34⅚	9 02⅔	8 03⅙	9 49⅓	9 12⅓	9 24½	9 13⅔	8 53⅓	8 52½	8 01⅚	8 12	5 45⅚	6 14	2 45⅓	3 25⅔	0 20⅚	0 23⅚
Deviation due to rotation.		1° 35′⅙		1° 30′⅓		1° 17′½		0° 59′½		0° 37′		0° 10′⅚		+ 0° 00′⅚		− 0° 10′⅙		−0° 28′⅙		− 0° 40′⅓		−0 44⅔	

Considering the centre of the plate to have been in longitude 180°, and consequently the deviations easterly in all the observations, and thus taking a mean of the deviations in the several latitudes, I obtain the following.

A. *Table of the easterly deviations of the needle, when the centre of the plate was in longitude 180°, or to the west of the needle, the plate having revolved in opposite directions.*

Latitude of the plate's centre.	0°		10°		20°		30°		40°		50°		54° 45′		60°		70°		80°		90°	
Direction of rotation of plate's upper edge.	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E
Deviation East.	+47′ 55″	−47′ 55″	3° 57′ 20″	2° 26′ 40″	6° 57′ 00″	5° 40′ 00″	9° 03′ 25″	8° 03′ 45″	9° 56′ 20″	9° 19′ 30″	9° 31′ 20″	9° 21′ 25″	9° 09′ 15″	9° 09′ 05″	8° 05′ 15″	8° 16′ 15″	5° 51′ 35″	6° 19′ 40″	2° 51′ 25″	3° 31′ 30″	−22′ 25″	+22′ 25″
Deviation due to rotation.	1° 35′ 50″		1° 30′ 40″		1° 17′ 00		0° 59′ 40″		0° 36′ 50″		0° 09′ 55″		0° 00′ 10″		−0° 11′ 00″		−0° 28′ 05″		−0° 40′ 05″		−0° 44′ 50″	

Two sets of observations which I had made more than two years before, had given me the following results; but as I afterwards suspected that the absolute deviations might have been affected by the proximity of a mass of iron, of which I was not aware at the time of making the observations, I considered it better to repeat them in a situation where no such influence could be exerted, although I did not conceive that this would materially affect the conclusions.

B. *Table of the mean easterly deviations of the needle, when the centre of the plate was in longitude 180°, or to the west of the needle, the plate having revolved in opposite directions;* deduced from two sets of observations made in November 1822 and February 1823.

Latitude of the plate's centre.	0°		10°		20°		30°		40°		50°		54° 45′		60°		70°		80°		90°	
Direction of rotation of plate's upper edge.	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E	E to W	W to E
Deviation East. 1st Set.	+50′ 25″	−50′ 25″	4° 35′ 55″	3° 00′ 55″	7° 39′ 25″	6° 19′ 00″	10° 07′ 10″	9° 04′ 20″	11° 11′ 15″	10° 33′ 00″	10° 48′ 55″	10° 36′ 55″	No observation.		9° 13′ 05″	9° 25′ 30″	6° 32′ 15″	7° 04′ 55″	3° 14′ 55″	4° 00′ 35″	−25′ 00″	+25′ 00″
Deviation East. 2d Set.	+50 32	−50 32	4 25 45	2 48 50	7 36 45	6 11 55	10 08 30	9 02 05	10 53 10	10 14 15	10 40 55	10 28 15	9 53 35	9 53 05	9 11 15	9 23 50	6 35 30	7 07 45	3 06 45	3 51 25	−24 38	+24 38
Mean deviation East.	+50 28	−50 28	4 30 50	2 54 32	7 38 05	6 15 27	10 07 50	9 03 12	11 02 12	10 23 37	10 44 55	10 32 35	9 53 35	9 53 05	9 12 10	9 24 40	6 33 52	7 06 20	3 10 50	3 56 00	−24 49	+24 49
Mean deviation due to rotation.	1° 40′ 56″		1° 35′ 58″		1° 22′ 38″		1° 04′ 38″		0° 38′ 35		0° 12′ 20″		c° 00′ 30′		−0° 12′ 30″		−0° 32′ 28″		−0° 45′ 108		−0° 49′ 38″	

Theoretical Investigations.

It has in general been considered that the different deviations of the horizontal needle, arising from the action of soft iron on it in different positions, can only be accounted for on the supposition, that the iron is polarised by position, the upper part being a north pole, and the lower a south one, each pole of the iron attracting the pole of the needle of the same name, and repelling that of a contrary name: but if we suppose that each particle of the iron simply attracts indifferently either pole of a magnetic particle, and refer the attraction of the iron to its centre, then if the angular deviations of a magnetic particle in the centre of the needle and in the line of the dip, arising from such attraction, be reduced to the horizontal plane, these reduced deviations will agree with the actual deviations of the horizontal needle. In investigating theoretically the effects that are produced by the *rotation* of a plate of iron, I will first suppose, that, independently of *rotation*, the iron acts in this manner, and that by the *rotation* it becomes polarised in a direction, making a certain angle with the magnetic axis, since from such a polarising of the iron, the law which I have shown to include all the phænomena, would evidently result. On this supposition, each pole of a magnetic particle in the centre of the needle would be urged by an attractive force towards *the centre* of the iron plate, by an attractive force towards the pole of a contrary name, and by a repulsive force from the pole of the same name in the iron.

Suppose now that the centre and plane of the plate are in the secondary to the meridian and equator, that its centre is

to the west of the needle, or in longitude 180°; and in south latitude, as in the observed deviations in Tables A and B, and that its upper edge revolves from east to west. Take the centre of the needle as the origin of the rectangular co-ordinates, the axis x being horizontal, and towards the west, that of z upwards, in the direction of the magnetic axis. We will indicate the north end of the magnetic particle, in the centre of the needle, on which the iron is supposed to act, by N, its south end by S; the north end of the line joining the poles in the plate by ν, its south end by σ, its centre by γ. Let the co-ordinates to the centre of this line be a, b; to its north end a_ν, b_ν; to its south end a_σ, b_σ; and let ψ be the angle which this line makes with a line passing through its centre, and parallel to the axis z. Also let r be half the length of the magnetic particle in the centre of the needle.

Since the effect of any force to turn the particle SN will be the same, whether it be supposed to act on N in a given direction, or on S in the contrary one, we may refer the action of the mass of the iron, and likewise the actions of its poles, wholly to the end S. Calling then m, the magnetic force of the earth acting in the direction of the dip; F the force of the mass of the iron, and f that of one of its poles on S, at the unity of distance; also the sum of all the forces on S resolved in the directions x and z, X and Z, we shall have

$$\left.\begin{array}{l} Fa\cdot\left(\frac{1}{S\gamma^3}-\frac{1}{N\gamma^3}\right) \\ +f\cdot\left\{a_\nu\cdot\left(\frac{1}{S\nu^3}+\frac{1}{N\nu^3}\right)-a_\sigma\cdot\left(\frac{1}{S\sigma^3}+\frac{1}{N\sigma^3}\right)\right\} \end{array}\right\}=X$$

$$\left.\begin{array}{l} 2m+F\cdot\left(\frac{b-r}{S\gamma^3}-\frac{b+r}{N\gamma^3}\right) \\ +f\cdot\left(\frac{b_\nu-r}{S\nu^3}+\frac{b_\nu+r}{N\nu^3}-\frac{b_\sigma-r}{S\sigma^3}-\frac{b_\sigma+r}{N\sigma^3}\right) \end{array}\right\}=Z$$

If ϕ' be the angle which, in consequence of these forces, the magnetic particle makes with the axis z, or line of the dip, that is, its angle of deviation towards the west, we shall have

$$\text{Tan. } \phi' = \frac{X}{Z} \qquad (1)$$

Let now λ be the latitude of the plate's centre; R the radius of the circle in which the centre of the plate is made to move, that is, its distance from the centre of the needle; ρ, half the distance between the poles in the iron plate: then

$$a = R \cos. \lambda, \qquad b = R \sin. \lambda;$$

$$a_\sigma = R \cos. \lambda + \rho \sin. \psi, \qquad b_\sigma = R \sin. \lambda + \rho \cos. \psi;$$

$$a_\nu = R \cos. \lambda - \rho \sin. \psi, \qquad b_\nu = R \sin. \lambda - \rho \cos. \psi;$$

$$S\gamma^2 = R^2 + r^2 - 2 R r \sin. \lambda; \quad N\gamma^2 = R^2 + r^2 + 2 R r \sin. \lambda;$$

$$S\nu^2 = R^2 + r^2 + \rho^2 - 2 R\rho \sin. (\psi + \lambda) - 2 r (R \sin. \lambda - \rho \cos. \psi);$$

$$N\nu^2 = R^2 + r^2 + \rho^2 - 2 R\rho \sin. (\psi + \lambda) + 2 r (R \sin. \lambda - \rho \cos. \psi);$$

$$S\sigma^2 = R^2 + r^2 + \rho^2 + 2 R\rho \sin. (\psi + \lambda) - 2 r (R \sin. \lambda + \rho \cos. \psi);$$

$$N\sigma^2 = R^2 + r^2 + \rho^2 + 2 R\rho \sin. (\psi + \lambda) + 2 r (R \sin. \lambda + \rho \cos. \psi).$$

Substituting these values in the expressions for X and Z, expanding the several fractions, and neglecting the terms in the series after the third, on account of r and ρ being small compared with R, the equation (1) will become,

$$\text{Tan. } \phi' = \frac{3 \sin. \lambda \cos. \lambda + \frac{2 f \rho}{F r} \left\{ 3 \sin. (\psi + \lambda) . \cos. \lambda - \sin. \psi \right\}}{\frac{m R^3}{F r} + 3 \sin.^2 \lambda - 1 + \frac{2 f \rho}{F r} \left\{ 3 \sin. (\psi + \lambda) . \sin. \lambda - \cos. \psi \right\}} \qquad (2)$$

If we call $\phi_{,}$ the angle of deviation of the magnetic particle when the plate revolves in the opposite direction, that is, its upper edge from west to east, we shall have

$$\text{Tan. } \phi_{,} = \frac{3 \sin. \lambda \cos. \lambda - \frac{2 f \rho}{F r} . \left\{ 3 \sin. (\psi - \lambda) . \cos. \lambda - \sin. \psi \right\}}{\frac{m R^3}{F r} + 3 \sin.^2 \lambda - 1 - \frac{2 f \rho}{F r} \left\{ 3 \sin. (\psi - \lambda) . \sin. \lambda + \cos. \psi \right\}} \qquad (3)$$

Also if φ is the angle of deviation of the magnetic particle due to the mass alone of the iron, then

$$\tan\varphi = \frac{3\sin.\lambda\cos.\lambda}{\frac{mR^3}{Fr} + 3\sin.^2\lambda - 1} \qquad (4),$$

Since when the *rotation* of the iron produces no effect, $\varphi' = \varphi = \varphi_{,}$ we must have in this case,

$$\frac{3\sin.(\psi+\lambda).\cos.\lambda - \sin.\psi}{3\sin.(\psi+\lambda).\sin.\lambda - \cos.\psi} = \frac{3\sin.(\psi-\lambda).\cos.\lambda - \sin.\psi}{3\sin.(\psi-\lambda).\sin.\lambda + \cos.\psi}.$$

whence

$$\sin.\psi.\cos.\psi = 0.$$

The value of ψ which satisfy this equation are 0°, 90°, 180°, 270°; and since 0° and 180° would in all cases give $\varphi' = \varphi_{,}$ these must be excluded; also $\psi = 270°$ would merely give the same value for φ' which $\psi = 90°$ would give for $\varphi_{,}$, and *vice versâ*; consequently we have $\psi = 90°$.

If now we reduce the deviations φ', φ, $\varphi_{,}$, of the magnetic particle in the line of the dip, to the horizontal plane, and call the corresponding horizontal deviations θ', θ, $\theta_{,}$, and the angle of the dip δ; then since tan. $\varphi = \cos.\delta\tan.\theta$, and $\psi = 90°$, the equations (2), (3), (4) will become

$$\text{Tan.}\,\theta' = \frac{1}{\cos.\delta}\cdot\frac{3\sin.2\lambda + \frac{2f\rho}{Fr}\cdot(3\cos.2\lambda+1)}{\frac{2mR^3}{Fr} - (3\cos.2\lambda - 1) + \frac{2f\rho}{Fr}\cdot 3\sin.2\lambda} \qquad (5),$$

$$\text{Tan.}\,\theta_{,} = \frac{1}{\cos.\delta}\cdot\frac{3\sin.2\lambda - \frac{2f\rho}{Fr}.(3\cos.2\lambda+1)}{\frac{2mR^3}{Fr} - (3\cos.2\lambda - 1) - \frac{2f\rho}{Fr}.3\sin.2\lambda} \qquad (6),$$

$$\text{Tan.}\,\theta = \frac{1}{\cos.\delta}\cdot\frac{3\sin.2\lambda}{\frac{2mR^3}{Fr} - (3\cos.2\lambda - 1)} \qquad (7),$$

The angles θ' θ, θ, are the horizontal angles of deviation of the *south* end of the magnetic particle towards the *west*; so

that the centre of the plate being to the *west* of the needle or in longitude 180°, the deviation of the *north* end of the horizontal needle ought to be towards the *east*, which agrees with the observations.

Having determined from observation the value of λ when the *rotation* of the plate produces no effect, and the corresponding value of θ, we may determine the value of $\frac{2mR^3}{Fr}$, independently of $f\rho$; and the value of $\frac{2f\rho}{Fr}$ may then be determined from the observed values of θ' and $\theta_{,}$ when $\lambda = 0$ or $90°$. Substituting these numerical values for $\frac{2Rm^3}{Fr}$ and $\frac{2f\rho}{Fr}$ in the equations (5) and (6), we may deduce the values of θ' and $\theta_{,}$ corresponding to different values of λ, and compare them with those actually observed.

From Tables A, B, and also Table VII, it appears that the *deviation due to the rotation* of the plate vanishes when the latitude of its centre is very nearly 54° 45′, or as nearly as can be determined when $3 \cos. 2\lambda + 1 = 0$, in which case $\lambda = 54° \, 44' \, 08''$. We shall therefore have from Table A,

$$\text{Tan. } 9° \, 09' \, 10'' = \frac{1}{\cos. 70° \, 15'} \frac{3 \sin. 109° \, 28' \, 16''}{\frac{2mR^3}{Fr} - 2}.$$

Whence $\frac{2mR^3}{Fr} = 49.9504 = 50$ very nearly.

When $\lambda = 0$, $\tan. \theta' = \frac{1}{\cos. \delta} \cdot \frac{4 \cdot \frac{2f\rho}{Fr}}{\frac{2mR^3}{Fr} - 2}$; and since $\theta = 47' \, 55''$ and $\delta = 70° \, 15'$, we obtain $\frac{2f\rho}{Fr} = .056524$.

Also when $\lambda = 90°$, $\tan. \theta' = \frac{1}{\cos. \delta} \cdot \frac{-2 \cdot \frac{2f\rho}{Fr}}{\frac{2mR^3}{Fr} + 4}$; where $\theta' = -22' \, 25''$; so that, $\frac{2f\rho}{Fr} = .059494$.

Taking a mean of the two values, we shall have,

$$\frac{2f\rho}{Fr} = .058009 = .058 \text{ very nearly.}$$

The equations (5) and (6) therefore become,

$$\text{Tan. } \theta' = \frac{1}{\cos. 70^\circ 15'} \cdot \frac{3 \sin. 2\lambda + .058 \times (3 \cos. 2\lambda + 1)}{51 - 3 \cos. 2\lambda + .058 \times 3 \sin. 2\lambda} \qquad (5_a),$$

$$\text{Tan. } \theta_{,} = \frac{1}{\cos. 70^\circ 15'} \cdot \frac{3 \sin. 2\lambda - .058 \times (3 \cos. 2\lambda + 1)}{51 - 3 \cos. 2\lambda + .058 \times 3 \sin. 2\lambda} \qquad (6_a).$$

From Table B, we shall in the same manner obtain

$$\frac{2mR^3}{Fr} = 46.0278 = 46 \text{ very nearly;}$$

$\frac{2f\rho}{Fr} = .054571$, when $\lambda = 0$, and $\frac{2f\rho}{Fr} = .060986$, when $\lambda = 90$; so that the mean value of $\frac{2f\rho}{Fr}$ is 0.57778 or .058 nearly, the same as before.

The equations (5) and (6) in this case become,

$$\text{Tan. } \theta' = \frac{1}{\cos. 70^\circ 15'} \times \frac{3 \sin. 2\lambda + .058 \times (3 \cos. 2\lambda + 1)}{47 - 3 \cos. 2\lambda + .058 \times 3 \sin. 2\lambda} \qquad (5_b),$$

$$\text{Tan. } \theta_{,} = \frac{1}{\cos. 70^\circ 15'} \times \frac{3 \sin. 2\lambda - .058 \times (3 \cos. 2\lambda + 1)}{47 - 3 \cos. 2\lambda - .058 \times 3 \sin. 2\lambda} \qquad (6_b).$$

We will first compare the situations of the points where the *deviation due to rotation* vanishes, as deduced from these equations, with the situation as determined by actual observation.

When the deviation due to rotation vanishes $\theta' = \theta = \theta_{,}$; we shall therefore have from equations (5_a) and (6_a)

$$\frac{3 \sin. 2\lambda}{51 - 3 \cos. 2\lambda} = \frac{3 \cos. 2\lambda + 1}{3 \sin. 2\lambda};$$

whence $\cos. 2\lambda = -.2800000$ and $\lambda = 53^\circ\ 07'\ 48''$.

The equations (5_b) and (6_b) give

$$\frac{3 \sin. 2\lambda}{47 - 3 \cos. 2\lambda} = \frac{3 \cos. 2\lambda + 1}{3 \sin. 2\lambda};$$

whence $\cos. 2\lambda = -.2753623$ and $\lambda = 52^\circ\ 59'\ 30''$.

Now I have uniformly found, in repeated observations which I have made at different distances, that when the centre

of the plate was in latitude 54° 45′, the *rotation* produced so little effect, that only in one instance did the mean values of θ' and $\theta_{,}$ differ by a minute; and therefore cannot but conclude that 54° 45 is, as nearly as can be ascertained by observation, the true value of λ when $\theta' = \theta_{,}$, and that the value of λ as determined by the theory differs from its value derived from observatien by $1\frac{1}{2}°$ or $1\frac{3}{4}°$.

Let us now compare the values of θ', $\theta_{,}$ and $\theta' - \theta_{,}$, deduced from the equations (5_a), (6_a), (5_b) (6_b), for different values of λ, with those actually observed. This comparison is made in the following tables, where I have computed the values of θ' and $\theta_{,}$ to seconds, not that either the observed or computed values can be determined to such a degree of accuracy, but because the omission of the seconds might in some cases affect the value of $\theta' - \theta_{,}$ by more than a minute.

Table of the values of θ', $\theta_{,}$ and $\theta' - \theta_{,}$, computed from the equations (5_a), (6_a) compared with their mean observed values in Table A.

λ	θ'			$\theta_{,}$			$\theta' - \theta_{,}$		
	Observed.	Computed.	Difference.	Observed.	Computed.	Difference.	Observed.	Computed.	Difference.
°	° ′ ″	° ′ ″	′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″	° ′ ″	′ ″
0	+ 47 55	+ 49 10	+ 1 15	— 47 55	— 49 10	+ 1 15	1 35 50	1 38 50	+ 2 30
10	3 57 20	4 22 35	+ 25 15	2 26 40	2 49 57	+ 23 17	1 30 40	1 32 28	+ 1 58
20	6 57 00	7 19 21	+ 22 21	5 40 00	6 02 21	+ 22 21	1 17 00	1 17 00	0 00
30	9 03 25	9 17 07	+ 13 18	8 03 45	8 22 06	+ 18 21	59 40	55 01	— 4 39
40	9 56 20	10 04 49	+ 8 29	9 19 30	9 34 13	+ 14 31	36 50	30 48	— 6 02
50	9 31 20	9 41 19	+ 9 59	9 21 25	9 34 25	+ 13 00	09 55	06 54	— 3 01
60	8 05 15	8 13 00	+ 7 45	8 16 15	8 26 50	+ 10 35	— 11 00	— 13 50	— 2 50
70	5 51 35	5 51 44	+ 0 09	6 19 40	6 21 41	+ 2 01	— 28 05	— 29 57	— 1 52
80	2 51 25	2 53 40	+ 2 15	3 31 30	3 33 52	+ 2 22	— 40 05	— 40 12	— 0 07
90	— 22 25	— 21 51	+ 0 34	+ 22 15	+ 21 51	— 0 34	— 44 50	— 43 42	+ 1 08

tions: and although the results of the theory agree in general very nearly with the observations, and the differences in the other values of $\frac{Fr}{2f\rho}$ are not greater than might possibly be attributed to errors of adjustment or observation, however little I may be disposed to admit the existence of errors to this extent, yet the uniform manner in which these values decrease, indicates that the effects are not produced in precisely the manner we have supposed. In one point our theory is unquestionably at variance with the actual circumstances of the case; for we have supposed that no partial magnetism exists in the iron, or that every part of it taken separately would equally affect the needle. It is, I believe, scarcely possible to procure iron that shall possess this uniformity of action, and it is evident that this was not the case with the plate of iron which I made use of. This species of polarity in iron is of so variable a nature, since by an accidental blow it will be transferred from one point to another, that it does not appear possible in any manner to submit its effects to calculation. It was to prevent these effects embarrassing the results, that I took the mean of twelve observations for each position of the plate; still it is possible that some of the differences between the observations and the results of the theory may have arisen from this cause.

As the results of the hypothesis which I have advanced do not precisely agree with the observations, it will be proper to enquire whether we shall obtain a more perfect agreement by means of the hypothesis commonly assumed, in order to account for the effects produced on the needle by a mass of soft iron, viz. that the upper part of every mass of iron acts as a north pole and the lower part as a south pole. Let us

then suppose such poles to exist in the iron plate, in the diameter in the direction of the dip, and that the rotation causes the line joining them to describe in the iron an angle ψ from this diameter. The whole effect being now produced by the action of these poles, F being equal to o in the equations (2) and (3), we shall, on this supposition, have,

$$\text{Tan. } \phi' = \frac{3 \sin. (\lambda + \psi) . \cos. \lambda - \sin. \psi}{\frac{M R^3}{2 f' \rho} + 3 \sin. (\lambda + \psi) . \sin. \lambda - \cos. \psi} \qquad (2'),$$

$$\text{Tan. } \phi_{\prime} = \frac{3 \sin. (\lambda - \psi) . \cos. \lambda + \sin. \psi}{\frac{m R^3}{2 f' \rho} + 3 \sin. (\lambda - \psi) . \sin. \lambda - \cos. \psi} \qquad (3'),$$

$$\text{Tan. } \phi = \frac{3 \sin. \lambda \cos. \lambda}{\frac{m R^3}{2 f' \rho} + 3 \sin.^2 \lambda - 1} \qquad (4').$$

These equations being reduced, give,

$$\text{Tan. } \theta' = \frac{1}{\cos \delta} . \frac{3 \sin. 2 \lambda + \tan. \psi (3 \cos. 2 \lambda + 1)}{\frac{m R^3}{f' \rho \cos. \psi} - (3 \cos. 2 \lambda - 1) + \tan \psi . 3 \sin. 2 \lambda} \qquad (5'),$$

$$\text{Tan. } \theta' = \frac{1}{\cos. \delta} . \frac{3 \sin. 2 \lambda - \tan. \psi (3 \cos. 2 \lambda + 1)}{\frac{m R^3}{f' \rho \cos. \psi} - (3 \cos. 2 \lambda - 1) - \tan. \psi . 3 \sin. 2 \lambda} \qquad (6'),$$

$$\text{Tan. } \theta = \frac{1}{\cos. \delta} . \frac{3 \sin. 2 \lambda}{\frac{m R^3}{f' \rho} - (3 \cos. 2 \lambda - 1)} \qquad (7'),$$

which will be precisely the same as ths equations (5), (6), (7), if $\tan. \psi = \frac{2 f r}{F r}$ and $f' \rho \cos. \psi = \frac{1}{2} F r$.

The numerical values which we should obtain for θ' and $\theta_{\prime}$, from these equations, would, in all cases, be exactly the same as those which we have already obtained from the equations (5) and (6): so that the agreement between the observations and the results from this theory would not be greater than in the former case.

In the explanation of the phænomena which take place on

presenting the different ends of a mass of iron to the poles of a magnetic needle, in addition to the hypothesis, that the upper part becomes a north, and the lower a south pole, by position, it is necessary to suppose also, that in every change of position of the iron there is a corresponding and immediate change of its poles; that is, the upper end becoming the lower, it also immediately becomes a south pole. Now it appears to me, that if we attempt to explain, on this hypothesis, the phænomena arising from the rotation of the iron, we shall find that there are circumstances which are wholly incompatible with it. If on turning a mass of iron end for end, the poles are immediately transferred from one end to the other, how can we suppose that the revolution of the iron will cause these poles to move forwards, so that the line joining them shall describe an angle from the line of the dip? or even granting that during the revolution of the iron they may be carried forward, they must, as soon as the iron ceases to revolve, resume their original position in the line of the dip, if they are so immediately transferred from one end of the iron to the other, as it is necessary to suppose in order to account for the phænomena which take place of attraction and repulsion, as they have been called. Immediately, then, that the iron becomes stationary in any position, the deviation of the needle ought, on this hypothesis, to become the same, whether the iron has been brought into that position by revolving in one direction, or in the contrary. It is hardly necessary for me to say that this would not be the case, since I have stated, that, in all the preceding observations, the iron was stationary previous to the observation being made.

Whatever are the effects produced on the iron by its revolution, so far from these effects being of the transient nature which we must suppose them to be on this hypothesis, they appear to have been quite permanent, that is, so long as the iron remained in the same position. The following observation will show the small changes which took place during 12 hours.

In order that the needle might be quite free to move, it was suspended in a balance of torsion by a brass wire, of the same diameter as the finest gold wire used for transits, free from torsion, 21.15 inches long. The plane of the plate was in the plane of the secondary to the equator and meridian, its centre in latitude 0° longitude 180°; and it was fixed to a wooden axis passing through its centre perpendicular to its plane: the ends of this axis, which revolved with the plate, being made of brass, that I might ascertain whether the effect was independent of friction on the plate itself. The plate was made to revolve in contrary directions, as usual, and the direction of the north end of the needle noted, when the point 180° on the plate coincided with the upper part of a plane parallel to the meridian, and passing through the plate's centre. After having made the plate revolve so that its upper edge moved from west to east, and noted the direction of the north end of the needle when 180° coincided with the above plane, it was made to revolve from east to west, and 180° being again brought to coincide with this plane, the direction of the north end of the needle was noted at different times for more than 12 hours, the plate remaining stationary during that time.

Direction of rotation of plate's upper edge.	W to E	E to W	Time of observation.	
	° '	° '	h m	
The several directions of the north end of the needle were observed when the point 180° on the plate coincided, above the centre, with the plane parallel to the meridian.	0 04W	2 50 E	9 35	During this time the plate was kept perfectly stationary, and care was taken that the apparatus should not be in the least disturbed.
		2 50	10 05	
		2 46	11 10	
		2 44	20 35	
		2 42	21 48	
	0 02 E		22 01	After $21^h\ 48^m$ the plate was made to revolve slowly once from W to E.
	0 02 E		22 17	After making the plate revolve several times and more rapidly.
		2 46	22 28	Making the plate revolve several times from E to W.
	0 04W		22 40	Making the plate revolve once so slowly that the time of rotation was $3^m\ 26''$.
	0 06W		24 05	The plate kept perfectly stationary since $22^h\ 40^m$.
	0 08W		25 35	
		1 22		Making the plate revolve through 30° from W to E, and then bringing it back 30° from E to W.
		2 42		Making the plate revolve through 90° from W to E, and then bringing it back 90° from E to W.
		2 42		Making the plate revolve repeatedly and rapidly.

From these investigations it appears, that the effect produced on the iron by its *rotation* is permanent, so long as the plate remains stationary; that it is independent of friction; that it is so far independent of velocity, that the iron can scarcely be moved so slowly that the whole effect shall not be produced; and that the whole effect is produced by making it perform only one fourth of a revolution.

Shortly after I had discovered these pecular effects to be produced by the rotation of iron, I pointed out the general nature of the phænomena and exhibited some of them to Mr. Barlow, and he has since made some experiments on the rotation of spherical shells, in which he has found that

phænomena somewhat analogous take place, but they appear to be dependent on the velocity with which the shell is made to revolve.

On computing the several values of $\frac{F r}{2 f \rho}$ from the equation (8), I found that if the term $2 \cos. \delta \sin. \theta' \sin. \theta_{,}$ were neglected; that is, if $\frac{F r}{2 f \rho}$ were equal to $\frac{3 \cos. 2 \lambda + 1}{3 \sin. 2 \lambda} \cdot \frac{\sin. (\theta' + \theta_{,})}{\sin. (\theta' - \theta_{,})}$ its numerical values, so determined, would agree very nearly with each other. I was in consequence led to expect that equations from which this value of $\frac{F r}{2 f \rho}$ might arise, would give values of $\theta' - \theta_{,}$ agreeing more nearly with the observations; and the result fully answered my expections.

If $$\text{Tan. } \theta' = \frac{3 \sin. 2 \lambda + \frac{2 f \rho}{F r} \cdot (3 \cos. 2 \lambda + 1)}{\frac{2 m R^3}{F r} \cdot \cos. \delta} \quad (9),$$

and $$\text{Tan. } \theta_{,} = \frac{3 \sin. 2 \lambda - \frac{2 f \rho}{F r} \cdot (3 \cos. 2 \lambda + 1)}{\frac{2 m R^3}{F r} \cdot \cos. \delta} \quad (10),$$

then we shall have

$$\frac{F r}{2 f \rho} = \frac{3 \cos. 2 \lambda + 1}{3 \sin. 2 \lambda} \cdot \frac{\sin. (\theta' + \theta_{,})}{\sin. (\theta' - \theta_{,})} \quad (11).$$

When the *deviation due to rotation* vanishes or $\theta' = \theta_{,}$ the equations (9) and (10) give $3 \cos. 2 \lambda + 1 = 0$ and $\lambda = 54° 44'$, which agrees perfectly with the observations.

From the observations in Table A, we have in this case,

$$\text{Tan. } 9° 09' 20'' = \frac{3 \sin. 109° 28' 16''}{\frac{2 m R^3}{F r} \cdot \cos. 70° 15'}$$

whence $\frac{2 m R^3}{F r} = 51.9504 = 52$ very nearly.

When $\lambda = 0$, $\frac{2 f \rho}{F r} = \frac{1}{4} \cdot \frac{2 m R^3}{F r} \cdot \cos. \delta \tan. \theta' = .061234$;

and when $\lambda = 90$, $\frac{2 f \rho}{F r} = -\frac{1}{2} \cdot \frac{2 m R^3}{F r} \cdot \cos. \delta \tan. \theta' = .057291$:

so that the mean value of $\frac{2 f \rho}{F r}$ is .059262, or nearly .059.

The equations (9) and (10 therefore become

$$\text{Tan. } \theta' = \frac{3 \sin. 2\lambda + .059 \times (3 \cos. 2\lambda + 1)}{52 \cos. 70^\circ 15'} \qquad (9_a);$$

$$\text{Tan. } \theta_{,} = \frac{3 \sin. 2\lambda - .059 \times (3 \cos. 2\lambda + 1)}{52 \cos. 70^\circ 15'} \qquad (10_a).$$

In the same manner the observations in Table B give, $\frac{2\,m\,R^3}{F\,r} = 48.0278 = 48$ nearly, and $\frac{2\,f\,\rho}{F\,r} = .059039 = .059$ nearly.

The equations (9) and (10), in this case, become,

$$\text{Tan. } \theta' = \frac{3 \sin. 2\lambda + .059 \times (3 \cos. 2\lambda + 1)}{48} \qquad (9_b);$$

$$\text{Tan. } \theta_{,} = \frac{3 \sin. 2\lambda - .059 \times (3 \cos. 2\lambda + 1)}{48} \qquad (10_b).$$

Table of the values of θ', $\theta_{,}$, and $\theta' - \theta_{,}$ computed from the equations (9_a), (10_a) compared with their observed values in Table A.

λ	θ'			$\theta_{,}$			$\theta' - \theta_{,}$		
	Observed.	Computed.	Difference.	Observed.	Computed.	Difference.	Observed.	Computed.	Difference.
°	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″	° ′ ″
0	+ 47 55	+ 46 10	— 1 45	— 47 55	— 46 10	+ 1 45	1 35 50	1 32 20	— 3 30
10	3 57 20	4 04 26	+ 7 06	2 26 40	2 36 30	+ 9 50	1 30 40	1 27 56	— 2 44
20	6 57 00	6 53 20	— 3 40	5 40 00	5 38 06	— 1 54	1 17 00	1 15 14	— 1 46
30	9 03 25	8 52 50	— 10 35	8 03 45	7 56 22	— 7 23	59 40	56 28	— 3 12
40	9 56 20	9 49 43	— 6 37	9 19 30	9 15 34	— 3 56	36 50	34 09	— 2 41
50	9 31 20	9 38 02	+ 6 42	9 21 25	9 27 16	+ 5 51	09 55	10 46	+ 0 51
60	8 05 15	8 18 59	+ 13 44	8 16 15	8 30 17	+ 14 02	— 11 00	— 11 18	— 0 18
70	5 51 35	6 00 57	+ 9 22	6 19 40	6 30 34	+ 10 54	— 28 05	— 29 37	— 1 32
80	2 51 25	2 59 35	+ 8 10	3 31 0	3 41 26	+ 9 56	— 40 05	— 41 51	— 1 46
90	— 22 25	— 23 05	— 0 40	+ 22 25	+ 23 05	+ 0 40	— 44 50	— 46 10	— 1 20

Table of the values of θ', $\theta_{,}$ and $\theta' - \theta_{,}$ computed from the equations (9_b) (10_b) *compared with their observed values in Table* B.

λ	θ'			$\theta_{,}$			$\theta' - \theta_{,}$		
	Observed.	Computed.	Difference.	Observed.	Computed.	Difference.	Observed.	Computed.	Difference.
°	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
0	+ 50 28	+ 50 01	— 0 27	— 50 28	— 50 01	+ 0 27	1 40 56	1 40 02	— 0 54
10	4 30 50	4 24 43	— 6 07	2 54 52	2 49 34	— 5 18	1 35 58	1 35 09	— 0 49
20	7 38 05	7 27 24	— 10 41	6 15 27	6 06 04	— 9 23	1 22 38	1 21 20	— 1 18
30	10 07 50	9 36 27	— 31 23	9 03 12	8 35 29	— 27 43	1 04 38	1 00 58	— 3 40
40	11 02 12	10 37 47	— 24 25	10 23 37	10 00 58	— 22 39	38 35	36 49	— 1 46
50	10 44 55	10 25 11	— 19 44	10 32 35	10 13 35	— 19 00	12 20	11 36	— 0 44
60	9 12 10	8 59 55	— 12 15	9 24 40	9 12 06	— 12 34	— 12 30	— 12 11	+ 0 19
70	6 33 52	6 30 47	— 3 05	7 06 20	7 02 48	— 3 32	— 32 28	— 32 01	+ 0 27
80	3 10 50	3 14 31	+ 3 41	3 56 00	3 59 50	+ 3 50	— 45 10	— 45 19	— 0 09
90	— 24 49	— 25 01	— 0 12	+ 24 49	+ 25 01	+ 0 12	— 49 38	— 50 02	— 0 24

The agreement between the computed and observed values of $\theta' - \theta_{,}$ also of θ' and $\theta_{,}$ in the first table is such, that had I been assured of the correctness of the formulæ, I should certainly not have expected it to be more perfect. In the second table, the agreement between the computed and observed values of $\theta' - \theta_{,}$ is equally close, but there is a greater difference between those values of the angles θ', $\theta_{,}$ themselves. In determining the value of $\frac{2mR^3}{Fr}$ from the observation when $\lambda = 54° 45'$, and $\theta' = \theta_{,}$, I had in the first instance in consequence of an error in computation, found it 47 instead of 48, and having computed the several values of θ' and $\theta_{,}$ from this value of $\frac{2mR^3}{Fr}$, I found that the difference between these and the observed values was less than 8′, except in two instances, in one of which it amounted to 11′ and in the other to 19′. Now the observations when $\lambda = 50°$ and $\lambda = 60°$ would give the value of $\frac{2mR^3}{Fr}$ even less than 47°, as will be

seen when we compute it for these values of λ, which would still further diminish these differences. I have therefore no doubt that the differences between the computed and observed values of θ' and $\theta_,$ in this table are to be attributed to an error of about $15'$ in the observed value of θ' when $\lambda = 54^\circ\ 45'$. The best criterion, however, of the correctness of the formulæ is in the agreement of the values of the constants derived from them by means of the observations.

If we eliminate $\frac{2 f \rho}{F r}$ from the equations (9) and (10), we shall obtain

$$\frac{2 m R^3}{F r} = \frac{6 \sin. 2 \lambda}{\cos. \delta} \cdot \frac{\cos. \theta' . \cos. \theta_,}{\sin. (\theta' + \theta_,)} \qquad (12).$$

Substituting in the equations (11) and (12) the several observed values of θ' and $\theta_,$ in tables A and B, we obtain the values of $\frac{2 F r}{f \rho}$ and $\frac{2 m R^3}{F r}$ contained in the following table.

Table of the values of the constants $\frac{2 m R^3}{F r}$ *and* $\frac{2 F r}{f \rho}$ *computed from the several observed values of* θ' *and* $\theta_,$ *in tables* A *and* B, *by means of the equations* (12) *and* (11).

λ	Observed values in Table A.		Computed values.		Observed values in Table B.		Computed values.	
	θ'	$\theta_,$	$\frac{2 m R^3}{F r}$	$\frac{2 F r}{f \rho}$	θ'	θ'	$\frac{2 m R^3}{F r}$	$\frac{2 F r}{f \rho}$
°	° ′ ″	° ′ ″			° ′ ″	° ′ ″		
0	+ 47 55	— 47 55	50.103	16.331	+ 50 28	— 50 28	47.571	16.798
10	3 57 20	2 26 40	54.301	15.733	4 30 50	2 54 52	46.767	17.240
20	6 57 00	5 40 00	51.615	16.681	7 38 05	6 15 27	46.834	17.085
30	9 03 25	8 03 45	51.077	16.321	10 07 50	9 03 12	45.492	16.819
40	9 56 20	9 19 30	51.516	15.852	11 02 12	10 23 37	46.203	16.759
50	9 31 20	9 21 25	52.587	18.188	10 44 55	10 32 35	46.512	16.412
60	8 05 15	8 16 15	53.492	16.939	9 12 10	9 24 40	46.915	16.894
70	5 51 35	6 19 40	53.452	17.397	6 33 52	7 06 20	47.609	16.849
80	2 51 25	3 31 30	54.463	16.902	3 10 50	3 56 00	48.846	16.712
90	— 22 25	+ 22 25	53.551	17.455	— 24 49	+ 24 49	48.372	17.081
Mean values			52.616	16.780		Mean values	47.112	16.865

On comparing together the several values of $\frac{2mR^3}{Fr}$ and also of $\frac{2Fr}{f\rho}$, contained in this table, there can, I think, be no doubt that if the formulæ (9) and (10), on which these values depend, be not absolutely correct, they will, at least, give in all cases, as close approximations to the values of θ' and θ, that would be obtained by actual observation, as the nature of the case appears to admit of. It is very possible that some modification in the theory which I have examined might lead to the omission of the terms $2 \cos. \lambda - 1 \pm \frac{2f\rho}{Fr} . 3 \sin. 2\lambda$ in the formulæ (5) and (6); and should this be the case, that the formulæ (9) and (10) were to be derived from the theory so modified, it would, I think, be a very strong presumption in favour of the truth of such a theory.

Since it appears from all the observations which I have detailed, that the direction of the magnetic polarity, which iron acquires by *rotation about an axis*, whether it be at right angles to the line of the dip, as would follow from the theory which I have investigated, or not, has always reference to the direction of the terrestrial magnetic forces, we must infer that this magnetism is communicated to it from the earth. It does not therefore appear from this, that a body can become polarised by rotation alone, independently of the action of another body: so that if from these experiments we might be led to attribute the magnetic polarity of the earth to its rotation, we must at the same time suppose a source from which magnetic influence is derived. Is it not then possible that the sun may be the centre of such influence, as well as the source of light and heat, and that by their rotation, the

earth and other planets may receive polarity from it? If so, further experiments and observations on the magnetic effects produced by the rotation of bodies may indicate the cause of the situations of the earth's magnetic poles, and of their progressive movements or oscillations.

Comparison of the magnetical effects produced by slow and by rapid rotation.

With the view of ascertaining how far the effects produced on a magnetic needle by a plate of iron during its rapid rotation, corresponded with those that I have described as nearly independent of the velocity of rotation, and as continuing after the rotation had ceased, I placed the same plate of iron, which I had used in my former experiments, in the plane of the magnetic meridian, on an axis perpendicular to its plane, and about which it could be made to revolve with any velocity, not exceeding 10 revolutions in a second. I then placed a small compass, with a light needle delicately suspended, on a platform wholly detached from the iron plate, in certain positions opposite to the edge of the plate, both to the east and to the west of it, as near to the surface as the compass box would admit. The compass being adjusted, the plate was made to revolve once, slowly, so that its upper edge moved from north to south, and the point o coinciding with the plane perpendicular to the plane of the plate, and passing through its centre and that of the needle, the direction of the north end of the needle was observed; and also when 180 coincided with the plane, the same observation was made. The plate was now made to revolve rapidly in the same direction, about 8 times in a second; and when the

needle became stationary during the rotation, the direction of its north end was observed. The point o on the plate was again made to coincide as quickly after the rapid rotation as possible, and the direction of the needle observed, in order to see if that rotation had produced any permanent change in the iron; the same was done when the point 180 again coincided. Observations precisely similar to these were made when the upper edge of the plate revolved from south to north.

Although the centre of the plate was stationary, and the needle was placed in certain positions with respect to it, I consider, as before, the situation of the centre of the plate with reference to the plane passing through the centre of the needle perpendicular to the dip; and its angular distance from this plane, the equator, was measured on a circle of 9 inches radius parallel to the meridian, passing through the centre of the needle, and at the distance 1.45 inches from it, so that the centre of the needle was always at this distance from the edge of the plate, east or west. As the needle was only two inches in length, and the rim of the compass divided into degrees, the direction of the needle could not be observed nearer than to 5′, and indeed scarcely to that degree of accuracy. The mode which I was under the necessity of adopting in adjusting the compass to the several positions did not admit of extreme accuracy, so that these positions may be considered as liable to errors amounting to 1°, or perhaps rather more, in angular distance from the equator; but as my principal object was the comparison of the deviation due to the slow and rapid rotation of the plate, when its centre was in precisely the same position with respect to that of the needle, this was not very material: it

will however account for any disagreements that may be noticed in the absolute deviations in corresponding positions, as the greatest accuracy of adjustment would be requisite for their perfect agreement, when the plate is so near to the poles of the needle.

Having ascertained, by the observations when the plate was to the west of the needle, that the rapid rotation produced no permanent change in the iron beyond that arising from the slow rotation, the deviations when any particular points of the plate were opposite to the needle being, as near as could be expected, the same after the rapid rotation as they were after the slow rotation in the first instance, the errors being sometimes in excess, sometimes in defect, as will appear by inspection of the first table, I did not repeat the observations on the effects of the slow rotation after the rapid, when the plate was to the east of the needle.

The following tables contain the observations. The first four columns of deviations, are those which were observed when the plate was stationary, after having very slowly revolved, and also those when the needle pointed steadily during the rapid rotations. The deviations in the 5th and 6th columns are obtained by taking half the difference between those in the 1st and 2nd, and between those in the 3rd and 4th columns, as the deviation due to the rotation when the plate's upper edge revolved from north to south.

Tables of the deviations of a magnetic needle, caused by the rapid rotation of a plate of iron, observed during rotation, compared with the deviations due to the slow rotation of the plate, and permanent after the rotation had ceased.

1*st.* ***The iron plate,*** 18 ***inches in diameter, to the west of the compass.***

Angular distance of the plate's centre from the equator. (Measured from the north when plate's centre in north lat. and from south when in south lat.)	Angular velocity of the plate, and points opposite to the needle when plate stationary.	Direction of the N. end of the needle when plate stationary after slow rotation, and also while the plate was rapidly revolving.				Deviation due to the rotation of the plate's upper edge from N to S, when plate's centre was	
		Plate's centre in N. lat.		Plate's centre in S. lat.			
		Upper edge of the plate revolving.					
		N to S	S to N	N to S	S to N	in N. lat.	in S. lat.
		° ′	° ′	° ′	° ′	° ′	° ′
0	Slow 0	2 45W	14 25 E	6 40 E	8 10W	8 59W	6 54 E
	Slow 180	4 55 E	23 40 E	14 15	2 05 E		
	8 rev. per sec.	3 55W	19 25 E	14 30	4 40W	11 40W	9 35 E
	Slow 0	2 55W	13 55 E	6 40	7 50W	8 45W	6 49 E
	Slow 180	6 30 E	24 40 E	14 50	2 05 E		
20	Slow 0	32 50W	27 30W	40 55	33 20	2 25	3 15
	Slow 180	27 40	23 00	42 25	37 00		
	8 rev. per sec.	30 55	24 35	41 25	32 40	3 10	4 22
	Slow 0	32 50	27 35	40 40	33 35	2 34	3 22
	Slow 180	28 00	22 40	42 25	36 45		
50	Slow 0	43 50	41 25	54 00	51 00	1 14	1 21
	Slow 180	40 30	38 00	53 05	50 40		
	8 rev. per sec.	42 25	39 15	53 30	49 00	1 35	2 15
	Slow 0	43 45	41 25	54 10	51 20	1 09	1 15
	Slow 180	40 15	38 00	53 10	51 00		
70	Slow 0	52 30	51 00	59 25	57 20	0 55	1 06
	Slow 180	49 10	47 00	59 00	56 40		
	8 rev. per sec.	51 05	48 20	57 40	54 40	1 23	1 30
	Slow 0	52 30	50 50	59 40	57 25	0 55	1 04
	Slow 180	48 55	46 55	58 40	56 40		
90	Slow 0	62 00	60 20	68 30	66 40	0 50	0 51
	Slow 180	57 40	56 00	67 45	66 10		
	8 rev. per sec.	60 35	57 25	66 55	64 20	1 35	1 17
	Slow 0	62 25	60 25	68 50	66 25	0 55	1 01
	Slow 180	58 00	56 20	67 55	66 15		
110	Slow 0	72 25	71 25	75 25	74 00	0 34	0 35
	Slow 180	68 10	66 55	76 00	75 05		
	8 rev. per sec.	70 20	68 15	74 40	72 40	1 02	1 00
	Slow 0	72 35	71 25	75 35	74 25	0 38	0 32
	Slow 180	68 00	66 40	76 00	75 00		
130	Slow 0	84 05	84 10	83 55	83 30	0 06	0 11
	Slow 180	80 50	80 20	85 00	84 40		
	8 rev. per sec.	82 20	81 00	83 00	81 45	0 40	0 37
	Slow 0	84 20	84 10	84 00	83 10	0 09	0 14
	Slow 180	80 40	80 15	85 00	84 55		
140	Slow 0	86 20	87 00	86 00	86 15	0 33 E	0 10W
	Slow 180	82 15	83 45	87 40	88 05		
	8 rev. per sec.	83 40	85 00	84 40	85 15	0 40 E	0 18W
	Slow 0	86 00	87 20	85 40	86 20	0 39 E	0 16W
	Slow 180	82 40	83 55	87 35	88 00		
160	Slow 0	75 20	83 55	76 00	83 00	7 05 E	3 18
	Slow 180	54 00	73 45	78 15	84 25		
	8 rev. per sec.	57 55	78 20	69 40	80 30	10 12 E	5 25
	Slow 0	76 10	83 40	76 05	82 00	6 45	3 01
	Slow 180	53 20	72 50	78 10	84 20		

2nd. The same iron plate to the east of the compass.

Angular distance of the plate's centre from the equator.	Angular velocity of the plate, and points opposite to the needle when plate stationary.	Direction of the N. end of the needle when plate stationary after slow rotation, and also while the plate was rapidly revolving. Plate's centre in N. lat. Upper edge of the plate revolving. N to S	Plate's centre in N. lat. S to N	Plate's centre in S. lat. N to S	Plate's centre in S. lat. S to N	Deviation due to the rotation of the plate's upper edge from N to S, when plate's centre was in N. lat.	in S. lat.
Measured from north when the plate's centre in north lat. and from south when in south lat.		° ′	° ′	° ′	° ′	° ′	° ′
0	Slow 0	9 55 E	1 25W	8 50W	5 20 E	5 56 E	6 29W
	Slow 180	5 00 E	7 25W	16 30	4 45W		
	8 rev. per sec.	11 05	4 25W	16 35	1 35 E	7 45 E	0 05W
20	Slow 0	35 15	29 25 E	39 40	33 30W	2 50	2 45
	Slow 180	31 55	26 25	42 20	37 30		
	8 rev. per sec.	34 50	27 25	42 25	34 55	3 43	3 45
50	Slow 0	42 30	40 05	53 45	50 50	1 09	1 16
	Slow 180	39 50	37 30	54 25	52 15		
	8 rev. per sec.	42 10	38 50	53 20	49 10	1 40	2 05
70	Slow 0	48 55	47 50	59 15	56 20	0 40	1 18
	Slow 180	45 30	43 55	59 30	57 15		
	8 rev. per sec.	48 35	45 45	58 50	55 40	1 25	1 35
90	Slow 0	59 45	57 55	66 35	64 40	0 50	0 57
	Slow 180	56 00	54 30	66 50	65 00		
	8 rev. per sec.	58 25	55 40	66 25	63 35	1 23	1 25
110	Slow 0	71 00	69 30	75 10	73 40	0 43	0 41
	Slow 180	67 10	65 50	75 10	73 55		
	8 rev. per sec.	70 40	68 30	74 20	72 10	1 05	1 05
130	Slow 0	83 25	83 30	81 10	80 45	0 00	0 06
	Slow 180	80 20	80 15	83 00	83 00		
	8 rev. per sec.	82 20	81 45	81 15	80 15	0 18	0 30
140	Slow 0	86 35	87 30	81 30	82 20	0 35W	0 20 E
	Slow 180	83 00	84 25	84 50	85 20		
	8 rev. per sec.	85 50	86 40	82 20	82 40	0 25W	0 10 E
160	Slow 0	77 00	85 20	68 00	77 20	8 31W	4 30
	Slow 180	47 30	73 15	72 35	81 15		
	8 rev. per sec.	51 40	81 40	67 10	79 40	15 00	6 15

From the inspection of these tables, it appears that the forces which are exerted on the needle during the rapid rotation of the plate, are always in the same direction as the forces which are derived from the slowest rotation, and which continue to act after the rotation has ceased; but that the

former forces are greater than the latter, there being only one instance of the contrary, and that in a position where the effects are so small, that a trifling error of observation would account for the difference. Taking a mean of all the observations, these forces appear to be in the ratio of 19 to 13, or very nearly 3 to 2. It is evident then that the polarising of the iron in the same direction will account for the phænomena in both cases, but that the intensity of the polarity during the rapid rotation is greater than of that which appears to be permanent after the rotation, whether slow or rapid, has ceased; and that the phænomena observed during rapid rotation are such as we should expect from those which I have so fully described as arising from rotation, without regard to its velocity.

XVII. *Some account of the transit instrument made by Mr.* Dollond, *and lately put up at the Cambridge Observatory. Communicated April* 13, 1825. *By* Robert Woodhouse, *A. M. F. R. S.*

Read May 19, 1825.

As I am inclined to hope that the observations to be made at the Observatory lately established at Cambridge may, at some future period, be useful to astronomical science, I beg leave to send a brief description of our transit telescope, the only large instrument which we are at present possessed of.

The annexed drawings, which I have caused to be made of the instrument, will explain its construction.

Its dimensions are nearly the same as those of the Greenwich transit made by Mr. Troughton.

	Ft.	In.
Its focal length is - - -	9	10
Its aperture - - -	0	5
The length of the axis between the piers	3	6

The weight of the instrument is 200lbs.

The instrument is counterpoised; and the whole lengths (2 inches) of the pivots rest on the Y *s*.

Seven fixed wires are placed in the focus of the object glass, and two other wires moveable by a micrometer screw; the interval of which wires is equal to the interval between any two of the fixed wires, and, *equatorially*, is $17^s.88$.

The two small graduated circles (see the figure) with their spirit levels, fixed near to the eye-piece, are for the

purpose of finding a star's place in the meridian. Each circle is furnished with two verniers; one for polar, the other for zenith distances.

I wish to add a few words respecting the determining the place of the transit room, and the adjusting the instrument to the plane of the meridian; which, as we had in the beginning no astronomical point to stand on, was a matter of some trouble.

Our first object was, if possible, so to fix the site of the transit room, that its meridian mark might be placed on the steeple of Granchester church, distant to the south about $2\frac{1}{2}$ miles from the field on which the observatory was to be built.

The first approximations to such site were made by adjusting the middle wire of a small transit telescope (18 inches long) to the spire, or iron rod of the steeple, and by comparing the sun's transit with the time brought up by chronometers from Mr. CATTON'S observatory at St. John's College. Our second approximations were made by observations of *high and low* stars with the small transit instrument above mentioned.

According to the results thus obtained the piers of the transit were placed; and when, in June 1824, the instrument was put upon them, were found to be placed with considerable exactness. From the above time observations have been constantly made with the instruments described in this paper, and with a clock made by MOLYNEUX and COPE.

The first operation was to determine the clock's *rate*, which was done by observations of the same stars on successive days: the next, to determine the clock's *error*, which was found in the usual way, by deducting the observed

passages of stars from their tabulated, or computed right ascensions.

The clock's error, as it was to be expected, was found, after allowing for its rate, different with different stars; which is a sign of the instrument being out of adjustment in some of its parts. The error might be in the line of collimation; in the axis not being horizontal; or, which was probably the chief cause of error, in the transit deviating from the plane of the meridian. Any one, or two, or all of these circumstances might occasion the noted difference in the clock's errors.

For instance, the clock being before sidereal time, its error from α Cygni was found to be less than from α Aquilæ. This might arise from the western end of the axis being too high, or from the line of collimation deviating to the east, or from the transit deviating to the west. A single observation such as this, or any number of the same stars, would leave us in doubt respecting the causes of the want of adjustment; but a third star would lessen this doubt. Thus, if, the clock's error after allowing for its rate being from α Cygni $4^s.721$, and from α Aquilæ $4^s.854$, we attributed the difference of errors to a defect of horizontality in the axis, the quantity of such defect would become known. Let it be expressed by H, the clock's error by ε; then for the latitude of Cambridge we should have two simple equations between H and ε, from which both may be found

$$-4.721+\varepsilon=1.39\,\text{H},\ \alpha\ \text{Cygni}$$
$$-4.854+\varepsilon=.75\,\text{H},\ \alpha\ \text{Aquilæ};$$

and, accordingly, $\text{H}=0^s.2$, nearly.

With this value of H, the error of time for α Urs. maj.

would be $0^s.4288$ ($= 0^s.2 \times 2144$); which not being found to agree with the observed error (or rather the difference of its observed passage and its computed right ascension), showed that the difference of the errors of the clock had been wrongly, or partially, assigned.

If we suppose the difference of the errors of the clock to arise from two causes—the want of horizontality and the deviation of the transit from the plane of the meridian, then, calling the latter deviation H, we have, instead of the former, these equations:

$$-4.721 + \epsilon = 1.39\,H + .185\,Z$$
$$-4.854 + \epsilon = .73\,H + .7\,Z$$

to which a third similar equation must be added for α Urs. maj.

If from such three equations we determined H and Z, we might proceed as before, and examine, by means of a fourth star, whether it were necessary to suppose the existence of a third cause (an error in the line of collimation for instance) to account for the differences in the clock's error.

If C should denote the error of collimation, dt the error of time, c the colatitude of the place of observation, δ the star's north polar distance, the general form of the equations for determining H, Z, &c. is

$$-dt + \epsilon = H\,\frac{\cos.(c-\delta)}{\sin.\delta} + Z\,.\,\frac{\sin.(c-\delta)}{\sin.\delta} + \frac{C}{\sin.\delta}$$

In this way we might consider the subject in *all its generality* (as foreign writers express themselves); and from observations alone, arrive at a knowledge of the defects of the instrument. And this mode of considering the subject is not without its use, since it may be applied to recorded and ancient observations; as Bessel has done in the case of

BRADLEY'S Observations. But no practical astronomer, I apprehend, can be so fond of encountering difficulties,* as to adopt this mode of adjusting his instrument; for, if from one set of equations he deduced the values of H, Z and C, he could not, by reason of his imperfect knowledge and management of the screws of his instrument, at once adjust it; but would be again and again obliged to repeat his observations, and the solutions of the resulting equations.

But this is not all. The differences of the clock's errors are the differences of the differences of the observed culminations of stars, and their tabulated or computed right ascensions, and therefore must partake of the uncertainties to which the latter quantities are subject. The point to be aimed at in adjusting an instrument is, to adjust it by means that do not rest on the results of astronomical science.

* As a kind of proof of the great *uncertainty* of determining the deviations of the instrument by the method of equations, I subjoin the following instance:

October 18, 1824.	Time by Clock.	Æ	Errors.
	h. m. s.	s.	s.
α Aquarii -	21 56 52.86	48.72	4.14
α Pegasi - -	22 56 8.74	4 .4	4.34
α Andromedæ -	23 59 27 .7	23.25	4.45

whence, the axis being horizontal, and the clock going sidereal time, we have these equations:

$$-4^s.14 + \iota = .8028\,Z + C$$
$$-4.34 + \iota = .6347\,Z + 1.032\,C$$
$$-4.45 + \iota = .4627\,Z + 1.114\,C$$

from which, $Z = 1''.48$, $C = 1''.52$; but if an error of $0^s.1$ had occurred in the observations, or if we suppose the tables to be erroneous to that degree, and the second equation had been

$$-4^s.44 + \iota = .6347\,Z + 1.032\,C,$$

then, instead of the preceding values of Z and C, we should have had these:

$$Z = 2''.935$$
$$C = 6.033$$

The old methods of adjusting a transit instrument do not rest on such results; and the old method of proceeding seems to me the most sensible one, that of separately and successively correcting each cause of defective adjustment.

The axis can be made horizontal, or its defect of horizontality known, by the level, the plumb line, or by reflection.

The line of collimation can be adjusted by means of a small object in, or near to, the horizon. In this operation a small defect in the horizontality of the axis will have scarcely any effect on the accuracy of the operation. If the mark should, for instance, be 2° above the horizon, and one end of the axis $5''$ higher than the other, the error in collimating from that cause would, in the latitude of Cambridge, be only $0''.1075$. The error in the same operation with the pole star, supposing it be fixed, would be $1'\ 11''.5$.

The third adjustment, which is the most troublesome, is to place the transit instrument in the plane of the meridian; and there are two methods of effecting this: one, by *high and low* stars; the other, by circumpolar stars, or, as it almost always happens in practice, by the pole star.

The essential difference in these two methods is, that the former rests on the results of astronomical science, whilst the latter does not so rest; and this circumstance gives the latter a decided advantage over the former, when it is necessary to make a nice adjustment. Yet there is not wanting considerable astronomical authority for placing the two methods on a level, the one with the other. Baron de ZACH, for instance, views each as an equally good method; and in his Tabulæ speciales Aberr^s. et Nut^s. gives instances of the adjustment of a transit instrument (a 5-feet one by DOLLOND); the first, by the comparison of the passages over the meridian of

Capella and Rigel; the second, by the passages of Capella above and below the pole: and the result, equal to a deviation of 12″.685 to the east, is in each case the same: a coincidence of marvellous accuracy; and which, if the observations were exactly noted, we must suppose to have arisen from a fortuitous balancing of the errors of the observations with those of the tables.

In the method of *high and low* stars I suppose, which is almost always the case, that the clock's error is found by subtracting from the observed passage of the star its computed right ascension. The error may indeed be found by *equal altitudes*, should the observer possess an altitude and azimuth instrument of sufficient accuracy for that purpose. But should he not, the adjustment of the transit instrument by high and low stars must partake of that uncertainty to which we are subject in computing the true apparent right ascensions of stars.

We have only to look at the catalogues of stars by different astronomers to be convinced of the existence of such uncertainty. If, indeed, the tabulated right ascensions differed only by a constant quantity, the *difference* of the errors of the clock, on which the method of high and low stars depends, would be the same, whether we employed BESSEL's or the Greenwich catalogue. But it is otherwise: to instance this, on the 8th October, 1824, the clock going very nearly sidereal time, the passages of Arcturus and β Urs. min. were as follow:

	Time by Clock.	Æ by N.A.	Error.	Æ by SCHUMACHER.	Error.
	h. m. s.	s.		s.	
Arcturus -	14 7 44.56	40.18	4.38	39.98	4.58
β Urs. min. -	14 51 20.43	14.67	5.76	14.95	5.48

Hence for determining the deviations of the transit instrument we have, respectively, the following equations:

By Nautical Almanack.

$$\left.\begin{array}{r} -4.38 + \epsilon = .5657\,Z \\ 5.77 - \epsilon = 1.48\,Z \end{array}\right\} ,\ Z = 1^{s}.52$$

By Schumacher's Tables.

$$\left.\begin{array}{r} -4.58 + \epsilon = .5657\,Z \\ 5.48 - \epsilon = 1.48\,Z \end{array}\right\} ,\ Z = 0^{s}.98.$$

The value of the deviation (Z) is uncertain then to the amount, and more, of half a second of time. From such kind of uncertainty, the method of circumpolar stars is entirely free; its characteristic excellence, as it has been already said, consists in its being independent of the results of astronomical science.

In what I have said, I must be supposed to speak of the exact adjustment of large instruments. The method of high and low stars is very convenient, and easily practised: it informs us, in the space of a few hours, of the nature and degree of the deviation of the instrument; and in some cases, when the transit instrument is prevented by its situation from being directed to stars beneath the pole, it is almost an indispensable method.

I wish to add a few words respecting the adjustment of the line of collimation by means of the reversion of the transit instrument during the passage of the pole star. This method has indeed the air of being philosophical; but, according to my opinion, is neither so easily practised, nor so certain as the old method. It is liable to the uncertainty of the times of the pole star's passages over the wires; and always requires, before and after the observation, the examination of

the horizontality of the axis. Without attention to this latter circumstance the method is worth nothing; for, if H should be the error in horizontality, the corresponding error in time would, in the latitude of Cambridge, be equal to about 28.6 H. When we adjust according to the old plan, the collimation by means of an object near the horizon, the operation of levelling is not required; which in large instruments is rather a troublesome one; and certainly is not, what M. Delambre states it to be, " the affair of an instant."*

The level indicating the degree of the defect of horizontality, enables us to correct the time;† and this correction is made on the supposition that the instrument is in the same state when the star is observed, as it was during its examination by the level. It is therefore, other things being equal, expedient to examine by the level, as nearly as it is possible, at the time of observation. But this I am unable to do; as I will show, by stating a circumstance rather deserving of attention. The tube of the telescope is braced to the axis (see the figure) by four tubes. The stations of the two

* Il ne faut pas commencer d'observations sans avoir rectifié l'horizontalité de l'axe, ce qui est l'affaire d'un instant. Astron. tom. i, p. 431.

† Mr. Dollond considers the value of 1 division of the scale of the level to be equal to 1″. I have determined its value astronomically. Previously to a star's culmination, I lowered the eastern end of the axis 10 or 12 divisions, and observed the star's passages across the four first wires. I then caused the western end to be lowered, and observed the star's passages across the three remaining wires, and then examined the level. The following are the results:

δ Cephei	-	-	1″.014	
α Cygni	-	-	0.9	
δ Draconis	-	-	1.005	
α Cephei	-	-	.855	
Polaris	-	-	0.9516	Mean 0″.9451.

persons who level are opposite, and contiguous to the south-west and north-east braces. Being in the constant habit of examining the meridian mark, in order to know what degree of stability the instrument possesses, I found, after levelling, that the south meridian mark was to the east of the middle wire. In about 10 minutes the middle wire returned to the meridian mark, and bisected it. I noted this circumstance a second, third, and fourth time, and then began to inquire whether I had conjectured rightly in attributing it to the expansion of the tubes or braces. For this end, I placed a heated blanket across the south-west and north-east braces, and found the meridian mark deviating to the east of the middle wire: a contrary effect was produced by placing the blanket across the south-east and north-west braces. In these trials the object glass was towards the south: contrary effects took place when it was turned to the north.*

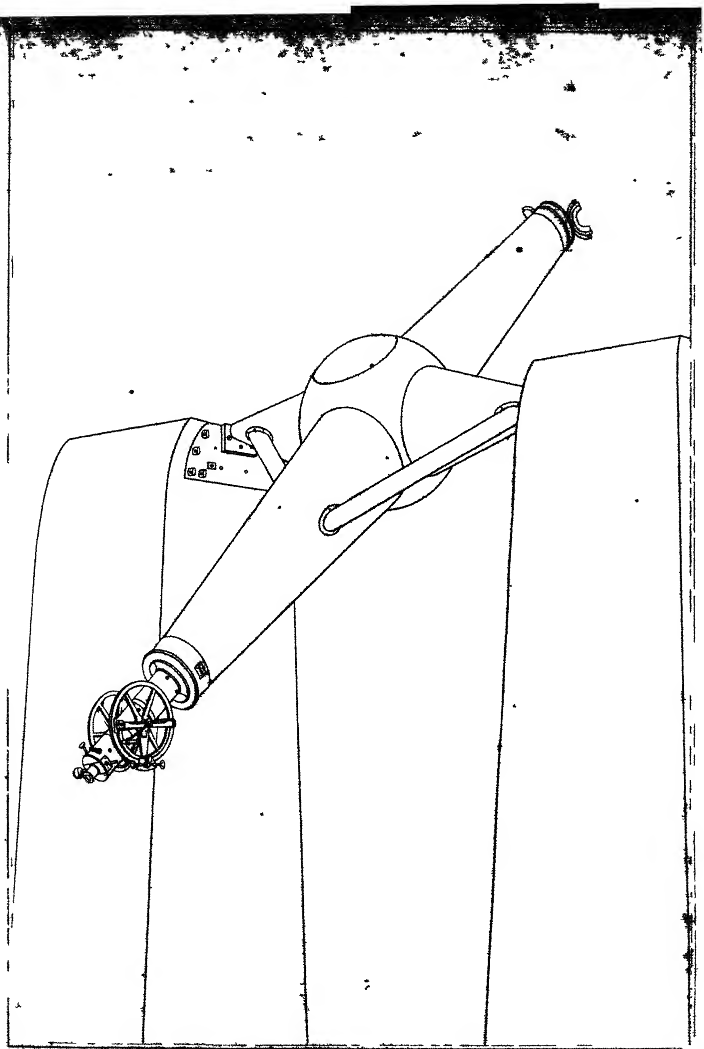
As yet I am unable to say whether or not the sun's rays falling on the braces, during an observation of his transit, affect the accuracy of the observation. I am enquiring into that point, and have ordered a screen to be made to protect the braces from the rays of the sun.

After repeated trials, I have been obliged to abandon the counterpoises;† instead of relieving the instrument, they render it unsteady. It has happened with them (as it has happened in cases of a different nature), they have overpowered what they were meant only to assist.

* The effect I have noted is somewhat of the same kind as that which was complained of in HALLEY's transit. See BRADLEY's Observations, vol. i. p. 2.

† They are now with Mr. DOLLOND, who is endeavouring to remedy their defects.

My chief study, since the fixing up of the instrument, has been to obtain a thorough knowledge of it: to find out its defects, should it have any, their nature and degree. The observations of stars have been chiefly made for, and have served that end; but they are not, I think, otherwise useful, nor worth registering.



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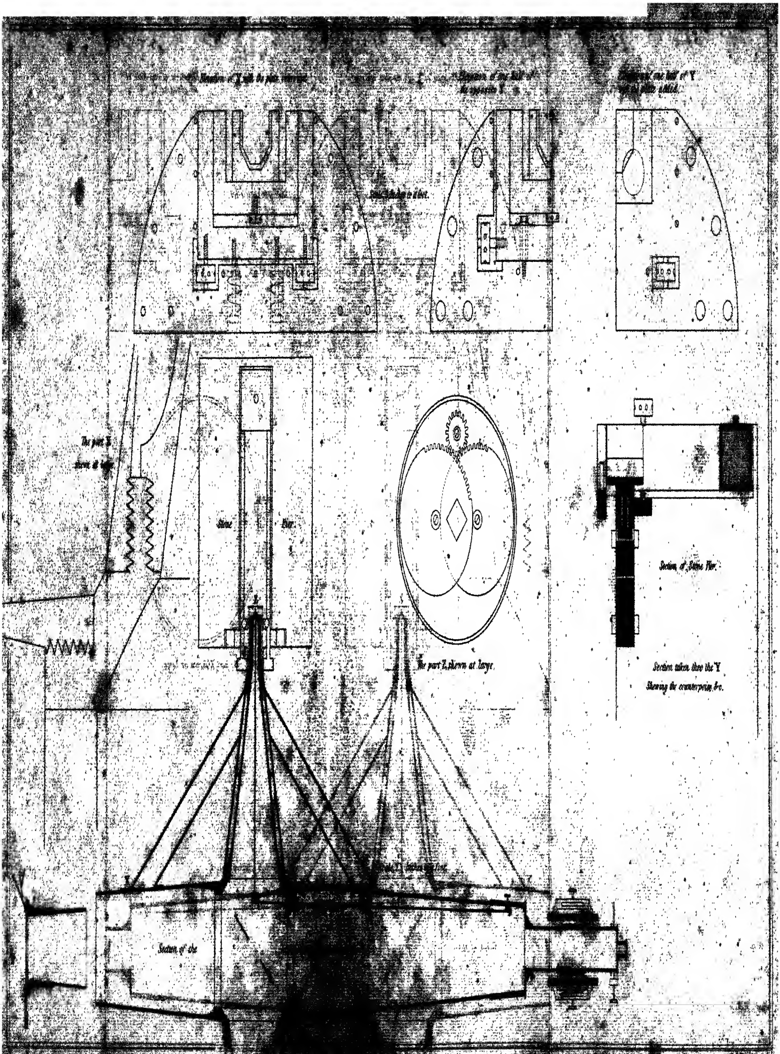

Stone
Pier
The part Z, shown at large.
Section of Stone Pier.
Section taken thro the Y
Showing the counterpoise &c.
Section of the

XVIII. *On the fossil Elk of Ireland.* By Thomas Weaver, *Esq. Member of the Royal Irish Academy, of the Royal Dublin Society, and of the Wernerian and Geological Societies.*

Read May 19, 1825.

Notwithstanding the frequent occurrence of the remains of the gigantic elk in Ireland, it is remarkable that precise accounts should not have been kept of all the peculiar circumstances under which they occur entombed in its superficial strata. To obtain an opportunity of examining these relations had long been my desire; and as fortunately, during my avocations last autumn in the north of Ireland, a discovery came to my knowledge that seemed likely to throw light on the subject, I proceeded to its investigation, intending, should the results be found deserving of attention, to place them on record. These results have proved the more interesting, as they apparently lead to the conclusion, that this magnificent animal lived in the countries in which its remains are now found, at a period of time which, in the history of the earth, can be considered only as modern.

I had advanced thus far when I became apprized of an analogous discovery made last year in the west of Ireland by the Rev. W. Wray Maunsell, Archdeacon of Limerick; which is not only confirmative of my own experience, but has the additional value of embracing particulars not hitherto noticed by any other observer. Mr. Maunsell's researches,

elucidated by the able assistance of Mr. JOHN HART, Member of the Royal College of Surgeons, have been communicated from time to time to the Royal Dublin Society in the form of letters, and have been entered upon their minutes; and, it is to be hoped, that a distinct publication on the subject may hereafter appear, illustrated by a description of the splendid specimen of the skeleton of the animal now deposited by the liberality of the Reverend Archdeacon in the museum of that Society. In the mean time I propose, after giving a concise account of my own inquiries, to refer briefly to the more prominent points in Mr. MAUNSELL's discoveries, in as far as they bear immediately on the question of the ancient or modern origin of those remains.

The spot which I examined is situated in the county of Down, about $1\frac{1}{2}$ mile to the west of the village of Dundrum. That part of the country consists of an alternating series of beds of clay slate and fine grained greywacke, with occasional subordinate rocks, which it is needless at present to mention; the whole distinguished by numerous small contemporaneous veins of calcareous spar and quartz, and traversed in some places by true rake veins that are metalliferous. Hills of moderate elevation, from 150 to 300 feet high, are thus composed. In a concavity between two of these hills is placed the bog of Kilmegan, forming a narrow slip, which extends about one mile in a nearly N. and S. direction. The natural hollow which it occupies appears formerly to have been a lake, which in process of time became nearly filled by the continued growth and decay of marshy plants, and the consequent formation of peat. The latter, however, from the flooded state of its surface, afforded little advantage as fuel,

until the present Marquis of DOWNSHIRE caused a level to be brought up from the eastward (part of it being a tunnel), and thus laid the bog dry. This measure was attended with a two-fold benefit to the tenantry, the provision of a valuable combustible, and the discovery of an excellent manure in the form of white marl beneath the peat. The latter extends from a few feet to twenty feet in depth; and the subjacent marl from one to three, four, and five feet in thickness. The marl when fresh dug has partly a grayish tinge, but on losing its moisture it becomes white.

In cutting down the peat to the bed of marl, the remains of the gigantic elk have frequently been met with; and invariably, as I am assured by the concurrent testimony of the tenantry, placed between the peat and the marl, or merely impressed in the latter. It is stated that at least a dozen heads with the branches, accompanied by other remains, have thus been found from time to time: but being unfortunately deemed of no value by the country people, they have for the most part been scattered and destroyed. It is to be hoped, however, that a sufficient inducement will lead them to bestow greater care on the preservation of whatever remains may be hereafter discovered.

The marl, upon examination, appears in a great measure composed of an earthy calcareous base, containing comminuted portions of shells; and that these are all derived from fresh water species, is proved by the myriads of these shells that remain in the marl, still preserving their perfect forms. They are however bleached, very brittle, and retain little of their animal matter; but in all other respects they have the characters of recent shells. After examining several

masses of the marl, I found the whole of the shells referable to three species, two univalves, and one bivalve: namely,

1. The *helix putris* of LINNÆUS. See DONOVAN's British Shells, Pl. 168, fig. 1, and LISTER, Conch. Tab. 123, fig. 23. N. B. Of the two, LISTER's figure is the more exact representation of the shell.
2. The *turbo fontinalis.* DONOVAN, Pl. 102.
3. The *tellina cornea.* DONOVAN, Pl. 96.

Of these shells some prevail more in one spot than in another; but generally speaking they appear distributed through the upper portion of the marl in nearly equal quantities; in the lower portion they are less frequent, if not altogether absent.

The circumstances which I have related seem to remove all idea of these remains of the Irish elk being of any other than comparatively recent origin. In seeking a cause for the nearly constant distribution of these remains in Ireland in swampy spots, may we not conjecture that this animal often sought the waters and the marshy land as a place of refuge from its enemies, and thus not unfrequently found a grave where it had looked for protection?

The foregoing conjecture appears supported by the following details of circumstances, observed by the Rev. Mr. MAUNSELL in the peat bog of Rathcannon, situated about four miles to the west of the town of Bruff, in the county of Limerick. This bog covers a space of about twenty plantation acres, occupying a small valley, surrounded on every side by a ridge of the carboniferous or mountain limestone, except on the S. W., where it opens into an extensive flat.

The peat is from one to two feet thick; and beneath this is a bed of white shell marl, varying from $1\frac{1}{2}$ to $2\frac{1}{2}$ feet in thickness, succeeded below by bluish clay marl, of an unascertained depth, but in one place it was found to exceed 12 feet. This bluish clay marl becomes white, and falls to powder on being dried. Coarse gravel is said to occur, partially at least, below the marl.

In this small valley portions of the skeletons of eight individuals were found, seven of adult, and one of a young animal, all belonging to the gigantic elk. With these also occurred the pelvis of an adult animal, probably referable to the red deer; and the skull of a dog, of the size of that of an ordinary water spaniel.

The bones that were first discovered were found at the depth of two or three feet below the surface; and Mr. Maunsell had the advantage of seeing them before they were displaced. Most of the above mentioned remains were lodged in the shell marl; many of them, however, appeared to rest on the clay marl, and to be merely covered by the shell marl. But parts of some of the bones were immersed in the peat also: these were tinged of a blackish colour, and were so extremely soft in consequence of the moisture they had imbibed, that it was with difficulty the horns found in this situation could be preserved entire; yet, when carefully handled and allowed to dry, they became as firm and hard as the rest.

Some of the bones of the elk showed marks of having been diseased; and one rib had evidently been broken, and afterwards reunited. Another rib exhibited a remarkable perforation of an oval form, about half an inch long, and

one-eighth of an inch broad, the longer axis being parallel to the side of the rib; the margin of this opening was depressed on the outer, and raised on the inner surface; while a bony point projected from the upper edge of the rib, which deviated from its natural line of direction to an extent equal to the length of the aperture. The only cause that could have produced this perforation is a wound by a sharp instrument, which did not penetrate deep enough to prove fatal, and between which event and the death of the animal a year at least must have elapsed, as the edges of the opening are quite smooth.

The bones are so well preserved, that in the cavity of one shank bone which was broken, marrow was found, having all the appearance of fresh rendered suet, and which blazed on the application of a lighted taper. They appear to contain all the principles to be found in fresh bones, with perhaps the addition of some carbonate of lime, imbibed with the moisture of the soft marl in which they had lain.

The remains of the eight individuals were disposed in such a manner as to prevent the possibility of referring the component parts exactly to each skeleton; but all the heads with their branches were found; and one specimen is particularly fine, displaying the broad expanded palms, with almost every antler and projecting point in a perfect state. By joining this head to a selection from the other remains, a nearly perfect skeleton of the largest size has been formed by Mr. HART; one rib, a few of the carpal and tarsal bones, and the bones of the tail being only wanting.

Of the shells found in the white marl many are preserved entire; but the greater part are broken into small fragments.

They are all univalves, and belong to fresh water species, which exist at the present day.

It is added, that so frequently have the remains of the fossil elk been discovered in the county of Limerick, that one gentleman enumerated thirty heads which had been dug up at different times within the space of the last twenty years.

From Professor HENSLOW's account of the curraghs, or peat bogs of the Isle of Man, it would appear that the remains of the gigantic elk are there also distributed in a manner analogous to that in which they are found in Ireland. That gentleman supposes a herd of elks to have perished there; and his description of the white, or grayish marl, in which their remains are found, answers in most respects to that of the white marl which so frequently forms the sub-stratum of the peat bogs in Ireland.

Upon the whole, the preceding details appear to justify the conclusion, that the extinction of the gigantic species of elk is attributable rather to the continued persecution it endured from its enemies, accelerated perhaps by incidental natural local causes, than to a general catastrophe which overwhelmed the surface of the globe. In a word, it may be inferred that these remains are not of diluvian, but of post diluvian origin.

T. WEAVER.

Kenmare, April 12, 1825.

XIX. *Microscopical observations on the Materials of the Brain, and of the Ova of Animals, to show the analogy that exists between them. By Sir* EVERARD HOME, *Bart. V. P. R. S. Read at the Society for promoting Animal Chemistry, April* 12, 1825.

Read at the Royal Society June 3, 1825.

HALF a century ago, when I began my professional education under Mr. HUNTER, he was deeply engaged in investigating the properties of the blood, and ascertaining the changes it underwent in different circumstances. His object in this inquiry was to prove that the blood possessed within itself a principle of life, by which all these changes were regulated.

By his direction I made the following experiment, which proved that when frozen and thawed it had undergone no change.

Two inches in length of the jugular vein distended with blood and secured at each end by a ligature, when immersed in a cooling mixture and frozen, was found after it was thawed to remain fluid, and to coagulate on exposure like recently drawn blood. From this fact, which is published in his work on the blood, corroborated by many others, he concluded that as the principle of life resided in the blood, and no change was produced in that fluid by the act of freezing, none were to be expected to arise from its action

on the other parts of the body; and had we been able to produce the necessary degree of cold, he certainly would have tried the experiment.

From the time of Mr. Hunter's death to that of the expedition to the polar circle being fitted out, the subject had never recurred to my mind; it was then revived; and I had no doubt of being fully informed upon its return, whether animals after being frozen could be revived; but in this I was disappointed.

In the winter before last an experiment was made in the presence of several Members of this Society, of freezing a frog, inclosed in tin foil, in a mixture cooled to zero. The frog recovered; but there was reason to doubt of the brain having been frozen; and this experiment was repeated by Mr. Faraday, in the laboratory of the Royal Institution, in the presence of Sir H. Davy, Professor Brande, and myself, in the following manner.

Two healthy frogs, nearly of the same size, were separately wrapped up in tin foil, and immersed in a cooling mixture at zero. At the end of four hours one of them was examined; the brain and heart were found completely frozen; the other was allowed to thaw gradually, but had no remains of life. Upon opening the skull the brain was dissolved, and the cavity contained nothing but a watery fluid, with some gelatinous matter.

By this experiment it is decided that an animal whose brain has been frozen can never be restored to life.

Having, in the Croonian Lecture for 1823, illustrated the more minute structure of the human brain by three drawings, magnified in different degrees by Mr. Bauer, made from a

healthy brain very recently after death, I became desirous of decomposing a similar portion of brain by the act of freezing, and then having drawings similar to the others made, to show the contrast between the two.

For this purpose I got Mr. Faraday to inclose in tin foil a thin slice of human brain soon after death, then weigh the tin foil in which it was enveloped in the balance belonging to the Royal Institution. After being thus accurately weighed, it was immersed in a cooling mixture as low as zero. When it had remained there for four hours it was taken out, and the tin foil unfolded that it might thaw gradually; a quantity of watery fluid had separated in the act of thawing from the portion of brain: this was allowed to drain off, and the tin foil with its contents was re-weighed, and had lost 20 per cent. from its decomposition. Mr. Bauer's drawings of it in this state, magnified in three different degrees, to correspond with the others, are annexed.

These two sets of drawings establish the real appearance of the more minute structure of the brain, and the changes that structure undergoes when exposed to the effects of having been frozen, and led me on to ascertain the effects of freezing upon the molecule of the pullet's egg after it has been impregnated, that I might ascertain whether the opinion I had formed, of its more minute parts corresponding with those of the brain, was correct; and as I have given drawings of the molecules highly magnified, similar drawings made after it had been frozen, would enable me to preserve the difference in appearance between the two.

To freeze the egg without disturbing the molecule, I enclosed it in a leaden case, with a cover exactly fitted to it;

Fig. 1.

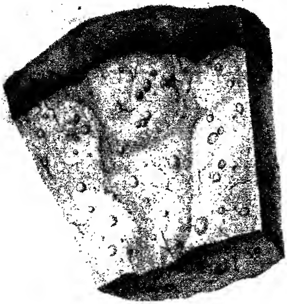


Fig. 2.

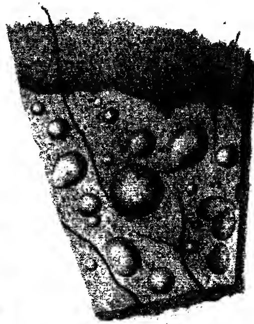


Fig. 3.

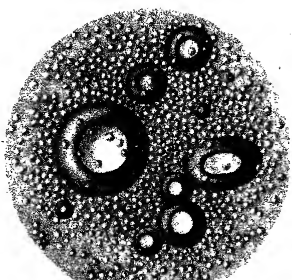
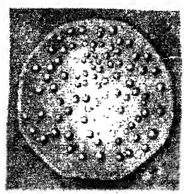


Fig. 4.



Franz Bauer del.t

F. Basire sculp.t

then exposed the molecule, put on the cover, and immersed the whole into a cold mixture, and carried it to Kew, that Mr. Bauer might represent the appearance.

EXPLANATION OF PLATE XXVII.

Fig. 1. A small portion of the cortical and medullary substance of the human brain that had been frozen and thawed, magnified 5 times.

Fig. 2. A part of the above, magnified 25 times.

Fig. 3. A still smaller part of the above, magnified 200 times.

These three drawings correspond with three that have a a place in the Philosophical Transactions: taken from the human brain recently after death, in a natural state.

Fig. 4. The molecule of the pullet's egg after impregnation, that had been frozen and thawed, magnified 10 times, to correspond with a similar drawing of the molecule in a natural state.

XX. *On new compounds of carbon and hydrogen, and on certain other products obtained during the decomposition of oil by heat. By* M. Faraday, *F.R.S. Cor. Mem. Royal Academy of Sciences of Paris, &c.*

Read June 16, 1825.

The object of the paper which I have the honour of submitting at this time to the attention of the Royal Society, is to describe particularly two new compounds of carbon and hydrogen, and generally, other products obtained during the decomposition of oil by heat. My attention was first called to the substances formed in oil at moderate and at high temperatures, in the year 1820; and since then I have endeavoured to lay hold of every opportunity for obtaining information on the subject. A particularly favourable one has been afforded me lately through the kindness of Mr. Gordon, who has furnished me with considerable quantities of a fluid obtained during the compression of oil gas, of which I had some years since possessed small portions, sufficient to excite great interest, but not to satisfy it.

It is now generally known, that in the operations of the Portable Gas Company, when the oil gas used is compressed in the vessels, a fluid is deposited, which may be drawn off and preserved in the liquid state. The pressure applied amounts to 30 atmospheres; and in the operation, the gas previously contained in a gasometer over water, first passes into a large strong receiver, and from it, by pipes, into the

portable vessels. It is in the receiver that the condensation principally takes place; and it is from that vessel that the liquid I have worked with has been taken. The fluid is drawn off at the bottom by opening a conical valve: at first a portion of water generally comes out, and then the liquid. It effervesces as it issues forth; and by the difference of refractive power it may be seen, that a dense transparent vapour is descending through the air from the aperture. The effervescence immediately ceases; and the liquid may be readily retained in ordinary stoppered, or even corked bottles; a thin phial being sufficiently strong to confine it. I understand that 1000 cubical feet of good gas yield nearly one gallon of the fluid.

The substance appears as a thin light fluid; sometimes transparent and colourless, at others opalescent, being yellow or brown by transmitted, and green by reflected light. It has the odour of oil gas. When the bottle containing it is opened, evaporation takes place from the surface of the liquid; and it may be seen by the striæ in the air that vapour is passing off from it. Sometimes in such circumstances it will boil, if the bottle and its contents have had their temperature raised a few degrees. After a short time this abundant evolution of vapour ceases, and the remaining portion is comparatively fixed.

The specific gravity of this substance is 0.821. It does not solidify at a temperature of $0°$ F. It is insoluble, or nearly so, in water; very soluble in alcohol, ether, and volatile and fixed oils. It is neutral to test colours. It is not more soluble in alkaline solutions than in water; and only a small portion is acted upon by them. Muriatic acid has no

action upon it. Nitric acid gradually acts upon it, producing nitrous acid, nitric oxide gas, carbonic, and sometimes hydrocyanic acid, &c. but the action is not violent. Sulphuric acid acts upon it in a very remarkable and peculiar manner, which I shall have occasion to refer to more particularly presently.

This fluid is a mixture of various bodies; which, though they resemble each other in being highly combustible, and throwing off much smoke when burnt in large flame, may yet by their difference of volatility be separated in part from each other. Some of it drawn from the condenser, after the pressure had been repeatedly raised to 30 atmospheres, and at a time when it was at 28 atmospheres, then introduced rapidly into a stoppered bottle and closed up, was, when brought home, put into a flask and distilled, its temperature being raised by the hand. The vapour which came off, and which caused the appearance of boiling, was passed through a glass tube at 0°, and then conducted to the mercurial trough; but little uncondensed vapour came over, not more than thrice the bulk of the liquid; a portion of fluid collected in the cold tube, which boiled and evaporated when the temperature was allowed to rise; and the great bulk of the liquid which remained, might now be raised to a comparatively high point, before it entered into ebullition.

A thermometer being introduced into another portion of the fluid, heat was applied, so as to keep the temperature just at the boiling point. When the vessel containing it was opened, it began to boil at 60° F. As the more volatile portions were dissipated, the temperature rose: before a tenth part had been thrown off, the temperature was above 100°.

The heat continued gradually to rise, and before the substance was all volatized, it had attained 250°.

With the hope of separating some distinct substances from this evident mixture, a quantity of it was distilled, and the vapours condensed at a temperature of 0° into separate portions, the receiver being changed with each rise of 10° in the retort, and the liquid retained in a state of incipient ebullition. In this way a succession of products were obtained; but they were by no means constant; for the portions, for instance, which came over when the fluid was boiling from 160° to 170°, when redistilled, began to boil at 130°, and a part remained which did not rise under 200°. By repeatedly rectifying all these portions, and adding similar products together, I was able to diminish these differences of temperature, and at last bring them more nearly to resemble a series of substances of different volatility. During these operations I had occasion to remark, that the boiling point was more constant at, or between 176° and 190°, than at any other temperature; large quantities of fluid distilling over without any change in the degree; whilst in other parts of the series it was constantly rising. This induced me to search in the products obtained between these points for some definite substance, and I ultimately succeeded in separating a new compound of carbon and hydrogen, which I may by anticipation distinguish as bi-carburet of hydrogen.

Bi-carburet of hydrogen.

This substance was obtained in the first instance in the following manner: tubes containing portions of the above rectified products were introduced into a freezing mixture at

liquid it may, like water and some saline solutions, be cooled much below that point before any part becomes solid. It contracts very much on congealing, 9 parts in bulk becoming 8 very nearly; hence its specific gravity in that state is about 0.956. At 0° it appears as a white or transparent substance, brittle, pulverulent, and of the hardness nearly of loaf sugar.

It evaporates entirely when exposed to the air. Its boiling point in contact with glass is 186°. The specific gravity of its vapour, corrected to a temperature of 60°, is nearly 40 Hydrogen being 1; for 2.3 grains became 3.52 cubic inches of vapour at 212°. Barometer 29.98. Other experiments gave a mean approaching very closely to this result.

It does not conduct electricity.

This substance is very slightly soluble in water; very soluble in fixed and volatile oils, in ether, alcohol, &c.; the alcoholic solution being precipitated by water. It burns with a bright flame and much smoke. When admitted to oxygen gas, so much vapour rises as to make a powerfully detonating mixture. When passed through a red hot tube it gradually deposits carbon, yielding carburetted hydrogen gas.

Chlorine introduced to the substance in a retort exerted but little action until placed in sun-light, when dense fumes were formed, without the evolution of much heat; and ultimately much muriatic acid was produced, and two other substances, one a solid crystalline body, the other a dense thick fluid. It was found by further examination, that neither of these were soluble in water; that both were soluble in alcohol—the liquid readily, the solid with more difficulty. Both of them appeared to be triple compounds of chlorine,

carbon, and hydrogen; but I reserve the consideration of these, and of other similar compounds, to another opportunity.

Iodine appears to exert no action upon the substance in several days in sun-light; it dissolves in the liquid in small quantity, forming a crimson solution.

Potassium heated in the liquid did not lose its brilliancy, or exert any action upon it, at a temperature of 186°.

Solution of alkalis, or their carbonates, had no action upon it.

Nitric acid acted slowly upon the substance and became red, the fluid remaining colourless. When cooled to 32°, the substance became solid and of a fine red colour, which disappeared upon fusion. The odour of the substance with the acid was exceedingly like that of almonds, and it is probable that hydrocyanic acid was formed. When washed with water, it appeared to have undergone little or no change.

Sulphuric acid added to it over mercury exerted a moderate action upon it, little or no heat was evolved, no blackening took place, no sulphurous acid was formed; but the acid became of a light yellow colour, and a portion of a clear colourless fluid floated, which appeared to be a product of the action. When separated, it was found to be bright and clear, not affected by water or more sulphuric acid, solidifying at about 34°, and being then white, crystalline, and dendritical. The substance was lighter than water, soluble in alcohol, the solution being precipitated by a small quantity of water, but becoming clear by great excess.*

* The action of sulphuric acid on this and the other compounds to be described, is very remakable. It is frequently accompanied with heat; and large quantities of

With regard to the composition of this substance, my experiments tend to prove it a binary compound of carbon and hydrogen, two proportionals of the former element being united to one of the latter. The absence of oxygen is proved by the inaction of potassium, and the results obtained when passed through a red hot tube.

The following is a result obtained when it was passed in vapour over heated oxide of copper. 0.776 grains of the substance produced 5.6 cubic inches of carbonic acid gas, at a temperature of 60°, and pressure 29.98 inches; and 0.58 grains of water were collected. The 5.6 cubic inches of gas are equivalent to 0.711704 grains of carbon by calculation, and the 0.58 grains of water to 0.064444 of hydrogen.

Carbon . .	0.711704	or	11.44
Hydrogen .	0.064444	or	1.

These quantities nearly equal in weight the weight of the substance used; and making the hydrogen 1, the carbon is not far removed from 12, or two proportionals.

those bodies which have elasticity enough to exist as vapours when alone at common pressures, are absorbed. No sulphurous acid is produced; nor when the acid is diluted, does any separation of the gas, vapour or substance take place, except of a small portion of a peculiar product resulting from the action of the acid on the substances, and dissolved by it. The acid combines directly with carbon and hydrogen; and I find when united with bases forms a peculiar class of salts, somewhat resembling the sulphovinates, but still different from them. I find also that sulphuric acid will condense and combine with olefiant gas, no carbon being separated, or sulphurous or carbonic acid being formed; and this absorption has in the course of 18 days amounted to 84.7 volumes of olefiant gas to 1 volume of sulphuric acid. The acid produced combines with bases, &c. forming peculiar salts, which I have not yet had time, but which it is my intention, to examine, as well as the products formed by the action of sulphuric acid on naphtha, essential oils, &c. and even upon starch and lignine, in the production of sugar, gum, &c. where no carbonization takes place, but where similar results seem to occur.

Four other experiments gave results all approximating to the above. The mean result was 1 hydrogen, 11.576 carbon.

Now considering that the substance must, according to the manner in which it was prepared, still retain a portion of the body boiling at 186°, but remaining fluid at 0°, and which substance I find, as will be seen hereafter, to contain less carbon than the crystalline compound, (only about 8.25 to 1 of hydrogen,) it may be admitted, I think, that the constant though small deficit of carbon found in the experiments is due to the portion so retained; and that the crystalline compound would, if pure, yield 12 of carbon for each 1 of hydrogen; or two proportionals of the former element and one of the latter.

$$\left.\begin{array}{lr} \text{2 proportionals carbon} & 12 \\ \text{1 hydrogen} & 1 \end{array}\right\} \text{13 bi-carburet of hydrogen.}$$

This result is confirmed by such data as I have been able to obtain by detonating the vapour of the substance with oxygen. Thus in one experiment 1092 mercury grain measures of oxygen at 62° had such quantity of the substance introduced into it as would entirely rise in vapour; the volume increased to 8505, hence the vapour amounted to 413 parts, or $\frac{1}{20.6}$ of the mixture nearly. Seven volumes of this mixture were detonated in an eudiometer tube by an electric spark, and diminished in consequence nearly to 6.1: these acted upon by potash were further diminished to 4, which were pure oxygen. Hence 3 volumes of mixture had been detonated, of which nearly 0.34 was vapour of the substance, and 2.65 oxygen. The carbonic acid amounted to 2.1 volumes, and must have consumed an equal bulk of oxygen gas; so that 0.55 remain as the quantity of oxygen which

has combined with the hydrogen to form water, and which with the 0.34 of vapour nearly make the diminution of 0.9.

It will be seen at once that the oxygen required for the carbon is four times that for the hydrogen; and that the whole statement is but little different from the following theoretical one, deduced partly from the former experiments. 1 volume of vapour requires 7.5 volumes of oxygen for its combustion; 6 of the latter combine with carbon to form 6 of carbonic acid, and the 1.5 remaining combine with hydrogen to form water. The hydrogen present therefore in this compound is equivalent to 3 volumes, though condensed into one volume in union with the carbon; and of the latter elements there are present six proportionals, or 36 by weight. A volume therefore of the substance in vapour contains

Carbon	-	$6 \times 6 = 36$
Hydrogen	-	$1 \times 3 = 3$
		39

and its weight or specific gravity will be 39, hydrogen being 1. Other experiments of the same kind gave results according with these.

Among the liquid products obtained from the original fluid was one which, procured as before mentioned, by submitting to 0° the portion distilling over at 180° or 190°, corresponded with the substance already described, as to boiling points, but differed from it in remaining fluid at low temperatures; and I was desirous of comparing the two together. I had no means of separating this body from the

bi-carburet of hydrogen, of which it would of course be a saturated solution at 0°. Its boiling point was very constantly 186°. In its general characters of solubility, combustibility, action of potassium, &c. it agreed with the substance already described. Its specific gravity was 0.86 at 60°. When raised in vapour 1.11 grain of it gave 1.573 cubic inches of vapour at 212°, equal to 1.212 cubic inches at 60°. Hence 100 cubic inches would weigh about 91.6 grains, and its specific gravity would be 43.25 nearly. In another experiment, 1.72 grains gave 2.4 cubic inches at 212°, equal to 1.849 cubic inches at 60°; from which the weight of 100 cubic inches would be deduced as 93 grains; and its specific gravity to hydrogen as 44 to 1. Hence probably the reason why, experimentally, the specific gravity of bi-carburet of hydrogen in vapour was found higher, than by theory it would appear to be when pure.

Sulphuric acid acted much more powerfully upon this substance than upon the bi-carburet: great heat was evolved, much discolouration occasioned, and a separation took place into a thick black acid, and a yellow lighter liquid, resisting any further action at common temperatures.

0.64 grains of this substance were passed over heated oxide of copper; 4.51 cubic inches of carbonic acid gas were obtained, and 0.6 grains of water. The carbonic acid and water are equivalent to

Carbon	-	0.573176, or	8.764
Hydrogen	-	0.066666	1.

but as the substance must have contained much bi-carburet of hydrogen, it is evident that, if in a pure state, the carbon would fall far short of the above quantity, and the compound

would approximate of course to a simple carburet of hydrogen containing single proportionals.

New carburet of hydrogen.

Of the various other products from the condensed liquor, the next most definite to the bi-carburet of hydrogen appears to be that which is most volatile. If a portion of the original liquid be warmed by the hand, or otherwise, and the vapour which passes off be passed through a tube at 0°, very little uncondensed vapour will go on to the mercurial trough; but there will be found after a time a portion of fluid in the tube, distinguished by the following properties. Though a liquid at 0°, it upon slight elevation of temperature begins to boil, and before it has attained 32°, is all resolved into vapour or gas, which may be received and preserved over mercury.

This gas is very combustible, and burns with a brilliant flame. The specific gravity of the portion I obtained was between 27 and 28, hydrogen being 1: for 39 cubic inches introduced into an exhausted glass globe were found to increase its weight 22.4 grains at 60° F. bar. 29.94. Hence 100 cubic inches weigh nearly 57.44 grains.

When cooled to 0° it condensed again, and inclosed in this state in a tube of known capacity, and hermetically sealed up, the bulk of a given weight of the substance at common temperatures was ascertained. This compared with water gave the specific gravity of the liquid as 0.627 at 54°. It is therefore among solids or liquids the lightest body known.

This gas or vapour when agitated with water is absorbed in small quantities. Alcohol dissolves it in large quantity; and a solution is obtained, which, upon the addition of water,

effervesces, and a considerable quantity of the gas is liberated. The alcoholic solution has a peculiar taste, and is neutral to test papers.

Olive oil dissolves about six volumes of the gas.

Solution of alkali does not affect it; nor does muriatic acid.

Sulphuric acid condenses the gas in very large quantity; 1 volume of the acid condensing above 100 volumes of the vapour. Sometimes the condensation is perfect, at other times a small quantity of residual gas is left, which burns with a pale blue flame, and seems to be a product of too rapid action. Great heat is produced during the action; no sulphurous acid is formed; the acid is much blackened, has a peculiar odour, and upon dilution generally becomes turbid, but no gas is evolved. A permanent compound of the acid with carbon and hydrogen is produced, and enters as before mentioned into combination with bases.

A mixture of 2 volumes of this vapour with 14 volumes of pure oxygen was made, and a portion detonated in an eudiometer tube. 8.8 volumes of the mixture diminished by the spark to 5.7 volumes, and these by solution of potash to 1.4 volumes, which were oxygen. Hence 7.4 volumes had been consumed, consisting of

Vapour of substance - -	1.1
Oxygen - - -	6.3
Carbonic acid formed - -	4.3
Oxygen in carbonic acid -	4.3
Oxygen combining with hydrogen	2.0
Diminution by spark - -	3.1

This is nearly as if 1 volume of the vapour or gas had required 6 volumes of oxygen, had consumed 4 of them in

producing 4 of carbonic acid gas, and had occupied the other 2 by 4 of hydrogen to form water. Upon which view, 4 volumes or proportionals of hydrogen = 4, are combined with 4 proportionals of carbon = 24, to form one volume of the vapour, the specific gravity of which would therefore be 28. Now this is but little removed from the actual specific gravity obtained by the preceding experiments; and knowing that this vapour must contain small portions of other substances in solution, there appears no reason to doubt that, if obtained pure, it would be found thus constituted.

As the proportions of the elements in this vapour appear to be the same as in olefiant gas, it became interesting to ascertain whether chlorine had the same action upon it as on the latter body. Chlorine and the vapour were therefore mixed in an exhausted retort: rapid combination took place, much heat was evolved, and a liquor produced resembling hydro-chloride of carbon, or the substance obtained by the same process from olefiant gas. It was transparent, colourless, and heavier than water. It had the same sweet taste, but accompanied by an after aromatic bitterness, very persistent. Further, it was composed of nearly equal volumes of the vapour and chlorine: it could not therefore be the same as the hydro-chloride of carbon from olefiant gas, since it contained twice as much carbon and hydrogen. It was therefore treated with excess of chlorine in sun-light: action slowly took place, more chlorine combined with the substance, muriatic acid was formed, and ultimately a fluid tenacious triple compound of chlorine, carbon, and hydrogen was obtained; but no chloride of carbon. This is a remarkable circumstance, and assists in showing, that though the

elements are the same, and in the same proportions as in olefiant gas, they are in a very different state of combination.

The tension of the most volatile part of the condensed oil gas liquid, and indeed of the substance next beneath olefiant gas in elasticity existing in the mixture constituting oil gas, appears to be equal to about 4 atmospheres at the temperature of 60.* To ascertain this a tube was prepared, like the one delineated in the sketch, Fig. 1, containing a mercurial

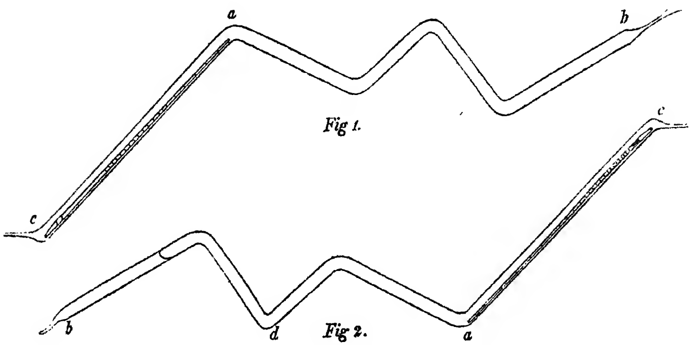


Fig 1.

Fig 2.

gauge at *a. c.* and the extremities being open. It was then cooled to 0° from *a* to *b*, and in that state made the receiver into which the first product from a portion of the original fluid was distilled. The part at *b* was then closed by a spirit lamp; and having raised enough vapour to make it issue at

**Note.* The particular inclination of the parts of the tube one to another was given, that the fluid when required might be returned from *a* to *d* without passing on to *b*.

c, that was also closed. The instrument now placed as at Fig. 2, had *a* and *d* cooled to 0°, whilst the fluid collected in *b* was warmed by the hand or the air; and when a portion had collected in *d* sufficient for the purpose, the whole instrument was immersed in water at 60°; and before the vapour had returned and been all dissolved by the liquid at *b*, the pressure upon the gauge within was noted. Sometimes the fluid at *d* was rectified by warming that part of the tube, and cooling *a* only, the reabsorption at *b* being prevented or rather retarded, in consequence of the superior levity of the fluid at *d*, so that the first portions which returned to *b* lay upon it in a stratum, and prevented sudden solution in the mass below. This difference in specific gravity was easily seen upon agitation, in consequence of the striæ produced during the mixture.

Proceeding in this way it was found, as before stated, that the highest elastic power that could be obtained from the substances in the tube, was about 4 atmospheres at 60°; and as there seems no reason to doubt, but that portions of the most volatile substances in oil gas beneath olefiant gas were contained in the fluid, inasmuch as even olefiant gas itself is dissolved by it in small proportions, it may be presumed that there is no substance in oil gas much more volatile than the one requiring a pressure of 4 atmospheres at 60°, except the well known compounds; or, in other words, that there is not a series of substances passing upwards from this body to olefiant gas, and possessing every intermediate degree of elasticity, as there seems to be from this body downwards, to compounds requiring 250° or 300° for their ebullition.

In reference to these more volatile products, I may state

that I have frequently observed a substance come over in small quantity, rising with the vapour which boils off at 50° or 60°, and crystallizing in spiculæ in the receiver at 0°. A temperature of 8° or 10° causes its fusion and disappearance. It is doubtless a peculiar and definite body, but the quantity is extremely small, or else it is very soluble in the accompanying fluids. I have not yet been able to separate it, or examine it minutely.

I ventured some time since upon the condensation of various gases,* to suggest the possibility of forming a vapour lamp, which containing a brilliantly combustible substance, liquid at a pressure of two, three, or four atmospheres at common temperatures, but a vapour at less pressure, should furnish a constant light for a length of time, without requiring high, or involving inconstant, pressure. Such a lamp I have now formed, feeding it with the substance just described; and though at present it is only a matter of curiosity, and perhaps may continue so, yet there is a possibility that processes may be devised, by which the substance may be formed in larger quantity, and render an application of this kind practically useful.

On the remaining portions of the condensed oil gas liquor.

It has been before mentioned, that by repeated distillations various products were obtained, boiling within limits of temperature which did not vary much; and which when distilled were not resolved into other portions, differing far from each other in volatility, as always happened in the earlier distillations. Though conscious that there were mixtures, perhaps

* Quarterly Journal of Science, XVI. 240.

of unknown bodies, and certainly in unknown proportions; yet experiments were made on their composition by passing them over oxide of copper, in hopes of results which might assist in suggesting correct views of their nature. They all appeared to be binary compounds of carbon and hydrogen, and the following table exhibits the proportions obtained: the first column expressing the boiling temperature at which the products were distilled, as before mentioned; the second the hydrogen, made a constant quantity; and the third the carbon.

140°	-	1	-	7.58
150°	-	1	-	8.38
160°	-	1	-	7.90
176°	-	1	-	8.25
190°	-	1	-	8.76
200°	-	1	-	9.17
210°	-	1	-	8.91
220°	-	1	-	8.46

These substances generally possess the properties before described, as belonging to the bi-carburet of hydrogen. They all resist the action of alkali, even that which requires a temperature above 250° for its ebullition; and in that point are strongly distinguished from the oils from which they are produced. Sulphuric acid acts upon them instantly with phenomena already briefly referred to.

Dr. HENRY, whilst detailing the results of his numerous and exact experiments in papers laid before the Royal Society, mentions in that read February 22, 1821,* the discovery

* Philosophical Transactions.

made by Mr. Dalton, of a vapour in oil gas of greater specific gravity than olefiant gas, requiring much more oxygen for its combustion, but yet condensible by chlorine. Mr. Dalton appears to consider all that was condensible by chlorine as a new and constant compound of carbon and hydrogen; but Dr. Henry, who had observed that the proportion of oxygen required for its combustion varied from 4,5 to 5 volumes, and the quantity of carbonic acid produced, from 2,5 to 3 volumes, was inclined to consider it as a mixture of the vapour of a highly volatile oil with the olefiant and other combustible gases; and he further mentions, that naphtha in contact with hydrogen gas will send up such a vapour; and that he has been informed, that when oil gas was condensed in Gordon's lamp, it deposited a portion of highly volatile oil.

A writer in the Annals of Philosophy, N. S. III. 37, has deduced from Dr. Henry's experiments, that the substance, the existence of which was pointed out by Mr. Dalton, was not a new gas sui generis, " but a modification of olefiant gas, constituted of the same elements as that fluid, and in the same proportions, with this only difference, that the compound atoms are triple instead of double:" and Dr. Thomson has adopted this opinion in his Principles of Chemistry. This, I believe, is the first time that two gaseous compounds have been supposed to exist, differing from each each other in nothing but density; and though the proportion of 3 to 2 is not confirmed, yet the more important part of the statement is, by the existence of the compound described at page 452, which though composed of carbon and

hydrogen in the same proportion as in olefiant gas, is of double the density.*

It is evident, that the vapour observed by Mr. DALTON and Dr. HENRY must have contained not only this compound, and a portion of the bi-carburet of hydrogen, but also portions of the other, as yet apparently indefinite substances; and there can be no doubt that the quantity of these vapours will vary from the point of full saturation of the gas, when

* In reference to the existence of bodies composed of the same elements and in the same proportions, but differing in their qualities, it may be observed, that now we are taught to look for them, they will probably multiply upon us. I had occasion formerly to describe a compound of olefiant gas and iodine (Phil. Trans. CXI. 72), which upon analysis yielded one proportional of iodine, two proportionals of carbon, and two of hydrogen, (Quarterly Journal, XIII. 429). M. SERRULAS, by the action of potassium upon an alcoholic solution of iodine, obtained a compound decidedly different from the preceding in its properties; yet when analysed, it yielded the same elements in the same proportions, (Ann. de Chimie, XX. 245, XXII. 172).

Again. MM. LIEBIG and GAY LUSSAC, after an elaborate and beautiful investigation of the nature of fulminating compounds of silver, mercury, &c. were led to the conclusion that they were salts, containing a new acid, and owed their explosive powers to the facility with which the elements of this acid separated from each other. (Annales de Chimie, XXIV. 294, XXV. 285). The acid itself being composed of one proportional of oxygen, one of nitrogen, and two of carbon, is equivalent to a proportional of oxygen + a proportional of cyanogen, and is therefore considered as a true cyanic acid. But M. WOHLER, by deflagrating together a mixture of ferro-prussiate of potash and nitre, has formed a salt, which, according to his analysis, is a true cyanate of potash. The acid consists of one proportion of oxygen, one of nitrogen, and two of carbon. It may be transferred to various other bases, as the earths, the oxides of lead, silver, &c.; but the salts formed have nothing in common with the similar salts of MM. LIEBIG and GAY LUSSAC, except their composition, (GILBERT'S Annalen, LXXIII. 157. Ann. de Chimie, XXVII. 190). M. GAY LUSSAC observes, that if the analysis be correct, the difference can only be accounted for by admitting a different mode of combination.

standing over water and oil, to unknown, but much smaller proportions. It is therefore an object in the analysis of oil and coal gas, to possess means by which their presence and quantity may be ascertained; and this I find may be done with considerable exactness by the use of sulphuric acid, oil, &c. in consequence of their solvent power over them.

Sulphuric acid is in this respect a very excellent agent. It acts upon all these substances instantly, evolving no sulphurous acid; and though, when the quantity of substance is considerable as compared with the acid, a body is left undecomposed by, or uncombined with the acid, and volatile, so as constantly to afford a certain portion of vapour; yet when the original substance is in small quantity, as where it exists in vapour in a given volume of gas, this does not interfere, in consequence of the solubility of the vapour of the new compound produced by the action of the acid in the acid itself in small quantities: and I found that when 1 volume of the vapour of any of the products of the oil gas liquor was acted upon, either alone, or mixed with 1, 2, 3, 4, up to 12 volumes of air, oxygen or hydrogen, by from half a volume to a volume of sulphuric acid, it was entirely absorbed and removed.

When olefiant gas is present, additional care is required in analytical experiments, in consequence of the gradual combination of the olefiant gas with the sulphuric acid. I found that 1 volume of sulphuric acid in abundance of olefiant gas, absorbed about 7 volumes in 24 hours in the dull light of a room; sun-shine seemed to increase the action a little. When the olefiant gas was diluted with air or hydrogen, the quantity absorbed in a given time was much diminished; and in those cases it was hardly appreciable in

two hours: a length of time which appears to be quite sufficient for the removal of any of the peculiar vapours from oil or coal gas.

My mode of operating was generally in glass tubes over clean mercury,* introducing the gas, vapour or mixture, and then throwing up the sulphuric acid by means of a bent tube with a bulb blown in it, passing the acid through the mercury by the force of the mouth. The following results are given as illustrations of the process:

Oil gas from a gasometer.

			in 8′	in 1 hour.	2 hours.	diminution.
188 vol.	+ 9.5 vol.	sulphuric acid diminished to	155	148.5	146.4	22.12 per cent.
107	+ 13.	- - - - - - -	88.5	84.5	82.0	23.33
138	+ 5.2	- - - - - - -	113.7	108.0	106.5	22.82

Oil gas from GORDON's lamp.

			15′	30′	3 hours.	
214	+ 6.8	- - - - - - -	183.3	180.8	176.	17.75
159	+ 5.9	- - - - - - -	137.5	136.0	130.4	17.98
113	+ 12.2	- - - - - - -	98.0	96.0	92.0	18.58

Coal gas of poor quality.

548.6	+ 27.6	- - - - - - -	533.3	529.2	529	3.57
273.6	+ 27.8	- - - - - - -	267.9	266	266	2.78
190.6	+ 13.1	- - - - . - -	186.	184.2	184.1	3.41

Oil may also be used in a similar manner for the separation of these vapours. It condenses about 6 volumes of the most elastic vapour at common temperatures, and it dissolves with greater facility the vapour of those liquids requiring higher temperatures for their ebullition. I found that in

* If the mercury contain oxidizable metals, the sulphuric acid acts upon it, and evolves sulphurous acid gas. It may be cleaned sufficiently by being left in contact with sulphuric acid for 24 hours, agitating it frequently at intervals.

mixtures made with air or oxygen for detonation, I could readily separate the vapour by means of olive oil; and when olefiant and other gases were present, its solvent power over them was prevented, by first agitating the oil with olefiant gas or with a portion of the gas to saturate it, and then using it for the removal of the vapours.

In the same way some of the more fixed essential oils may be used, as *dry* oil of turpentine; and even a portion of the condensed liquor itself, as that part which requires a temperature of 220° or 230° for its ebullition: care being taken to estimate the expansion of the gas by the vapour of the liquid, which may readily be done by a known portion of common *air preserved over* the liquid as a standard.

With reference to the proportions of the different substances in the liquid as obtained by condensation of oil gas, it is extremely difficult to obtain any thing like precise results, in consequence of the immense number of rectifications required to separate the more volatile from the less volatile portions; but the following table will furnish an approximation. It contains the loss of 100 parts by weight of the original fluid by evaporation in a flask for every 10° in elevation of temperature, the substance being retained in a state of ebullition.

100 parts at 58°	parts.	differences.
had lost at 70°	1.1	1.9
80°	3.0	2.2
90°	5.2	2.5
100°	7.7	2.4
110°	10.1	3.1

120°	13.2	2.9
130°	16.1	3.2
140°	19.3	3.1
150°	22.4	3.2
160°	25.6	3.4
170°	29.0	15.7
180°	44.7	23.4
190°	68.1	23.4
200°	84.2	16.1
210°	91.6	7.4
220°	95.3	3.7
230°	96.6	1.3

The residue 3.4 parts was dissipated before 250° with slight decomposition. The third column expresses the quantity volatilized between each 10°, and indicates the existence of what has been described as bi-carburet of hydrogen in considerable quantity.

The importance of these vapours in oil gas, as contributing to its very high illuminating powers, will be appreciated, when it is considered that with many of them, and those of the denser kind, it is quite saturated. On distilling a portion of liquid, which had condensed in the pipes leading to an oil gas gasometer, and given to me by Mr. HENNEL, of the Apothecaries' Hall, I found it to contain portions of the bi-carburet of hydrogen. It was detected by submitting the small quantity of liquid which distilled over before 190° to a cold of 0°, when the substance crystallized from the solution. It is evident therefore, that the gas from which it was deposited must have been saturated with it. On distilling a portion of recent

coal gas tar, as was expected, none could be detected in it, but the action of sulphuric acid is sufficient to show the existence of some of these bodies in the coal gas itself.

With respect to the probable uses of the fluid from compressed oil gas, it is evident in the first place, that being thus volatile, it will if introduced into gas which burns with a pale flame, give such quantity of vapour as to make it brightly illuminating; and even the vapour of those portions which require temperatures of 170° 180° or higher for their ebullition, is so dense as to be fully sufficient for this purpose in small quantities. A taper was burnt out in a jar of common air over water; a portion of fluid boiling at 190° was thrown up into it, and agitated; the mixture then burnt from a large aperture with the bright flame and appearance of oil gas, though of course many times the quantity that would have been required of oil gas for the same light was consumed: at the same time there was no mixture of blueness with the flame, whether it were large or small. Mr. Gordon has I understand proposed using it in this manner.

The fluid is also an excellent solvent of caoutchouc, surpassing every other substance in this quality. It has already been applied to this purpose.

It will answer all the purposes to which the essential oils are applied as solvents, as in varnishes, &c. and in some cases where volatility is required, when rectified it will far surpass them.

It is possible that, at some future time, when we better understand the minute changes which take place during the decomposition of oil, fat, and other substances by heat, and have more command of the process, that this substance,

among others, may furnish the fuel for a lamp, which remaining a fluid at the pressure of two or three atmospheres, but becoming a vapour at less pressure, shall possess all the advantages of a gas lamp, without involving the necessity of high pressure.

Royal Institution, June 7, 1825.

XXI. *Account of the repetition of* M. ARAGO's *experiments on the magnetism manifested by various substances during the act of rotation.* *By* C. BABBAGE, *Esq. F. R. S. and* J. F. W. HERSCHEL, *Esq. Sec. R. S.*

Read June 16, 1825.

1. THE curious experiments of M. ARAGO described by M. GAY LUSSAC during his visit to London in the spring of the present year, in which plates of copper and other substances set in rapid rotation beneath a magnetized needle, caused it to deviate from its direction, and finally dragged it round with them, naturally excited much attention, and the investigation of their various circumstances, and of their connexion with the effects observed by Mr. BARLOW in December, to be produced by the rotation of masses of iron, and described by him in a paper read to the Society,* became an object of considerable interest. Accordingly, having erected at Mr. BABBAGE's house, in Devonshire-street, an apparatus for setting a copper plate in rotation about a vertical axis by the aid of a turning lathe, we proceeded to try its effect on a magnetized needle suspended over it. The first attempt failed from the use of too small a needle; but this being replaced by a magnetic bar of considerable weight delicately suspended by a silk thread, we had the satisfaction of seeing it deviate several degrees from its point of rest in a direction

* See N°. XIV. of the present volume.

corresponding with that of the rotation of the copper plate; and on employing instead of this bar, a very delicate azimuth compass, belonging to and the invention of Captain KATER, the influence of zinc, brass, and lead was similarly rendered sensible.

2. In this first trial, having neither the command of a very rapid rotation, nor of massive metallic discs, the deviation of the compass observed did not exceed 10 or 11 degrees. In order therefore to enlarge the visible effect, and at the same time disencumber ourselves of the limit set to it by the polarity of the needle, it occurred to us to reverse the experiment, and ascertain whether discs of copper or other non-magnetic substances (in the usual acceptation of the word) might not be set in rotation if freely suspended over a revolving magnet. In order to make this experiment, we mounted a powerful compound horse-shoe magnet, capable of lifting 20 pounds, in such a manner as to receive a rapid rotation about its axis of symmetry placed vertically, the line joining the poles being horizontal and the poles upwards. A circular disc of copper, 6 inches in diameter and 0.05 inch thick, was suspended centrally over it by a silk thread without torsion, just capable of supporting it. A sheet of paper properly stretched was interposed, and no sooner was the magnet set in rotation than the copper commenced revolving in the same direction, at first slowly, but with a velocity gradually and steadily accelerating. The motion of the magnet being reversed, the velocity of the copper was gradually destroyed; it rested for an instant, and then immediately commenced revolving in the opposite direction, and so on alternately, as often as we pleased.

3. The rotation of the copper being performed with great regularity, it was evident that by noting the times of successive revolutions, we should acquire a precise and delicate measure of the intensity of the force urging it, provided we took care to neutralize the torsion of the suspending thread. To make the experiment strictly comparable proved however a matter of much delicacy, as the slightest change in the distance of the plate from the magnet was found to produce a material alteration in the time of its gyration.

4. Our first enquiry was directed to ascertain the effect of the interposition of different bodies as screens in cutting off or modifying the peculiar rotatory effect. The substances tried were, paper, glass, wood, copper, tin, zinc, lead, bismuth, antimony, and tinned iron plate. The comparative effects of these may be seen by the following tabulated observations, in making which we had the advantage of Mr. BARLOW's and Mr. CHRISTIE's presence and assistance.

TABLE I.

No. of revolut. performed.	Times of their performance.					
	Nothing interposed.	Paper interposed.	Wood interposed.	Antimony interposed. 1.	Antimony int. 2d trial.	Antimony int. 3d trial.
0	0.0	0.0	0.0	0.0	0.0	0.0
1	34	36.2	37.2	37.0	36.0	35.0
2	48	51.0	52.2	51.0	50.5	50.0
3	59	62.0	63.5	62.0	61.5	61.5
4	68	71.5	73.0	72.5	71.0	71.0
5	76.5	80.0	81.5	80.5	79.2	79.7
6	83.5	87.5	89.0	88.0	86.5	87.2
7	90 0	95.0	96.2	95.3	93.7	94.0
8	97.0	101.0	103.0	—	100 0	101.0
9	103.5	107.5	109.8	108.0	106.5	107.5
10	109.0	113.5	115.5	114.0	112.5	113.7

TABLE II.

No. of rev. performed.	Times of their performance.				
	Zinc interposed.	Bismuth interposed.	Copper interposed.	Lead interposed.	Tin interposed.
0	0.0	0.0	0.0	0.0	0.0
1	32.0	31.5	32.2	32.0	32.0
2	44.5	44.7	46.0	46.0	45.5
3	—	54.3	56.0	56.0	55 2
4	64.0	63.0	64.7	64.7	64.0
5	72.0	70.7	72.7	72.6	71.5
6	79.0	77.5	79.5	80.0	79.0
7	86.0	84.0	86.0	86.0	85.0
8	92.0	89.8	92.0	92.0	91.2
9	97.5	95.5	97.5	98.0	96.5
10	—	101.6	103.0	103.5	102.2

5. The metallic plates here interposed, as also the wooden ones, were circular discs of 10 inches in diameter and half an inch in thickness, the metals being all cast for the purpose, the wooden disc serving for a pattern. Such only are arranged together as were made under such circumstances as to be strictly comparable. It will be seen by these results that the various substances examined exert no sensible interceptive power, the slight excess of velocity in table 1. col. 1. when nothing was interposed, being evidently referable to the eddy caused in the air by the revolving magnet. Glass in like manner had no effect; but when the substance interposed was iron, the case was widely different, the magnetic influence being greatly diminished by one, and almost annihilated by two thicknesses of common tinned iron plate, as the following table will shew.

TABLE III.

Revolutions performed.	Time occupied.		
	Paper interposed.	One sheet of tinned iron interposed.	Two sheets of tinned iron interposed.
0	0.0"	0.0"	0.0"
½	- -	89.7	164.7
1	22.5	128.2	—
1½	- -	159.5	—
2	31.5	186.7	—
2½	- -	211.5	—
3	38.5	234.7	—

When the poles of the revolving magnet were connected by a piece of soft iron, the rotation of the copper disc was in like manner almost entirely annihilated.

6. Resuming now the original form of the experiment, the copper disc of 10 inches diameter and $\frac{1}{2}$ inch thick, was placed on the vertical axis, and made to revolve with a velocity of 7 turns in a second, a velocity which it was found convenient to give, and easy to maintain, corresponding as it did with one stroke per second of the treadle of the lathe; and this velocity, unless the contrary is mentioned, is to be understood of all the rotations so communicated, spoken of in the remainder of this account.

7. The copper plate thus revolving, the disc of copper mentioned in Art. 2 was suspended over it; but though at first it seemed to be very slightly affected, yet on frequent and most careful repetition of the experiment, with every precaution to guard against currents of air, not the most trifling effect could be perceived. This remarkable result, while it stands opposed to any theory of magnetic vortices generated by the rotation of one body, and transferring a part of its motion to others, is, on the other hand, perfectly consonant with, and indeed a necessary consequence of the view which will be taken of the subject in the sequel.

8. In like manner a bar of hardened, but not magnetised steel, was very slightly, if at all, set in rotation by the revolving copper, not more than probably would correspond to the small degree of magnetism unavoidably developed in it in the act of hardening; but when magnetised to saturation, it was made to revolve rapidly. This experiment appears decisive as to the origin of the magnetic virtue exhibited by

the copper and other bodies in these experiments. It is obviously *induced* by the action of the magnetic bar, compass needle, &c. on their molecules.

9. Our next enquiry was directed to the degree in which this developement of magnetic virtue takes place in different metals and other bodies. For this purpose two different processes were adopted. The first consisted in securing each of the 10-inch discs already spoken of successively on the vertical axis of our machine (which was now fitted up more firmly). Giving them thus a rotation in their own planes, the azimuth compass above mentioned was placed on a convenient stand centrally over each at the same distance. The deviations observed, and the ratios of their sines to that of the deviation produced by one of them (copper) chosen as a standard, were as follows.

TABLE IV.

Name of the revolving body.	(Motion of the disc direct, or screwing.)	(Motion retrograde, or unscrewing.)	Mean.	Ratio of the force to that of copper.
	° ′	° ′	° ′	
Copper -	11 30	11 17	11 24	1.00
Zinc - -	10 7	10 15	10 11	0.90
Tin - -	5 30	5 12	5 21	0.47
Lead - -	2 50	2 55	2 53	0.25
Antimony -	1 12	1 17	1 16	0.11
Bismuth -	0 6	0 6	0 6	0.01
Wood - -	0 0	0 0	0 0	0.00

The experiment was repeated (some weeks afterwards), placing the compass (by a more advantageous adjustment of the apparatus) much nearer the revolving disc. The results were as follows.

TABLE V.

Name of the revolving substance.	Mean of deviations screwing and unscrewing.	Ratio of force to that of copper.
	° ′	
Copper -	28 54	1.00
Zinc - -	26 42	0.93
Tin - -	12 54	0.46
Lead - -	7 0	0.25
Antimony -	2 27	0.09
Bismuth - -	0 32	0.02

Agreeing as nearly as could possibly have been expected with the foregoing.

10. The extension of the same mode of examination to other simple and compound bodies, differing widely in their relations to heat, electricity, gravity, and other chemical and mechanical agents, presents an extensive and most interesting field of enquiry, and one which promises a nearer insight into the nature of magnetism, both permanent and transient, than we have yet attained. Our examination has necessarily been limited, partly from the imperfection of our apparatus, but chiefly from want of time; indeed on reperusing the present notice, it is impossible not to regard it as in many respects imperfect and hasty; and nothing certainly but the strong interest of the subject, and the uncertainty whether we shall have it in our power to prosecute it with greater assiduity in future, could induce us to present our results in their present state. Such as they are, however, we shall give them.

11. Of the other metals, silver appears to hold a high rank, and gold a very low one in the scale of magnetic energy. Indeed the latter metal rendered standard by copper was

scarcely more powerfully set in rotation than seemed fairly attributable to the quantity of its alloy.

12. The examination of mercury presented peculiar interest from its fluidity, and the facility with which iron might be excluded from the experiment; to make which, a flat ring of box-wood was cemented with wax between two circular glass discs, so as to form a hollow cylinder, 2 inches in internal diameter, and 0.10 in its interior height. This being suspended, empty, by a long delicate silk thread over the horse-shoe magnet, was not in the slightest visible degree affected by its rotation, however long continued. It was then detached and filled with mercury, which, from having been thrice distilled, and afterwards having stood upwards of a twelvemonth in a bottle in contact with a solution of the nitrate of that metal, might assuredly be regarded as absolutely free from iron. Being again suspended as before, it now readily, though feebly, obeyed the rotation of the magnet in either direction, being fully commanded by it, and set in motion, stopped, or reversed in its gyrations at pleasure by merely continuing or changing properly the motion of the magnet. This experiment was witnessed, among others, by our illustrious President. The place which mercury appears to hold in the scale of magnetic energy was judged to be between antimony and bismuth, certainly superior to the latter, and certainly inferior to lead.

13. In wood, glass, wax, rosin, sulphur, sulphuric acid, water, &c. we have not hitherto succeeded in obtaining unequivocal traces of magnetism. The experiment with unannealed glass succeeded no better than with annealed. In the case only of one non-metallic body (unless a minute portion

of iron present may have deceived us) a decisive result has been obtained; and, what is very singular, this body is carbon, in that peculiar state in which its density, lustre, degree of hardness, and high conducting quality, both as regards heat and electricity, seem to give it some title to a place among the metals. This is the state in which it is precipitated by a red heat from coal-gas. It is found in thick masses encrusting the interior of the retorts, gradually blocking them up, and in time rendering them useless. It is composed of coats frequently curved round a centre, and exhibiting a radiated structure, but oftener in laminæ of a fine close grain, a beautiful gray colour, and in some varieties of a shining metallic brilliancy, between that of plumbago and hardened steel; some portions yield readily to the knife, but others of a darker hue and dull earthy fracture, resist obstinately, and give copious sparks with steel. The two sorts are found alternating or intermixed in the same specimen. The magnetism developed in this singular substance is, however, too feeble to admit of precise measurement, and is only rendered barely sensible by delicate management.

14. The second process alluded to as employed by us to compare the relative magnetic forces of the different bodies examined, consists in suspending magnetised bars over revolving discs of them, and observing, not the point of equilibrium but the velocity generated, or the time required for the description of certain spaces; in other words, by measuring not the statical, but the dynamical effect. These methods, for distinction's sake, may be called he statical and dynamical methods of observation.

In the original experiment of M. ARAGO, a magnetic

needle was made to deviate or revolve by the rotation of a plate beneath it. The motion of the needle must of course be rendered irregular by the effects of its polarity, and subject to periodical accelerations or retardations; and it is obvious, that in the case of a very weak magnetic force in the plate it can never execute an entire revolution, but must oscillate backwards and forwards till reduced to rest by the friction and resistance of the air. It occurred to us, however, that much more regular and uniform results might be obtained by this means, could the polarity of the needle be destroyed without at the same time destroying its magnetism; in other words, could the earth's action on it be so precisely neutralised as to allow of its resting indifferently in all directions. The obvious mode of doing this, by the approach of a powerful magnet acting in opposition to the earth, proved much too coarse for our purpose, which however, after a few trials, we found might be accomplished to any required degree of precision by the following simple contrivance.

If two exactly equal and similar magnets of equal strength be placed parallel to each other, but in a reverse position, and at such a distance as not mutually to affect each others' magnetism, and if in this situation they be firmly attached to a piece of wood, glass, &c. the system so formed will have no polarity, i. e. no tendency to rest in one rather than another situation, however suspended. This is clear; because whatever be the inclination (θ) of one of the magnets to the line of dip, that of the other will necessarily be ($180 + \theta$), and the directive forces being represented by the sines of these two angles will always be equal and opposite, so that each magnet urges the system with equal force, but in opposite

directions. The truth of this proposition, it is no less evident, is independent of the axis of suspension, which may pass through a part of the system any how situated with respect to the magnets, in virtue of the property of a magnet whose force to turn a system of which it makes a part, round a fixed centre, is the same wherever in the system it is placed, and the same as if it were in the centre.

Hence it follows, that if two equal and similar magnets be laid parallel to each other, but in a reversed position on a horizontal glass plate freely suspended by a thread, the system will be devoid of any polar tendency, (which we shall express by calling such a system *neutral).* It is difficult however to procure two magnets exactly equal, and of equal force. But fortunately this is of no consequence, as a slight deviation from perfect neutrality may be corrected by inclining the stronger needle a little more or less to the plane of the plate. In fact the proposition is general; and by a proper adjustment of the positions of two magnets however unequal, with respect to the axis and to each other, they may be made to neutralize each other.

15. As this adjustment however is nice, and as magnets influence each other, and our object moreover called for the utmost delicacy, we adopted a more refined application of the principle just detailed. A circular glass disc was prepared, 8 inches in diameter, and suspended by three silk threads from a filament of silk, descending along the axis of a copper tube about 5 feet long, passing with stiff friction through collars in the cieling of the apartment, and serving nicely by means of an index to regulate the height of the glass disc.

At the opposite extremities of two diameters at right angles to each other, four equal small bar magnets were fixed in a vertical position, having alternately their north and south poles downwards. This position promised to present two material advantages; first, that in neutralizing the system we have not the whole polarity of the magnets to contend with, but only the small remains of directive tendency which arises from the magnetic axis in each not being precisely coincident with its axis of figure, since it is evident that an infinitely thin magnetic cylinder placed perpendicularly to the horizon, would from that cause alone be indifferent as to situation; 2dly, That in this situation their poles interfere with each other's action on the plate revolving below them, less than in any other. Instead of four we might (and as will be seen) occasionally did place a greater number of magnets round the circle, or within its area, but for the experiments now in view four were enough.

16. The system so constructed was found to require no after adjustment, being to all appearance perfectly neutral, so that this part of our purpose was completely accomplished, and the earth's action eliminated from the enquiry. The irregular torsion of the silk thread however still embarrassed us a good deal. But though this undoubtedly caused individual results to differ more from the mean than we had expected, it is not sufficient to account for a singular anomaly observed not only in the mean results of a great number of trials, but in all individual cases; viz. that by this mode of observation, zinc was invariably found to stand above copper in the scale of magnetic action, whereas in the determination by the statical method, where the deviation of

the compass was observed, the former metal was as invariably found to be placed below the latter, the other metals retaining their order. A possible explanation of this anomaly (should future experiments show that the fact depends on no fallacy) may be found in the principles hereafter to be explained, but we wish to be understood as speaking with reserve on this point.

17. The following table is constructed in the same way as Tables I. II. III. with the addition only of the accelerating forces deduced on the supposition of uniform acceleration from the expression $\frac{s}{t^2}$.

TABLE VI.

No of revolutions or parts,	Times of their performance. For					Forces deduced from the expression $f = 1000000 \frac{s}{t^2}$				
$s =$	Copper $t =$	Zinc $t =$	Tin $t =$	Lead $t =$	Antimo. $t =$	Copper $f =$	Zinc $f =$	Tin $f =$	Lead $f =$	Antim. $f =$
0.25	38.3	36.1	51.7	70.9	109.6	170	192	93	42	21
0.50	54.2	51.7	74.8	102.5	157.9	170	187	89	48	20
0.75	68.5	63.9	92.8	128.0	197.4	160	184	87	46	19
1. 0	79.8	74.0	107.8	151.2	232.4	157	183	86	44	19
2. 0	110.6	106.2	156.8	221.8	351.7	164	177	81	41	16
3. 0	136.9	131.4	195.5	281.3	460.7	160	174	78	38	14
4. 0	160.0	152.8	229.5	335.0	—	156	171	76	36	—
5. 0	180.4	172.8	260.3	385.6	—	153	167	74	34	—
Mean of all - -						161	179	83	41	—
Mean of first six - -						163				18

The effect of torsion, resistance and friction, is very evident in the apparent diminution of the accelerating force in each revolution, so that only the numbers in the same horizontal lines can be regarded as comparable. Comparing accordingly the means of all for copper, zinc, tin, lead, and of the six first

for copper and antimony, the proportional intensity of magnetic action for each respectively will be

Zinc - -	1.11
Copper - -	1.00
Tin - -	0.51
Lead - - -	0.25
Antimony - -	0.01

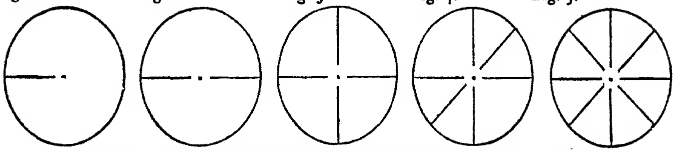
The smallness of the number for antimony is here also very remarkable. That for bismuth deduced by this means would be still more minute, so small indeed that the torsion of the thread would not allow of its magnitude being fairly determined, the suspended system merely performing extensive oscillations in very long times.

18. This method however requires us to operate on very considerable quantities of the substances under examination, a great disadvantage, as it cannot be applied to the scarcer metals, and does not admit of the use of the common ones in a state of rigorous purity. A method at once more simple and expeditious, and allowing of our acting on small quantities of matter, is to suspend portions of the different bodies we would try, similar in form and exactly equal in size, over the revolving magnet, and noting either, dynamically the times of successive revolutions, or, statically the point of equilibrium between the rotatory force and the torsion of the string. This method we pursued in a very interesting part of the enquiry, viz. in investigating (after M. ARAGO) the effect of a solution of continuity, partial or total in the mass acted on.

19. A disc of lead of 2 inches in diameter and $\frac{1}{10}$ thick, was suspended in a small thin wooden tray at a given distance from the horse-shoe magnet, revolving with the usual velo-

city, at first entire, and then successively cut with a chisel in radii nearly up to the centre, as here represented.

Fig. 1. Fig. 2. Fig. 3. Fig. 4. Fig. 5.



The times observed and forces deduced in the several cases were as follows:

Rev.	Disc uncut.		Disc cut as in Fig. 1.		Disc cut as in Fig. 2.		Disc cut as in Fig. 3.		Disc cut as in Fig. 4.		Disc cut as in Fig. 5.	
	$t=$	$f=$	$t=$	$f=$	$t=$	$f=$	$t=$	$f=$	$t=$	$f=$	$t=$	$f=$
1	28.2	1258	30.9	1047	33.1	913	42.1	564	48.1	432	55.6	324
2	41.2	1178	44.5		47.4		59.8		69.0		81.4	302
3	50.6	1172	55.0		59.0		74.7		86.6		103.3	281
4	58.7	1161	63.9		68.3		88.0		102.1		124.5	258
5	66.4	1134	72.0		77.2		100.0		116.8		145.9	235

Similar effects were observed in other metals, but in different degrees. For instance, in the case of soft tinned iron, the same number of cuts, made in the same manner, produced a very slight diminution of force, while in copper the effect of the same operation was to reduce the force in the ratio of 1 to 0.20.

20. A thin disc of copper suspended at a given distance over the revolving magnet, performed 6 revolutions from rest in 54ˢ. 8. It was then cut in 8 places in the direction of radii nearly up to the centre and 45° asunder, by which operation its magnetic virtue was so weakened, that it now required 121ˢ.3 to execute the same number of revolutions. The cuts were now soldered up *with tin*, and the magnetic action was now found to be so far restored as to enable it to

perform its six revolutions in $57^s.3$, that is to say, very nearly in the same time as when entire. This is the more remarkable, since tin, as we have seen, is not above half so energetic as copper when acting directly. This indirect mode of action therefore affords us a means of magnifying small magnetic susceptibilities which may hereafter prove very valuable.

21. To illustrate this more strongly, we suspended a brass disc of $2^{in}.25$ in diameter, and $0^{in}.15$ in thickness, as in the last case, and noted the time of its performing successive revolutions, as follows:

1 rev.	2 rev.	3 rev.	4 rev.	5 rev.
$20^s.2$	29.1	35.2	40.8	45.7

It was now cut, as in the last case but it being necessary for this purpose to use a saw, the abraded portions, which were pretty copious, were strewed over it with the intervention of a piece of thin paper, to obviate the effect of loss of weight, as nearly as might be. The times were now found increased as follows:

1 rev.	2 rev.	3 rev.	4 rev.	5 rev.
41.1	57.9	71.0	83.0	93.7

being almost exactly doubled, and of course the force was reduced in the ratio of about 4 to 1.

The cuts were now cleanly soldered with bismuth; and though, as we have seen, the direct force of bismuth is so small as to be scarce perceptible, yet its indirect effect in restoring the magnetism of the brass was such as to cause the same arcs to be described in the following numbers of seconds.

1 rev.	2 rev.	3 rev.	4 rev.	5 rev.
28.2	39.7	48.4	56.3	63.0

which require the exertion of an accelerating force more than double of that developed in the last trial.

The bismuth was now melted out, and the cuts being carefully washed with melted tin, were filled with fresh tin, which was allowed to fix, and the disc being trimmed, and replaced, the times were now found to be

21.7	30.8	38.0	43.5	48.7

The restoration of energy, as in the case of the copper disc is here very manifest, the times of rotation being nearly reduced to their original magnitude. The comparison of these, reduced by the formula $f = 1000000 \frac{s}{t^2}$, and the means of the five results taken in each case, gives for the accelerating forces

Brass, uncut	1.00	Copper uncut	1.00
cut	0.24	cut	0.20
soldered with bismuth	0.53	soldered with tin	0.91
soldered with tin	0.88		

The effects of soldering with lead and with fusible metal were also tried, and found to be both represented on the same scale by the same fraction, viz. 0.85, being but very little inferior to tin.

22. When the soldering is imperfect, the effect in restoring the magnetic action is proportionally weaker, but the influence of ever so small a free metallic communication is sensible.

23. A disc of lead cut in 8 radii as above was found to make one revolution in 58^s.3. It was then wetted so as to fill the cuts with sulphuric acid, and the time of revolution was found to be 57.3; so that the influence of sulphuric acid,

even when thus magnified, is still equivocal; and its magnetism, if it exist, can hardly be estimated at a thousandth part of that of copper, and is probably still lower.

24. The reduction of the metals to filings or to powder, was found to produce a still more striking diminution of their magnetic energy; and a class of experiments of great interest, as to the effect of the agglutination of these powders by metallic and non-metallic cements and liquids, immediately presents itself, into which want of leisure only has hitherto prevented our entering, as well as on the important subject of the magnetism of metallic alloys and atomic combinations, with which this branch of the enquiry is essentially connected.

25. When we come to reason on the above facts, much caution is doubtless necessary to avoid over-hasty generalization. Whoever has considered the progress of our knowledge respecting the magnetic virtue, which, first supposed to belong only to iron and its compounds, was at length reluctantly conceded to nickel and cobalt, though in a much weaker degree—then suspected to belong to titanium, and now extended, apparently with an extraordinary range of degrees of intensity to all the metals—will hardly be inclined to stop short here, but will readily admit, at least the probability, of all bodies in nature participating in it more or less. Yet if the electro-dynamical theory of magnetism be well founded, it is difficult to conceive how that internal circulation of electricity, which has been regarded as necessary for the production of magnetism, can be excited or maintained in non-conducting bodies. Without pretending to draw a line however, in what is perhaps at last only a question of degree, one thing is certain, that all the unequivocal cases of mag-

netic action observed by us, lie among the best conductors of electricity.* Another feature, no less striking, is the extreme feebleness of this species of action compared with that which takes place in cases of sensible attraction and polarity. This will appear more evidently, if we consider the mode of action which probably obtains in these experiments, and the mechanism, if we may so express it, by which the effects of such almost infinitesimal forces are rendered perceptible in them.

26. The rationale of these phænomena, as well as of those observed by Mr. Barlow in the rotation of iron, which form only a particular case (though certainly the most prominent of any) of the class in question, seems to depend on a principle which, whether it has or has not been before entertained or distinctly stated in words, it may be as well, once for all, to assume here as a *postulatum*, viz. that *in the induction of magnetism, time enters as an essential element*, and that *no finite degree of magnetic polarity can be communicated to, or taken from any body whatever susceptible of magnetism, in an instant.*†

* The meagre statements and imperfect reports which have hitherto reached us of M. Arago's researches, had prepared us to expect a much more appreciable amount of magnetic force in non-metallic bodies than we have observed. Glass, wood, water, ice, and indeed every description of substance, have been included in the list of bodies capable of producing a notable deviation from the magnetic meridian in a suspended bar, by their rotation. This naturally renders us desirous of seeing that eminent philosopher's own account of the means employed by him to render sensible such very minute forces, which must have been unusually delicate. This may perhaps be the proper place to mention, that the numerical estimates in this paper are merely intended to be received as gross approximations, valuable only in the absence of all other information of the kind. The metals used were those of commerce, no pains having been taken to free them from iron. Much refinement would have been thrown away on such materials.

† It is now some years since one of the authors of this account (Mr. Babbage)

27. This principle will, if we mistake not, be found to afford at least a plausible explanation of most, if not all the phænomena above described, without the necessity of calling in any additional hypothesis, or new doctrine in magnetism. For the other principle we shall have occasion to employ, that magnetic bodies differ exceedingly, both in *susceptibility* of this quality and in the degree of the pertinacity with which they retain it (which may be called their *retentive* power), is not an hypothesis, but an acknowledged fact. It is only in the mode of its extension to new cases of *magnetics* that we can be led into any fallacies. Whether these two qualities (susceptibility and retentive power) be, or be not mutually dependent, this is not the place to enquire. Probably they are not so, at least directly: and the new facts almost convert this probability into certainty; at all events, at present we shall for greater generality suppose them independent.

observed the following facts, which set the principle stated in the text in a very clear light. A natural magnet, armed with soft iron, terminating in a cylindrical surface, was made to support a load hooked on to another piece of soft iron, terminating also in a cylindrical surface, so that its contact with the former was limited to a physical line. The weight it would usually support varied from 27 to 33 lbs., according to the caution used in increasing the load. On loading it with 30 lbs. it became necessary to add the remaining weight by degrees, a quarter of a pound at a time, and to wait a short time after each addition. At about 32½ lbs. the weights usually fell from the magnet, and it was observed on replacing them, that it would no longer support more than 30, and that some minutes must be suffered to elapse before it could be brought to its former load. It was thus evident that the magnet required time for the developement of its full virtue. Again, having loaded the magnet by degrees up to 32 lbs., if the contact was broken for an instant by seizing the iron to which the load was suspended with both hands, detaching it suddenly and instantly restoring it, the magnet now continued to suspend 32 lbs., though, had the separation been of longer duration, 30 only would have been suspended. Time therefore is required to lose, as well as to gain magnetism.

28. Conceive now a plate of any thickness, and of indefinite superficial extent, of a metal or other magnetic, whose retentive power is very small. If either pole (suppose the north) of a magnet be brought vertically over a point in its surface, it will there produce a pole of the contrary name in the plate, the maximum of polarity being immediately under the magnet. Now let the magnet be moved horizontally along the surface, preserving the same distance from it. The points over which in succession it becomes vertical, not *instantly* receiving all the magnetism of which they are susceptible, will not have reached their maximum of polarity at the precise moment of nearest appulse, but will continue to receive fresh accessions during the whole of that certain small portion of time when the distance (being at or near its minimum) undergoes no change, or only a certain very minute one. In like manner, the points which have attained their maximum of polarity, being left behind by the magnet, will by degrees lose their magnetism; but the loss not being sudden, they will continue near their maximum for a certain finite time, during the whole of which the magnet continues receding from them, and leaving them farther and farther behind. Thus from both causes, there will be always in arrear of the magnet a space both more extensive and more strongly impregnated with the opposite polarity, than in advance of it; and as the magnet moves forward, the point of actual maximum (or the pole) of the plate, instead of keeping pace with it and being always precisely under it, will lag behind. There will thus arise an oblique action between the pole of the magnet and the opposite pole of the plate so lagging behind it; and were the plate free to move in its own plane, the resolved portion

of this action parallel to its surface, would continually urge it in the direction of the magnet's motion.

29. But besides the attracting pole of the opposite name (south) produced by the (north) pole of the magnet at the spot immediately under it, there will also be developed a corresponding repulsion or north polarity in the plate. This however will not, like the attractive, be concentrated nearly in one spot immediately below the magnet, but must of necessity be diffused round it in a much less intense and more uniform state throughout the more distant parts of the mass, and may be conceived as arranged in spherical or other concave strata about the point vertically under the magnet as a centre. Now when the magnet by its motion is carried out of the axis of these strata, it is obvious that the resultant force of each of them will be less and less oblique to the surface as its radius is greater. The general resultant therefore of all the repulsive forces exerted throughout the whole extent of the plate is necessarily less oblique to the surface than that of the attractive ones, whose influence, from this cause alone, must therefore preponderate, and must necessarily produce a *dragging* or oblique action, such as above described. This force, however minute, acting constantly, must at length produce a finite and sensible velocity, provided the whole mass of the plate to be set in motion be finite, and the force of the magnet sufficient to overcome friction, resistance, &c.

30. Vice versa, if the plate be drawn along in its own plane, and the magnet be free to move in a horizontal direction, the former ought to drag the latter along in the same

direction with a velocity continually accelerating, till they move on together with equal velocities.

31. It is manifest that, cæteris paribus, the greater the relative velocity, the more will the pole developed in the plate lag behind the magnet, or the magnet (in the reverse case) behind the pole. The more oblique therefore will be the action, and the greater the resolved part of the force, and the velocity produced by it dato tempore. The same effect must also be produced by an increase in the absolute force, or lifting power of the magnet; so that in such experiments there is an advantage in using large magnets which have great lifting powers, over small ones with intense directive forces, and this is perfectly consonant to experience.

32. Hitherto we have only considered the case of rectilinear motion. If we regard the magnetism of the plate as very transient, and the velocity moderate, the whole space occupied by the magnetised portion of the plate will still be small, and confined to the immediate neighbourhood of the point vertically under the magnet. If the motion of the latter change its direction, the momentary pull communicated to the plate will always be in the direction of a tangent to the curve described. If therefore it describe a circle, it will tend at every instant to impress a gyratory motion on the plate about a centre vertically under the centre of its own motion, and vice versâ, if the plate be made to revolve about a centre, it will tend to drag the magnet round with a continually accelerated motion, provided its rectilinear recess from the centre of motion (or its centrifugal force) be prevented by a proper mechanism. The former is the case of a disc of copper suspended by its centre, and set in rotation by a

magnet revolving beneath it. The latter is that of a compass needle, or of our neutralised system of vertical magnets suspended over a revolving disc of copper. A very pretty illustration of the direction of these forces is obtained by suspending a circular disc of zinc or copper from the end of a counterbalanced arm, which is itself suspended by its middle, thus constituting a kind of double balance of torsion. If the length of the arm be so adjusted, that the circumference of the disc shall be an exterior tangent to the circle described by the poles of a revolving magnet, the whole disc will be swept round in an orbit concentric with the motion of the magnet, while it at the same time acquires a rotatory motion on its own centre in the contrary sense. The centrifugal force is here overcome by the arm and the weight of the disc, and the velocity goes on accelerating till the increase of resistance puts a stop to further accessions.

33. In Mr. BARLOW'S experiments, the earth is our inducing magnet; its two poles both act on every particle of the revolving shell employed in that gentleman's experiments, and their action when complete produces two poles, a north and a south, at opposite extremities of the diameter parallel to the dip. This is the case when the shell is at rest. Let it now be set in motion about any axis, anyhow inclined to the dip. If the communication and loss of magnetism were instantaneous, the places of the poles (i. e. the points of maximum polarity) would be unaffected by the rotation; but as that is not the case, these points, in virtue of the principles already stated, will shift their places, and decline from the direction of the dip in the same direction as the shell's motion, that is to say, in the direction of a tangent to a small

circle, whose axis is the axis of rotation, and whose circumference passes through the extremities of the diameter parallel to the dip. The extent of this declination will depend on the velocity of rotation and the diameter of this small circle, and will be proportional to both, that is, to the velocity of rotation multiplied into the sine of the angle made by the axis of rotation with the direction of the dip. It will therefore be a maximum when the axis of rotation is perpendicular to the magnetic meridian, and vanish when the shell is made to revolve on an axis parallel to the line of dip. These consequences are perfectly consonant to the results obtained by Mr. BARLOW in his paper; and in fact, the general result announced by him in (page 326 of this volume) comes to the very same thing as above stated; for it is obvious, that the new axis of polarization there spoken of, acting in combination with the original, or, as we may call it, the primary axis developed in the quiescent state of the shell, will exert a compound force on the needle, such as would be exerted by a single equivalent axis situated intermediately between them, but much nearer to the more intense than to the more feeble one. The position of this equivalent axis will necessarily be in the great circle passing through the two component ones. Now the small circle described by the point which was first the pole of the stronger or primary axis about the axis of rotation is a tangent to this great circle, and the equivalent axis (being but little removed from the primary one, by reason of the small intensity of the other), will therefore have its pole situate indifferently in either circle. Or conversely, the single axis produced in our view of the subject being resolved into two; one of which is that corresponding to the quiescent state of

the shell, and the other 90° removed from it in the same place, this latter will be identical with Mr. Barlow's secondary axis.

34. In what has been said, the velocity of rotation has been supposed commensurate to the velocity with which magnetism is propagated through the iron of the shell. But if we conceive in this, or in the general case, either the retentive power of the shell, disc, or lamina great, or the velocity of motion excessive, it may be instructive to consider the modifications thus introduced into the effect. It is evident that the induced pole will lag farther and farther behind the magnet in proportion as either of these conditions obtains. In the case of rectilinear motion, this will, up to a certain point, increase the oblique action, and the dragging effect will be strengthened; but if the velocity be excessive, or the retentive force considerable, as in steel, the pole may lag so far behind as to carry it altogether out of the sphere of the magnet's attraction; and the magnetised portion, remaining within its limits, may have not had time enough to acquire a high degree of polarity. From both causes the drag (the expression, though uncouth, is convenient) should be weakened. In the case of circular motion this effect may go so far, that a complete circumference shall have been described before the polarity of any one point shall have been either completely induced, or completely destroyed. In this case the effect observed will be a general weakening of the total polarity of the disc or sphere; and (supposing the latter of iron, or soft steel) a directive virtue on a small compass needle placed near it, not probably towards any particular place, but to a resultant imaginary point depending on the

situation of the compass, the dip, and the axis of rotation, by laws not very easy to assign. This will explain some expressions quoted by Mr. Barlow from his correspondence with one of the authors of this paper, which may appear otherwise to militate against the general view here taken.

35. This diminution of the total effect by a more general distribution of the magnetism, was imitated by sticking a great number of needles vertically through a light cork circle, all being strongly magnetised, and having their north poles downwards, so as to form a circle, or, as it were, a coronet of magnets. This apparatus suspended centrally over a revolving copper disc, was not sensibly set in rotation. In this case, when at rest, the south polarity induced in the plate would be disposed in spots accumulated under each needle; but these spots, elongated and blended by the effect of rotation, must produce a nearly uniform circle of south polarity, whose equal and contrary actions on all the needles would keep up the equilibrium, and prevent the coronet from acquiring a tendency either way.

36. One consequence of this reasoning, which deserves trial, is this—that if the axis of rotation of an iron shell be situated in the direction of the dip, the spots occupied by its poles will not change their places by rotation, and consequently no deviation of the compass ought to take place from that cause. The experiment however is very delicate; and care must be taken to remove any magnetised bodies whose influence might induce subordinate poles in the shell, whose places would shift by rotation. The compass therefore in this case cannot be neutralized by a magnet;* but we must

* In Mr. Barlow's experiments, the large and powerful bar magnets used to

have recourse to some neutral system, such as that described in the foregoing pages, in its place, or it may be left unneutralized. It ought too to be so small, or so remote, as not to produce induced polarity in the shell, which would react on itself when the sphere is set in motion, and destroy the success of the experiment.

37. The effect of a solution of continuity in the revolving bodies comes next to be considered. It is difficult; but the difficulty is not a consequence of our principles of explanation, but of our ignorance of the very complicated laws which regulate the distribution and communication of magnetism in bodies of irregular figure. So far however as the operation of the general principle can be traced, its results are consonant to observation.

38. In the first place, it is obvious that where one or more slits are cut in a metallic plate, over which the pole of a magnet is revolving, that immediate and free communication between particle and particle, on which probably the rapid, and certainly the intense developement of magnetism depends, is destroyed. The induced pole (by which we mean now the whole of that space in which sensible magnetism is developed, and which is, of course, a spot of sensible, and probably considerable magnitude—of a figure more or less elongated according to the velocity of the motion)—instead of travelling regularly round, retaining a constant magnetism and force, will now be in a perpetual state of change. Instead of being carried uniformly across the slit, it will die away in intensity, and shrink into a point in dimension on the hinder

neutralize the earth's action on the compass needle, cannot be without some disturbing influence of this kind.

side, and be again renewed on the side in advance, but at first not in its full intensity; so that it is not merely the diminution of surface arising from the abstraction of a part of the metal, but a much more considerable defalcation of magnetic force which takes place on either side of the slit, that operates. Now this operation is always to weaken the drag between the magnet and the disc, and no reason, a priori, can be assigned why this effect should not take place to any extent.

39 The validity of this reasoning is shown by taking the extreme case in which the substance acted on is in the state of powder. Each particle of this becomes necessarily a feeble magnet, and its north and south poles, being at the same distance (almost precisely) from the pole of the magnet, counteract each other's action. The extreme feebleness of their magnetism prevents the particles from affecting each other by induction across the intervals which separate them; so that each acts as an individual, and destroys in great measure its own effect. The moment however a *metallic*, i. e. *a magnetic contact* is established between them, their mutual induction acts, and the result is a general developement of one polarity in the region adjacent to the magnet; and of the other, feebler and more diffused, in the parts of the mass remote from it. This is probably the rationale of the restoration of virtue which takes place when a cut disc is soldered up. And it is not difficult to conceive that a weak magnetism may be thus very faithfully transmitted through substances, such as bismuth and lead, whose direct action is very small, because, as we have seen, the intensity of their direct action depends, for one of its causes, on the retentive

power of the substance, which is out of question in the indirect mode of action here considered. In fact, if the retentive power of the solder were reduced to nothing, i. e. if it gained and lost magnetism instantaneously, it would still act as a conductor, and probably the better for this quality; so that the communication between opposite sides of a slit, or contiguous portions of two adjacent particles of a powder, would still be kept up by it, provided it were susceptible of magnetism at all. The observed and very striking fact then of the powerful action of bismuth as a conductor, while its action as a magnet is so extremely feeble, is in itself a strong argument for the independence of these two qualities, which we have designated by the expressions—*susceptibility*, and *retentive power*, and may possibly be made the foundation of a mode of distinguishing and measuring their degrees in different substances.

C. B.

J. F. W. H.

XXII. *On the magnetism developed in copper and other substances during rotation. In a Letter from* SAMUEL HUNTER CHRISTIE, *Esq. M. A. &c. to* J. F. W. HERSCHEL, *Esq. Sec. R. S. Communicated by* J. F. W. HERSCHEL, *Esq.*

Read June 16, 1825.

DEAR SIR,

As you inform me that you are drawing up an account of your magnetical experiments, I send you a brief account of those which I have made: they may possibly bear upon some of the points which you have had under consideration; and in this case you will not be displeased at being able to compare independent results.

After having made experiments with a thin copper disk suspended over a horse-shoe magnet, similar to those which I witnessed at Mr. BABBAGE's, I made the following.

A disk of drawing paper was suspended by the finest brass wire (No. 37) over the horse-shoe magnet, with a paper screen between. A rapid rotation of the magnet (20 to 30 times per second) caused no rotation in the paper, but it occasionally dipped on the sides, as if attracted by the screen, which might be the effect of electricity excited in the screen by the friction of the air beneath it.

A disk of glass was similarly suspended over the magnet: no effect produced by the rotation.

A disk of mica was similarly suspended: no effect.

The horse-shoe magnet was replaced by two bar magnets, each 7.5 inches long, and weighing 3 oz. 16 dwt. each, placed

horizontally parallel to each other, and having their poles of the same name contiguous. These produced quick rotation in a heavy disk of copper 6 inches in diameter, and suspended by a wire, No. 20.

A bar magnet 4 inches long, and having both its ends south poles, was made to revolve rapidly under a copper disk. The disk revolved in the same direction as the magnets.

The two bar magnets before mentioned were adjusted to the axis of rotation, so that their upper ends were at the distance of 5 inches from each other, and their lower ends 1.8 inch apart. They were first made to revolve rapidly under the copper disk with poles of the same name nearest to the disk, and then with poles of a contrary name: the times in which the several rotations of the disk took place were as nearly as possible the same in the two cases.

No. of Revolutions.	Poles of the same name nearest to the disk.		Poles of a contrary name nearest to the disk.	
	Screw.*	Unscrew.*	Screw.	Unscrew.
	Time.	Time.	Time.	Time.
1	15 sec.	15 sec.	15 sec.	15.4
2	21	21	21	21.5
3	26	26	26	26.3
4	30	30	30	30.0

In the first three, I could only remark the time to the nearest second, having no assistance. Should the times agree precisely, which I have very little doubt they would be found to do, the result would, I think, be singular. It would

* These expressions refer to the direction with respect to the spectator in which the rotation was performed.

show that the magnetism in the disk is instantaneously developed by one pole of the magnet, and as instantaneously destroyed, and a contrary magnetism developed by the contrary pole; or rather it would indicate, that the time during which the disk retained the induced magnetism was less than the time of half a revolution of the magnet.

The same two bar magnets were laid horizontally by the side of each other, $\frac{4}{10}$ inch a part. They were first made to revolve rapidly under the disk with their poles of the same name adjacent, and then with those of a contrary name adjacent.

No. of revolutions.	Poles of the same name adjacent.		Poles of a contrary name adjacent.	
	Screw.	Unscrew.	Screw.	Unscrew.
	Time.	Time.	Time.	Time.
	m. s.	m. s.	m. s.	m. s.
1	29	28	31	32
2	40	57	42	46
3	48	46	52	58
4	56	53	1 01	1 07
5	1 03	1 00	1 11	1 15
6	1 10	1 06	1 18	1 23
7	1 16	1 12	1 25	1 30
8	1 21	1 17	1 31	1 36
9	1 26	1 22	1 37	1 43
10	1 31	1 27	1 43	1 49

From these it appears that the effect was but little diminished by placing poles of a contrary name so close to each other.

The adjacent poles being of the same name, they were connected by a piece of soft iron $\frac{1}{8}$ inch thick, and $\frac{1}{2}$ inch wide. After $4\frac{1}{2}$ revolutions of the disk (screw), the torsion of the wire was equal to the force of the magnets, and the

same was the case at $4\frac{3}{4}$ revolutions (unscrew). So that although the effects were greatly diminished by connecting the poles, they were by no means destroyed.

The magnets were now placed over each other, first with poles of a contrary name, and then with those of the same name contiguous.

No. of revolutions.	Poles of a contrary name contiguous.		No. of revolutions.	Poles of the same name contiguous.	
	Screw.	Unscrew.		Screw.	Unscrew.
	Time.	Time.		Time.	Time.
	m. s.	m. s.		s.	s.
1	1 48	1 32	1	21	21
2	3 20	2 40	2	30	29
$2\frac{1}{12}$	3 50	(Rev. $2\frac{1}{3}$) 3 45	3	36	34
			4	42	39
			5	47	44
			6	51	48

At $2\frac{1}{12}$ (screw) and $2\frac{1}{3}$ (unscrew) the torsion of the wire was equal to the force of the magnets.

So that although the upper magnet was nearer to the disk, by its own thickness, than in the 4th experiment, the effect when poles of contrary name contiguous was not half what it was when they were connected by the iron.

A thick copper plate 8 inches in diameter and 1 inch thick, was placed on the axis of rapid rotation, its plane horizontal. A thin copper disk 4 inches diameter, and weighing 23.5 dwts. was very delicately suspended over it by a fine brass wire (No. 37), with a paper screen between the plate and the disk. The distance between the surfaces of the plate and disk $\frac{5}{10}$ inch. The plate being put in rapid rotation, no sensible effect was produced on the disk.

A bar magnet was placed on the screen under the disk: still no effect produced by the rotation.

A light needle, weight 42.5 grains, 6 inches long, on a pivot in a compass box, being placed over the plate, the rotation caused a deviation of 20°; but when a heavy needle, weighing 197 grains, and of the same length, was similarly placed over the plate, it immediately revolved rapidly with the plate.

A bar magnet, weighing 3 oz. 15 dwts. 19 grs. suspended by a wire, No. 20, revolved rapidly with the plate.

A horse-shoe magnet, weighing nearly a pound, and suspended by the same wire, revolved with the disk.

The following experiments were made with the view of ascertaining whether the effects increased nearly according to any power of the decrease of the distance.

A strong needle, 6 inches in length, weighing 197 grains, and vibrating 22 times in a minute, delicately suspended on an agate within a rim accurately graduated, was placed with its centre exactly over that of the copper plate, and being accurately adjusted, so that the distance between the centre of the copper and that of the needle was such as I required for the observation, the copper was made to revolve rapidly (always as nearly as possible 12 times per second), and when the needle became stationary, the direction of its south end (being that most convenient for observation) was noted. This was done with the copper revolving in both directions, " screw" and " unscrew." The direction of the south end of the needle was also observed before the rotation.

Distance.	4.0 in.	3.5 in.	3.0 in.	2.5 in.	2.0 in.	
Screw - -	1° 46′W	3° 20′W	6° 20′ W	14° 30′W	29° 40′W	Direction of south end of the needle.
Unscrew -	1 32 E	3 08 E	6 00 E	13 50 E	29 00 E	
Mean - -	1 39	3 14	6 10	14 10	29 20	

On diminishing the distance to 1.5 inch, the needle revolved with the plate, and very shortly so rapidly, that it had the appearance of an entire circle.

After this I replaced the needle by others which were lighter, letting every thing else remain the same, that is, the distance still 1.5 inch.

	Needle weighing 42.5 grs.	Needle weighing 25.5 grs.
Screw - -	24° 40′W	10° 30′W
Unscrew -	25 20 E	10 40 E

(I should mention that the needles were not at all neutralized).

From the latter observations, it is evident that the effect produced depends upon the intensity of the magnetism in the needle employed; and this I think proves clearly that the effect arises from the magnetism induced in the copper from the needle itself.

If we suppose the tang. of the deviation to vary as $\frac{1}{(\text{dist.})^n}$, then θ and θ' being two deviations at the distances d and d', we shall have $n = \frac{\log.\tan.\theta' - \log.\tan.\theta}{\log.d - \log.d.}$.

Computing n from this, by a comparison of every two observations we have the following values of n:

When distance is measured from centre of copper.	When distance is measured from surface of copper.
5.04	4.37
4.60	3.93
4.62	3.88
4.29	3.51
4.20	3.60
4.45	3.64
4.10	3.31
4.65	3.80
4.07	3.23
3.59	2.78
Mean 4.361	Mean 3.605

If we suppose that the poles of the needle are urged by forces in the direction of the motion of the copper, which being constant in the copper, would affect the needle reciprocally as the square of the distance; then these forces in the copper being derived from the needle itself, we must suppose that their intensity will vary also reciprocally as the square of the distance: so that the force on the needle arising from this mutual action, would vary reciprocally as the fourth power of the distance. Taking the mean between the mean values of n above, when the distance is measured from the centre of the copper and from its surface, would give the value of n for an intermediate point 3.983, which is as near to 4, supposing that such ought to be the value, as we could expect the observations to give.

The next experiments which I made were with the view of determining the law of force as regards the distance, when magnets act upon a copper disk. For this purpose I made use of the suspending wire as a balance of torsion. The results which I have obtained in this manner give a much less rapid diminution of the force, as the distance increases, than appears to take place when a thick copper plate acts upon a small magnet, as in the former experiments, which agrees with what you have mentioned as following from your

results. The results obtained in the former case appear to indicate, that every particle in the copper urges the needle from the magnetic meridian with a force varying as $\frac{\text{vel. of particle}}{(\text{distance})^4}$, which law would arise from the magnetism in the needle developing the magnetism in the particles of copper, so that its intensity would vary as $\frac{1}{(\text{dist.})^2}$, and this magnetism again acting on the poles of the needle with a force varying as $\frac{1}{(\text{dist.})^2}$. Supposing this to be the case, if z is the distance of a lamina of copper from the plane of the needle, s the arc of a circle in this lamina at the distance r from the axis of rotation, R the radius of the copper cylinder, t its thickness, c the distance of its upper surface from the needle, and a the distance of the pole of the needle from its centre: then the whole force with which the cylinder urges the needle will be proportional to

$$\iiint \frac{r\,d\,s\,d\,r\,d\,z}{\left\{z^2 + (a-r)^2\right\}^2}$$

Although this may be integrable, the integral would be in so complicated a form, that it would be very ill suited for comparison with the results obtained from observation; but if we consider only the annulus of the copper immediately under the pole of the needle, which will be the most efficient part, we may readily make this comparison. For calling θ the deviation, we should have $\sin.\ \theta = \int \frac{dz}{z^4} \times \text{const.}$ or $\sin.\ \theta = \left(\frac{1}{c^3} - \frac{1}{(c+t)^3}\right) \times \text{const}$; and consequently $\frac{\sin.\ \theta}{\frac{1}{c^3} - \frac{1}{(c+t)^3}}$ $= \text{const.}$

From my experiments t being 1, I should obtain the following values of $\dfrac{\sin.\theta}{\frac{1}{c^3}-\frac{1}{(c+t)^3}}$

C	θ	$\dfrac{\sin.\theta}{\frac{1}{c^3}-\frac{1}{(c+t)^3}}$	
	° ′		
3.5	1 39	2.3316	Mean 2.505
3.0	3 14	2.6341	
2.5	6 10	2.6409	
2.0	14 10	2.8144	
1.5	29 20	2.1040	

Although there is a considerable difference in the numbers, especially the last, yet as the parts whose action is not considered have here the greatest effect, and all the observations are liable to errors arising from the difficulty of making the copper revolve with the same velocity in all cases, I think the agreement is sufficiently near to indicate that the copper acts as I have supposed. A thick copper ring would be best adapted for obtaining results for comparison; and when I have leisure I propose making use of one.

For the purpose of determining the law according to which magnets act upon a copper disk at different distances, I suspended, successively, two copper disks over the bar magnets placed horizontally by the side of each other, with their poles of the same name adjacent. The magnets were made to revolve until the torsion of the wire caused the disk to return in the contrary direction, when I considered that the force of torsion would be double the force with which the magnets urged the disk. The time in which this took place was noted, and also the degree of torsion. After this the magnets were made to revolve again with the same velocity, and the

torsion noted where the disk remained stationary by the action of the opposite forces of torsion and of the magnets. This was done at several distances; and those distances, between the magnets and the disk ascertained very accurately. In the observations with the disk which I have named A, the magnets were made to revolve with two different velocities; one of nearly 12 revolutions per second, the other of nearly 24 revolutions per second; but with the disk C the magnets always revolved with the velocity 24 revolutions per second, as I found that I could keep more steadily to this velocity than to the other. The length of the suspending wire (No. 22) was the same in both cases 34.25 inches. The thickness of the magnets is $\frac{1}{5}$ inch, so that I have added $\frac{1}{10}$ to the measured distances between the upper surface of the magnets and the copper, to reduce them to the distances between the plane of the copper and a horizontal plane passing through the axes of the magnets. The following tables contain the results.

Disk A, *weight* = 1305 *grains.*

Distance.	Screw.				Unscrew.			
	Unscrews.		Arc of torsion = force.		Screws.		Arc of torsion = force.	
	Arc.	Time.	Vel. 12.	Vel. 24.	Arc.	Time.	Vel. 12.	Vel. 24.
	°		°	°	°	m. s.	°	°
0.6	1330	Not obs.	760	1870	1160	1 09	700	1710
1.1	480	1m 10s	270	656	455	1 12	250	604
1.6	275	1 10	118	270	260	1 09	95	236
2.1	135	1 10	60	142	110	1 07	48	120
2.6	80	1 06	44	72	78	1 12	36	56

Disk C, *weight* = 2724 *grains.*

Distance.	Screw.				Unscrew.			
	Unscrews.		Arc of torsion = force.		Screws.		Arc of torsion = force.	
	Arc.	Time.	Vel. 12.	Vel. 24.	Arc.	Time.	Vel. 12.	Vel. 24.
	°	m. s.		°	°	m. s.		°
0.6	7270	1 44		3642	7750	1 43		3874
1.1	3465	1 42		1770	3380	1 40		1680
1.6	1670	1 40		834	1456	1 40		726
2.1	700	1 39		347	680	1 39		354
2.6	308	1 38		184	320	1 39		180

It is evident from these results, that the force with which the magnets urge the disk, as the distance increases, decreases much less rapidly than in the case of the copper plate revolving. If we suppose it to vary as $\frac{1}{\text{dist.}^n}$, then calling c and c' two distances and T and T′ the corresponding torsions, which are equal to the forces of the magnets, $n = \frac{\log. T - \log. T'}{\log. c' - \log. c}$.

Comparing the preceding results, the several values of n will be,

	Disk A.	Disk C.
Values of n	1.723	1.285
	1.995	1.556
	2.087	1.864
	2.271	2.065
	2.436	2.118
	2.429	2.406
	2.658	2.614
	2.420	2.803
	2.831	2.998
	3.354	3.246

These differ too widely from each other for us to suppose that the force varies as any exact power of the distance;

but the approximation is evidently towards the inverse square.

With regard to the forces with which different disks are urged at the same distance, they appear to be very accurately proportional to the weights of the disks when their distances from the magnets are small; but as the distances are increased, the forces appear to increase in a greater ratio than that of the weights of the disks.

Distance	.6	1.1	1.6	2.1	2.6	
$\frac{\text{Torsion}}{\text{Weight}}$ =	1.372	.483	.194	.100	.049	Disk A.
$\frac{\text{Torsion}}{\text{Weight}}$	1.380	.633	.286	.134	.067	Disk B.

As it was only by a rough estimate, that I considered the velocity with which the magnets revolved under the disk A, was double in one case of what it was in the other, I would not, from these observations, pretend to determine the ratio of the forces as depending upon the velocities, but I should have little doubt that they are proportional.

From these experiments it appears, that the time in which the disk begins to return, by the torsion of the wire, is the same at all distances; and from another experiment it appeared to be independent of the velocity of rotation. This ought to be the case, the force accelerating the disk being constant; and the retarding force, the torsion, varying as the distance from a fixed point.

I fear that I have trespassed too long on your time by this account of the experiments which I have made, but had no idea of rendering it so long when I began. I shall be happy if any of these experiments throw any light upon the

subject; and I beg you will make whatever use you think proper of them, and likewise of the account I before sent you.*

I am, dear Sir,

very truly your's,

S. H. CHRISTIE.

* In a former letter, dated May 11: the experiments related in which are embodied in this communication. (H.)

Royal Military Academy,
12th June, 1825.

XXIII. *On the annual variations of some of the principal fixed Stars.* *By* J. Pond, *F.R.S. Astron. Royal.*

Read June 16, 1825.

Whenever any difference of opinion exists on philosophical subjects depending on experiment or observation, it is much more useful simply to state facts, than to reason on them prematurely. Having this principle in view, I am induced to transmit to the Society the annexed small Table, which contains the annual variations of some of the fixed stars, as deduced both from Dr. Brinkley's observations and my own, and by which each may be compared with the annual variations determined by very distant observations, according to the more usual method. Of sixteen stars south of the zenith, observed at Dublin, it will be seen, by the table, that thirteen of them either indicate, or at least are not inconsistent with that irregularity which I have noticed under the name of southern deviation; of these thirteen, about half indicate rather a greater deviation than I have assigned to them, the other half deviate less. The three remaining stars, Castor, α Aquilæ, α Cygni, deviate in a contrary direction. The difference in α Cygni is considerable, and not easily to be accounted for, as this star is one of those most frequently observed at each observatory, and is so near the zenith as not to be easily affected by the uncertainty of astronomical refraction.

I fear the examination of these tables will rather increase than diminish that tendency to scepticism which does and

indeed ought to exist, relative to the determination of such very small quantities by astronomical observation; but I deem it peculiarly incumbent on any one, placed in the situation which I hold, not to be influenced by these considerations: on the contrary, the difficulty and perplexity of the subject should only act as an incentive to contrive more powerful methods of investigation.

Nothing has ever been farther from my intention, than to place this subject in a controversial point of view. It would be worse than useless so to do, since the difficulty will in the space of a very few years in all probability be satisfactorily explained.

		Dr. Brinkley, 1813.	Dr. Brinkley, 1810.	Annual Variation from Dublin Obs. of 1813 and 1810.	Annual Variation from Greenwich Obs. of 1813 and 1823.	Annual Variation from Greenwich Obs. of 1756 and 1813.
1	α Cassiopeiæ.	34.29.22.59	34.27.23.47	— 19.85	19.70	19.85
2	Polaris.					
3	α Arietis.	67.25.36.76	67.23.53.25	— 17.25	17.22	17.40
4	α Ceti.					
5	α Persei.					
6	Aldebaran.	73.52.35.98	73.51.49.22	— 7.79	7.77	7.92
7	Capella.					
8	Rigel.					
9	β Tauri.	61.33.44.22	61.33.21.76	— 3.74	3.72	3.80
10	α Orionis.	82.38. 9.23	82.38.15.94	— 1.12	1.15	1.36
11	Sirius.					
12	Castor.	57.42.47.54	57.43.29.94	+ 7.07	7.22	7.12
13	Procyon.	84.18.15.33	84.19. 8.42	+ 8.85	8.92	8.63
14	Pollux.	61.31.56.07	61.32.44.98	+ 8.15	8.04	8.02
15	α Hydræ.					
16	Regulus.	77. 7.23.06	77. 9. 7.45	+ 17.40	17.28	17.23
17	α Ursæ maj.					
18	β Leonis.	74.22.56.44	74.24.57.91	+ 20.24	20.08	20.04
19	γ Ursæ maj.	35.15.56.22	35.17.55.15	+ 19.82	19.95	19.98
20	Spica Virg.					
21	η Ursæ maj.	39.44.58.37	39.46.47.18	+ 18.13	18.16	18.15
22	Arcturus.	69.50.19.33	69.52.13.66	+ 19.05	19.01	18.97
23	β Ursæ min.					
24	α Cor. Bor.	62.38.55.51	62.40.10.46	+ 12.49	12.51	12.45
25	α Serpentis.	82.58.38.81	82.59.49.73	+ 11.82	11.73	11 72
26	Antares.					
27	α Herculis.					
28	α Ophiuchi.	77.17.40.39	77.17.58.23	+ 3.31	3.16	3.08
29	γ Draconis.	38.29. 3.70	38.29. 7.51	+ 0.635	0.69	0.67
30	α Lyræ.	51.23. 0.84	51.22.42.84	— 3.00	2.94	3.02
31	α Aquilæ.	81.36.59.85	81.36. 5.11	— 9.12	8.93	9.06
32	α Cygni.	45.22.58.30	45 21.42.30	— 12.65	12.47	12.63
33	α Cephei.	28.12.13.90	28.10.42.74	— 15.19	14.99	15.07
34	β Cephei.	20.15.31.41	20.13.57.05	— 15.73	15.66	15.68
35	α Aquarii.	91.13.21.75	91.11.39.40	— 17.06	17.00	17.27
36	α Pegasi.					
37	α Andromed.					

The first and second columns of the above table are taken from two papers of Dr. Brinkley, the one printed in the Irish Transactions, the other in the Philosophical Transactions for 1821.

XXIV. *On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of Life Contingencies. In a Letter to* FRANCIS BAILY, *Esq. F.R.S. &c. By* BENJAMIN GOMPERTZ, *Esq. F. R. S.*

Read June 16, 1825.

DEAR SIR,

THE frequent opportunities I have had of receiving pleasure from your writings and conversation, have induced me to prefer offering to the Royal Society through your medium, this Paper on Life Contingencies, which forms part of a continuation of my original paper on the same subject, published among the valuable papers of the Society, as by passing through your hands it may receive the advantage of your judgment.

I am, Dear Sir, yours with esteem,

BENJAMIN GOMPERTZ.

9th June 1825.

CHAPTER I.

ARTICLE 1. IN continuation of Art. 2. of my paper on the valuation of life contingencies, published in the Philosophical Transactions of this learned Society, in which I observed the near agreement with a geometrical series for a short period of time, which must pervade the series which expresses the number of living at ages in arithmetical progression, pro-

ceeding by small intervals of time, whatever the law of mortality may be, provided the intervals be not greater than certain limits: I now call the reader's attention to a law observable in the tables of mortality, for equal intervals of long periods; and adopting the notation of my former paper, considering L_x to express the number of living at the age x, and using λ for the characteristic of the common logarithm; that is, denoting by $\lambda(L_x)$ the common logarithm of the number of persons living at the age of x, whatever x may be, I observe that if $\lambda(L_n) - \lambda(L_{n+m})$, $\lambda(L_{n+m}) - \lambda(L_{n+2m})$ $\lambda(L_{n+2m}) - \lambda(L_{n+3m})$, &c. be all the same; that is to say, if the differences of the logarithms of the living at the ages n, $n+m$; $n+m$, $n+2m$; $n+2m$, $n+3m$; &c. be constant, then will the numbers of living corresponding to those ages form a geometrical progression; this being the fundamental principle of logarithms.

Art. 2. This law of geometrical progression pervades, in an approximate degree, large portions of different tables of mortality; during which portions the number of persons living at a series of ages in arithmetical progression, will be nearly in geometrical progression; thus, if we refer to the mortality of DEPARCIEUX, in Mr. BAILY's life annuities, we shall have the logarithm of the living at the ages 15, 25, 35, 45, and 55 respectively, 2,9285; 2,88874; 2,84136; 2,79379; 2.72099, for $\lambda(L_{15})$; $\lambda(L_{25})$; $\lambda(L_{35})$; &c. and we find $\lambda(L_{25}) - \lambda(L_{35}) =$,04738 $\lambda(L_{35}) - \lambda(L_{45}) =$,04757, and consequently these being nearly equal (and considering that for small portions of time the geometrical progression takes place very nearly) we observe that in those tables the numbers of

living in each yearly increase of age are from 25 to 45 nearly, in geometrical progression. If we refer to Mr. MILNE's table of Carlisle, we shall find that according to that table of mortality, the number of living at each successive year, from 92 up to 99, forms very nearly a geometrical progression, whose common ratio is $\frac{3}{4}$; thus setting out with 75 for the number of living at 92, and diminishing continually by $\frac{1}{4}$, we have to the nearest integer 75, 56, 42, 32, 24, 18, 13, 10, for the living at the respective ages 92, 93, 94, 95, 96, 97, 98, 99, which in no part differs from the table by $\frac{1}{37}$th part of the living at 92.

Art. 3. The near approximation in old age, according to some tables of mortality, leads to an observation, that if the law of mortality were accurately such that after a certain age the number of living corresponding to ages increasing in arithmetical progression, decreased in geometrical progression, it would follow that life annuities, for all ages beyond that period, were of equal value; for if the ratio of the number of persons living from one year to the other be constantly the same, the chance of a person at any proposed age living to a given number of years would be the same, whatever that age might be; and therefore the present worth of all the payments would be independent of the age, if the annuity were for the whole life; but according to the mode of calculating tables from a limited number of persons at the commencement of the term, and only retaining integer numbers, a limit is necessarily placed to the tabular, or indicative possibility of life; and the consequence may be, that the value of life annuities for old age, especially where they are

deferred, should be deemed incorrect, though indeed for immediate annuities, where the probability of death is very great, the limit of the table would not be of so much consequence, for the present value of the first payment would be nearly the value of the annuity.

Such a law of mortality would indeed make it appear that there was no positive limit to a person's age; but it would be easy, even in the case of the hypothesis, to show that a very limited age might be assumed to which it would be extremely improbable that any one should have been known to attain.

For if the mortality were, from the age of 92, such that $\frac{1}{4}$ of the persons living at the commencement of each year were to die during that year, which I have observed is nearly the mortality given in the Carlisle tables between the ages 92 and 99,* it would be above one million to one that out of three millions of persons, whom history might name to have reached the age of 92, not one would have attained to the age of 192, notwithstanding the value of life annuities of all ages above 92 would be of the same value. And though the limit to the possible duration of life is a subject not likely ever to be determined, even should it exist, still it appears interesting to dwell on a consequence which would follow, should the mortality of old age be as above described. For, it would follow that the non-appearance on the page of history of a single circumstance of a person having arrived

* If from the Northampton tables we take the numbers of living at the age of 88 to be 83, and diminish continually by $\frac{1}{4}$ for the living, at each successive age, we should have at the ages 88, 89, 90, 91, 92, the number of living 83; 61.3; 45.9; 34.4; 25.8; almost the same as in the Northampton table.

at a certain limited age, would not be the least proof of a limit of the age of man; and further, that neither profane history nor modern experience could contradict the possibility of the great age of the patriarchs of the scripture. And that if any argument can be adduced to prove the necessary termination of life, it does not appear likely that the materials for such can in strict logic be gathered from the relation of history, not even should we be enabled to prove (which is extremely likely to be the state of nature) that beyond a certain period the life of man is continually becoming worse.

Art. 4. It is possible that death may be the consequence of two generally co-existing causes; the one, chance, without previous disposition to death or deterioration; the other, a deterioration, or an increased inability to withstand destruction. If, for instance, there be a number of diseases to which the young and old were equally liable, and likewise which should be equally destructive whether the patient be young or old, it is evident that the deaths among the young and old by such diseases would be exactly in proportion of the number of young to the old; provided those numbers were sufficiently great for chance to have its play; and the intensity of mortality might then be said to be constant; and were there no other diseases but such as those, life of all ages would be of equal value, and the number of living and dying from a certain number living at a given earlier age, would decrease in geometrical progression, as the age increased by equal intervals of time; but if mankind be continually gaining seeds of indisposition, or in other words, an increased liability to death (which appears not to be an unlikely supposition with respect to a great part of life, though

the contrary appears to take place at certain periods) it would follow that the number of living out of a given number of persons at a given age, at equal successive increments of age, would decrease in a greater ratio than the geometrical progression, and then the chances against the knowledge of any one having arrived to certain defined terms of old age might increase in a much faster progression, notwithstanding there might still be no limit to the age of man.

Art. 5. If the average exhaustions of a man's power to avoid death were such that at the end of equal infinitely small intervals of time, he lost equal portions of his remaining power to oppose destruction which he had at the commencement of those intervals, then at the age x his power to avoid death, or the intensity of his mortality might be denoted by aq^x, a and q being constant quantities; and if L_x be the number of living at the age x, we shall have $a L_x \times \overset{x}{q}.\dot{x}$ for the fluxion of the number of deaths $= -(L_x)^{\cdot}$; $\therefore abq^x = -\frac{\dot{L}_x}{L_x}$, $\therefore abq^x = -$ hyp. log. of $b \times$ hyp. log. of L_x, and putting the common logarithm of $\frac{1}{b} \times$ square of the hyperbolic logarithm of $10 = c$, we have $c.q^x =$ common logarithm of $\frac{L_x}{d}$; d being a constant quantity, and therefore L_x or the number of persons living at the age of $x = d.\overline{g|}^{q^x}$; g being put for the number whose common logarithm is c. The reader should be aware that I mean $\overline{g|}^{q^x}$ to represent g raised to the power q^x and not g^q raised to the x power; which latter I should have expressed by $\overline{g^q|}^x$, and which would evidently be equal to g^{qx}. I take this opportunity to make this observation, as algebraists are sometimes not sufficiently precise in their notation of exponentials.

This equation between the number of the living, and the age, becomes deserving of attention, not in consequence of its hypothetical deduction, which in fact is congruous with many natural effects, as for instance, the exhaustions of the receiver of an air pump by strokes repeated at equal intervals of time, but it is deserving of attention, because it appears corroborated during a long portion of life by experience; as I derive the same equation from various published tables of mortality during a long period of man's life, which experience therefore proves that the hypothesis approximates to the law of mortality during the same portion of life; and in fact the hypothesis itself was derived from an analysis of the experience here alluded to.

Art. 6. But previously to the interpolating the law of mortality from tables of experience, I will premise that if, according to our notation, the number of living at the age x be denoted by L_x, and λ be the characteristic of a logarithm, or such that $\lambda(L_x)$ may denote the logarithm of that number, that if $\lambda(L_a) - \lambda(L_{a+r}) = m$, $\lambda(L_{a+r}) - \lambda(L_{a+2r}) = mp$, $\lambda(L_{a+2r}) - \lambda(L_{a+3r}) = m^2p$; and generally $\lambda(L_{a+\overline{n-r}}) - \lambda(L_{a+n}) = m \cdot p^{\frac{n}{r}-1}$; that by continual addition we shall have $\lambda(L_a) - \lambda(L_{a+n}) = m(1+p+p^2+p^3+\ldots p^{\frac{n}{r}-1}) = m \cdot \frac{1-p^{\frac{n}{r}}}{1-p}$; and therefore if $p^{\frac{1}{r}} = q$, and ε be put equal to the number whose common logarithm is $\frac{m}{1-q^n}$, we shall have $\lambda(L_{a+n}) = \lambda(L_a) - \lambda(\varepsilon) \times (1-q^n) = \lambda\left(\frac{L_a}{\varepsilon}\right) + \lambda(\varepsilon) . q^n$; $\therefore L_{a+n} = \frac{L_a}{\varepsilon} \times \varepsilon^{q^n}$; and this equation, if for $a+n$ we write x, will give $L_x = \frac{L_a}{\varepsilon} . \varepsilon^{q^{-a} \times q^x}$; and consequently if $\frac{L_a}{\varepsilon}$ be put

$= d$, and $\overline{\varepsilon|}^{q^{-a}} = g$, the equation will stand $L_x = d \,.\, \overline{g|}^{q^x}$, and $\lambda(g) = \lambda(\varepsilon) \times q^{-a} = \frac{m\,q^{-a}}{1-q^r}$; and I observe that when q is affirmative, and $\lambda(\varepsilon)$ negative, that $\lambda(g)$ is negative. The equation $L_x = d \,.\, \overline{g|}^{q^x}$ may be written in general $\lambda(L_x) = \lambda(d) \pm$ the positive number whose common logarithm is $\{\lambda^2(g) + x\,\lambda(g)\}$, the upper or under sign to be taken according as the logarithm of g is positive or negative, λ^2 standing for the characteristic of a second logarithm; that is, the logarithm of a logarithm, $\lambda(q) = \frac{1}{r} \times \lambda(p)$, $\lambda^2(g) = \lambda^2(\varepsilon) - a.\lambda(q) = \lambda\left(\frac{m}{1-p}\right) - a\,.\,\lambda(q) = \lambda(m) - \lambda(1-p) - a\,\lambda(q)$; also $\lambda(d) = \lambda(L_a) - \frac{m}{1-p}$.

Art. 7. Applying this to the interpolation of the Northampton table, I observe that taking $a = 15$ and $r = 10$ from that table, I find $\lambda(L_a) - \lambda(L_{a+r}) = ,0566 = m$, $\lambda(L_{a+r}) - \lambda(L_{a+2r}) = ,0745$, $\lambda(L_{a+2r}) - \lambda(L_{a+3r}) = ,0915$, and $\lambda(L_{a+3r}) - \lambda(L_{a+4r}) = ,1228$; now if these numbers were in geometrical progression, whose ratio is p, we should have respectively $m = ,0566$; $mp = ,0745$; $mp^2 = ,0915$; $mp^3 = ,1228$. No value of p can be assumed which will make these equations accurately true; but the numbers are such that p may be assumed, so that the equation shall be nearly true; for resuming the first and last equations we have $p^3 = \frac{1228}{566}$; $\therefore$ logarithm of $p = \frac{1}{3}$(logarithm of 1228 — logarithm of 566) $= ,11213$, $\therefore \lambda(q) = ,011213$ and $p = 1,2944$. And to examine how near this is to the thing required, continually to the logarithm of ,0566 namely $\bar{2},75282$, adding ,11213 which is the logarithm of p, we have respectively for the

logarithms of mp, of mp^2, of mp^3 the values $\bar{2},8649$, $\bar{2},9771$, $\bar{1},0892$; the numbers corresponding to which are ,07327; ,09486; ,1228; and consequently m, mp, mp^2, and mp^3 respectively equal to ,0566; ,07327; ,09486, and ,1228 which do not differ much from the proposed series ,0566; ,07327; ,09486, and ,1228; and according to our form for interpolation, taking $m = ,0566$ and $p = 1,2944$; we have $\frac{m}{1-p} = -\frac{,0566}{,2944} = -,1922$; and $\lambda(L_{15})$ agreeably to the Northampton tables, being $= 3,7342$ we have $\lambda(d) = 3,7342 + ,1922 = 3,9264$, $d = 8441$, $\lambda^2(q)$, that is to say, the logarithm of the logarithm of $q = \lambda\left(\frac{m}{1-p}\right) - a\,\lambda(q) = \bar{1},28375 - ,16819 = \bar{1},1156$, $\lambda(g) = -\,,130949 = \bar{1},8695$, the negative sign being taken because $\lambda(g) = \lambda(\epsilon) \times q^{-a} = \frac{m}{1-q} \cdot q^{-a}$, and $g = ,7404$. And therefore x being taken between the limits, we are to examine the degree of proximity of the equation $L_x = 8441 \times \overline{7404}|^{1,0261^x}$ or $\lambda(L_x)$, that is, the logarithm of the number of living at the age $x = 3,9264$ — number whose logarithm is $(\bar{1},11556 + x \times .011213)$, as the logarithm of g is negative. The table constructed according to this formula, which I shall lay before the reader, will enable him to judge of the proximity it has to the Northampton table; but previously thereto shall show that the same formula, with different constants, will serve for the interpolations of other tables.

Art. 8. To this end let it be required to interpolate DEPARCIEUX's tables, in Mr. BAILY's life annuities, between the ages 15 and 55.

The logarithms of the living at the age of

15 are	2,92840	differences $=$,03966	$=\lambda(L_{15})-\lambda(L_{25})$
25	2,88874	,04738	$=\lambda(L_{25})-\lambda(L_{35})$
35	2,84136	,04757	$=\lambda(L_{35})-\lambda(L_{45})$
45	2,79379	,07280	$=\lambda(L_{45})-\lambda(L_{55})$
55	2,72099		

Here the three first differences, instead of being nearly in geometrical progression are nearly equal to each other, showing from a remark above, that the living, according to these tables, are nearly in geometrical progression; and the reader might probably infer that this table will not admit of being expressed by a formula similar to that by which the Northampton table has been expressed between the same limits, but putting,

on the supposition of the possibility, though the thing cannot be accurately true,

$$\left\{\begin{array}{ll} \lambda(L_{15})= \quad . \quad . \quad . \quad . \quad . \quad . & =2{,}92840 \\ \lambda(L_{25})=\lambda(L_{15})-m \quad . \quad . \quad . & =2{,}88874 \\ \lambda(L_{35})=\lambda(L_{15})-m-mp \quad . \quad . & =2{,}84136 \\ \lambda(L_{45})=\lambda(L_{15})-m-mp-mp^2 \quad . & =2{,}79379 \\ \lambda(L_{55})=\lambda(L_{15})-m-mp-mp^2-mp^3 & =2{,}72099 \end{array}\right\}$$

and we shall have

$\lambda(L_{15})-\lambda(L_{35})$ or its equal $m+mp=,08704$, and $\lambda(L_{35})-\lambda(L_{55})$ or its equal $p^2\times\overline{m+pm}=,12037$; $\therefore p^2=\frac{12037}{8704}$ and the log. of $p=\frac{\text{log. of } 12037 - \text{log. of } 8704}{2}=,0703997$ and $p=1,176$, $m=\frac{,08704}{1+p}=\frac{,08704}{2,176}=,04$. And to see how these values of m and p will answer for the approximate determination of the logarithms above set down of the numbers of living at the ages 15, 25, 35, 45, and 55, we have the following easy calculation by continually adding the logarithm of p

Logarithm of $m = \bar{2},6020600$
Log. of $p = 0,0703997$ therefore $mp = ,047039$
Log. of $mp = \bar{2},6724597$ $mp^2 = ,055317$
Log. of $mp^2 = \bar{2},7428594$ $mp^3 = ,065051$
Log. of $mp^3 = \bar{2},8128591$

$$\begin{array}{r} \lambda\left(\mathrm{L}_{15}\right) = 2,92840 \\ -m = -\ ,04 \\ \hline \lambda\left(\mathrm{L}_{25}\right) = 2,88840 \\ -mp = -\ ,04704 \\ \hline 2,84136 \\ -mp^2 = -\ ,05532 \\ \hline 2,78604 \\ -mp^3 = -\ ,06505 \\ \hline 2,72099 \end{array}$$

These logarithms of the approximate number of living at the ages 15, 25, 35, 45 and 55, are extremely near those proposed, and the numbers corresponding to these give the number of living at the ages 15, 25, 35, 45 and 55, respectively, 848; 773,4; 694; 612,3; and 526; differing very little from the table in Mr. BAILY's life annuities; namely, 848; 774; 694; 622 and 526. And we have $a = 15$, $r = 10$, $m = ,04$; $\lambda(m) = \bar{2},60206$; $1 - p = -,176$; $\lambda q = \frac{1}{10}\lambda(p) = ,00703997$; $\lambda(g) = \frac{m q^{-a}}{1-p} = -\frac{,04 \times q^{-a}}{,176}$, and is negative; $\lambda\lambda(g) = \lambda(,04) - 15 \times ,00704 - \lambda(,176) = \bar{1},25095$; $\lambda(d) = \lambda(\mathrm{L}_a) - \frac{m}{1-p} = 2,9284 + ,22727 = 3,1557$; $\therefore \lambda(\mathrm{L}_x) = 3,1557 -$ number whose log. is $(\bar{1},25095 + ,00704\,x)$, for the logarithm of living in DEPARCIEUX' table in Mr. BAILY's annuities, between the limits of age 15 and 55. The table which we shall insert will afford an opportunity of appreciating the proximity of this formula to the table.

Art. 9. To interpolate the Swedish mortality among males between the ages of 10 and 50, from the table in Mr. BAILY's annuities:

Here $\lambda(L_{10}) = 3,779091$

$\lambda(L_{20}) = 3,746868$ to be assumed $= \lambda(L_{10}) - m$

$\lambda(L_{30}) = 3,703205 \quad . \quad . \quad = \lambda(L_{10}) - m - mp$

$\lambda(L_{40}) = 3,648165 \quad . \quad . \quad = \lambda(L_{10}) - m - mp - mp^2$

$\lambda(L_{50}) = 3,564192 \quad . \quad . \quad = \lambda(L_{10}) - m - mp - mp^2 - mp^3$

Consequently $m + mp = \lambda(L_{10}) - \lambda(L_{30}) = ,075886$, and $\lambda(L_{30}) - \lambda(L_{50}) = p^2 \times \overline{m + mp} = ,139013$; therefore $p^2 = \frac{139013}{75886}$, and $\lambda(p) = ,1314468$; $\therefore p = 1,3535$; $m = \frac{,075886}{1+p} = \frac{,075886}{2,3535}$; $\lambda(m) = \bar{2},5084775$; $m = ,032244$; $a = 10$; $r = 10$; $\lambda(q) = ,01314468$; $\lambda g = \frac{m \cdot q^{-10}}{-,3535}$, negative; $\lambda\lambda(g) = \lambda(m) - 10\,\lambda(q) - \lambda(,3535) = \bar{2},82861$; $\lambda(d) = \lambda L_a - \frac{m}{1-p} = 3,779091 + 091218, = 3,8703$; consequently this will give between the ages 10 and 50 of Swedish males,

$\lambda(L_x)$ or the logarithm of the living at the age of $x = 3,8703$ — number, whose logarithm is $(\bar{2},82861 + ,013145\,x)$.

A table will also follow to show the proximity of this with Mr. Baily's table.

Art. 10. For Mr. Milne's table of the Carlisle mortality we have, as given by that ingenious gentleman,

$$\lambda(L_{10}) = 3,81023$$
$$\lambda(L_{20}) = 3,78462$$
$$\lambda(L_{30}) = 3,75143$$
$$\lambda(L_{40}) = 3,70544$$
$$\lambda(L_{50}) = 3,64316$$
$$\lambda(L_{60}) = 3,56146$$

And the difference of these will form a series nearly in geometrical progression, whose common ratio is $\frac{4}{3}$, and in consequence of this, the first method may be adopted for the

interpolations. Thus because $\lambda\left(L_{10}\right)-\left(L_{20}\right)=,02561$, the first term of the differences, and $\lambda\left(L_{50}\right)-\lambda\left(L_{60}\right)=,0817$, the fifth term of the differences: take the common ratio $=\left.\frac{817}{256}\right|^{\frac{1}{4}}$, and $m=,0256$; $\therefore \lambda(m)=\bar{2},40824$. These will give $\lambda(p)=,126$; $p=1,3365$; $a=10$, $r=10$, $\lambda(q)=,0126$, $\lambda(\varepsilon)=\frac{m}{1-p}=-\frac{,0256}{,3365}$; $\therefore \lambda(g)$ negative; $\lambda\lambda g=2,40824-\lambda(,3365)-,126=\bar{2},75526$; and $\lambda(d)=\lambda\left(L_{10}\right)+\frac{,0256}{,3365}=3,88631$, and accordingly, to interpolate the Carlisle table of mortality for the ages between 10 and 60, we have for any age x,

$$\lambda\left(L_{x}\right)=3,88631-\text{number whose logarithm is }(\bar{2},88126+,0126\,x).$$

Here we have formed a theorem for a larger portion of time than we had previously done. If by the second method the theorem should be required from the data of a larger portion of life, we must take r accordingly larger; thus if a be taken 10, $r=12$, then the interpolation would be formed from an extent of life from 10 to 58 years; and referring to Mr. MILNE's tables, our second method would give $\lambda\left(L_{x}\right)=3,89063$ — the number whose logarithm is $(\bar{2},784336+,0120948\,x)$; this differs a little from the other, which ought to be expected.

If the portion between 60 and 100 years of Mr. MILNE's Carlisle table be required to be interpolated by our second method, we shall find $p=1,86466$; $\lambda(m)=\bar{1},30812$; $m=,20329$, &c. and we shall have $\lambda\left(L_{x}\right)=3,79657$ — the number whose logarithm is $(\bar{3},74767+,02706\,x)$.

This last theorem will give the numbers corresponding to the living at 60, 80, and 100, the same as in the table; but for the ages 70 and 90, they will differ by about one year:

the result for the age of 70 agreeing nearly with the living corresponding to the age 71; and the result for the age 90, agreeing nearly with the living at the age 89 of the Carlisle tables.

Art. 11. Lemma. If according to a certain table of mortality, out of a, persons of the age of 10, there will arrive b, c, d, &c. to the age 20, 30, 40, &c.; and if according to the tables of mortality, gathered from the experience of a particular society, the decrements of life between the intervals 10 and 20, 20 and 30, 30 and 40, &c. is to the decrements in the aforesaid table between the same ages, proportioned to the number of living at the commencement of those intervals respectively, as 1 to n, 1 to n', 1 to n'', &c. it is required to construct a table of mortality of that society, or such as will give the above data.

Solution. According to the first table, the decrements of life from 10 to 20, 20 to 30, 30 to 40, &c. respectively, will be found by multiplying the number of living at the commencement of each period by $\frac{a-b}{a}$, $\frac{b-c}{b}$, $\frac{c-d}{c}$, &c., and therefore, in the Society proposed, the corresponding decrements will be found by multiplying the number of living at those ages by $\frac{a-b}{a}n$; $\frac{b-c}{b}n'$; $\frac{c-d}{c}n''$ &c.; and the number of persons who will arrive at the ages 20, 30, 40, &c. will be the numbers respectively living at the ages 10, 20, 30, &c. multiplied respectively by $\frac{\overline{1-n}.a+nb}{a}$, $\frac{\overline{1-n'}.b+n'c}{b}$, $\frac{\overline{1-n''}.c+n''d}{c}$, &c.; hence out of the number a, living at the age 10, there will arrive at the age 10, 20, 30, 40, 50, &c. the numbers $\overline{1-n}.a+nb$; $\overline{\overline{1-n}.a+nb}\times\frac{\overline{1-n'}.b+nc}{b}$; $\overline{\overline{1-n}.a+nb}\times\frac{\overline{1-n'}.b+nc}{b}\times\frac{\overline{1-n''}.c+n''.d}{c}$; &c. and the numbers for the intermediate ages must be found by interpolation.

In the ingenious Mr. MORGAN's sixth edition of PRICE's Annuities, p. 183, vol. i. it is stated, that in the Equitable Assurance Society, the deaths have differed from the Northampton tables; and that from 10 to 20, 20 to 30, 30 to 40, 40 to 50, 50 to 60, and 60 to 80, it appears that the deaths in the Northampton tables were in proportion to the deaths which would be given by the experience of that society respectively, in the ratios of 2 to 1; 2 to 1; 5 to 3; 7 to 5, and 5 to 4. According to this, the decrements in 10 years of those now living at the ages 10, 20, 30, and 40, will be the number living at those ages multiplied respectively by ,0478; ,0730; ,1024; ,1284; and the deaths in twenty years of those now living at the age of 60, would be the number of those living multiplied by ,3163. And also, taking, according to the Northampton table, the living at the age of 10 years equal to 5675, I form a table for the number of persons living at

the ages . .	10	20	30	40	50	60	70	80
being . . .	5675	5403,5	5010	4496	3919	3116	*	197
and the log. of the number of persons living	3,75612	3,73268	3,69984	3,65283	3,59318	3,49360	*	

Consequently, if $a = 20$, $r = 10$, we have $\lambda\left(L_{20}\right) = 3{,}73268$; $\lambda\left(L_{40}\right) = \lambda\left(L_{20}\right) - m - mp = 3{,}65283$; $\lambda\left(L_{60}\right) = L_{20} - m - mp - mp^2 - mp^3 = 3{,}49360$; $m.\overline{1+p} = {,}07985$; and $mp^2 \times \overline{1+p} = 3{,}65283 - 3{,}49360 = {,}15923$; hence $\lambda(p) = \frac{1}{2}\lambda\left(\frac{,15923}{,07985}\right) = {,}149875$; and $p = 1{,}412131$; $\lambda(m) = \lambda({,}07985) - \lambda(2{,}41243) = \bar{2}{,}519874$; and $m = {,}033013$; $\therefore \lambda(\varepsilon) = \frac{-m}{,412131}$ negative; $\therefore \lambda(g)$ is negative; $\lambda\lambda(g) = \lambda m - \lambda{,}412131 - {,}0149875 \times 20 = \bar{2}{,}6051$; $\lambda(d) = \lambda\left(L_{20}\right) - \lambda(\varepsilon) = 3.73268 - {,}080302 = 3{,}813$ sufficiently near; and our formula for the

mortality between the : ;es of 20 and 60, which appears to me to be the experience of the Equitable Society, is $\lambda\left(L_x\right) = 3{,}813$ — the number whose log. is $(\bar{2}.6051 + {,}0149875\,x)$.

This formula will give

At the ages	10	20	30	40	50	60	70	80
No. of living	5703,2	5403,5	5007	4496	3862	3116	*	1500
Differs from the proposed by	28,2	0	+ 3	0	− 57	0	*	303

In the table of Art. 12, the column marked 1, represents the age; column marked 2, represents the number of persons living at the corresponding age; column marked 3, the error to be added to the number of living deduced from the formula, to give the number of living of the table for which the formula is constructed; column marked 4, gives the error in age, or the quantity to be added to the age in column 1, that would give the number of living in the original table, the same as in column 2. It may be proper to observe, that where the error in column 3 and 4 is stated to be 0, it is not meant to indicate that a perfect coincidence takes place, but that the difference is too small to be worth noticing.

Art. 12. $\lambda\left(\underset{x}{L}\right) = \lambda(d) -$ number whose logarithm is $(\lambda^2(g) + x\,\lambda\, q)$.

	Northampton.			Deparcieux.			Sweden.			Carlisle.			Formula of supposed experience of the Equitable, Compared with supposed exp.		Compared with Carlisle.			
1	2	3	4	2	3	4	2	3	4	2	3	4	2	3	2	3	4	1
10							6013	0		6460	0	0	5703	−28	6460			
11							5974	−16	− 1/3	6427	+ 4	+ 1/8	5677	not compared.	6431			
12							5935	−22	− ½	6393	+ 7	+ ¼	5650		6400			
13							5894	−26	− 2/3	6358	+10	+ 1/3	5622		6368½	− ½	− 1/16	
14							5852½	−24½	− 3/8	6322	+13	+ 1/3	5594		6336	− 1	− 1/35	14
15	5423			848	0	0	5810	−22½	− 3/8	6286	+14	+ 1/3	5564		6302	− 2	− 1/21	15
16	5360	+13	+ ¼	841	+ 1	+ 1/6	5767	−18	− 1/5	6248	+13	+ 4/13	5534		6268½	− 7½	− 1/6	16
17	5297	+23	+ ½	833½	+1½	+ ¼	5722	−12	− 1/3	6210	+ 9	+ 1/5	5503		6233	−14	− 1/3	17
18	5233	+29	+ ½	826	+ 2	+ 1/3	5677	− 6	− 1/7	6171	+ 5	+ 1/9	5470		6196½	−20½	− ½	18
19	5168	+31	+ ½	819	+ 2	+ 1/3	5630	− 3	− 1/14	6131	+ 2	+ 1/21	5437		6159	−26	− 3/5	19
20	5102	+30	+ ½	811	+ 3	+ 1/3	5583	− 0	0	6090	0	0	5403	0	6120½	−30½	− 5/7	20
21	5036	+24	+ 1/3	804	+ 2	+ ¼	5534	− 1	0	6048	− 1	0	5368	not compared.	6081	−34	− 5/6	21
22	4969	+16	+ 1/5	796	+ 2	+ ¼	5484	− 1	0	6005	+ 0	0	5333		6040½	−35½	− 5/6	22
23	4899	+11	+ 1/7	789	+ 1	+ 1/8	5434	− 1	0	5962	+ 1	0	5295		5998½	−35½	− 5/6	23
24	4830	+ 5	+ 1/13	781	+ 1	+ 1/8	5382	− 4	− 1/13	5917	+ 4	+ 1/10	5258		5955½	−34½	− 5/6	24
25	4762	− 2	− 1/37	774	+ 0	0	5329	− 6	− 1/9	5870½	+ 8½	+ 1/5	5218		5911	−32	− 5/7	25
26	4689	− 4	− 1/19	766	+ 0	0	5275	− 7	− 1/8	5825	+11	+ ¼	5178		5866	−30	− 5/7	26
27	4616	− 6	− 1/13	757	+ 1	+ 1/8	5220	− 7	− 1/8	5777	+16	+ 5/14	5137		5819	−26	− 5/9	27
28	4545	−10	− 1/7,5	750	+ 0	0	5164	− 6	− 1/9	5729	+19	+ 3/8	5095		5771	−23	− ½	28
29	4472	−12	− 1/6	742	+ 0	0	5107	− 4	− 1/14	5679	+19	+ 1/3	5051		5722	−24	− 4/9	29
30	4403	−22	− 1/3	734	+ 0	0	5049	− 0	0	5628½	+13½	+ ¼	5007	− 3	5671½	−29½	− 4/7	30
31	4325	−15	− 1/5	726	+ 0	0	4989½	− 1½	− 1/37	5578	+ 7	+ 1/8	4961	not compared.	5620	−34	− 3/5	31
32	4250	−15	− 1/5	718	+ 0	0	4929	− 1	0	5524	+ 4	+ 1/14	4914		5567	−39	− 5/7	32
33	4174	−14	− 1/5	710	+ 0	0	4868	− 0	0	5470½	+ 1½	+ 1/37	4866		5512½	−42½	− ¾	33
34	4098	−13	− 1/5	702	+ 0	0	4805	+ 3	+ 1/20	5416	+ 1	0	4817		5456½	−39½	− 5/7	34
35	4021	−11	− 1/7	694	+ 0	0	4741	+ 7	+ 1/9	5360	+ 2	+ 1/27	4767		5399	−37	− ½	35
36	3944	−11	− 1/7	686	+ 0	0	4676	+12	+ 1/5	5303	+ 4	+ 1/14	4715		5341	−34	− 3/5	36
37	3866	− 6	− 1/12	678	+ 0	0	4611	+17	+ ¼	5245½	+ 5½	+ 1/10	4662		5281	−30	− 4/7	37
38	3788	− 3	− 1/27	669	+ 2	+ ¼	4544	+24	+ 2/5	5187	+ 7	+ 1/8	4608		5219½	−25½	− 4/9	38
39	3709	+ 1	+ 1/75	661	+ 3	+ 1/5	4476	+28	+ ½	5127	+ 9	+ 1/6	4553		5157	−21	− 1/3	39
40	3630	+ 5	+ 1/15	653	+ 4	+ ½	4407	+41	+ 2/3	5065	+10	+ 1/6	4496	0	5093	−18	− ¼	40
41	3551	+ 8	+ 1/9	645	+ 5	+ ½	4337	+46	+ 2/3	5004	+ 5	+ 1/13	4438	not compared.	5027	−18	− ¼	41
42	3473	+11	+ 1/7	636	+ 7	+ 1	4266	+45	+ 5/9	4941	− 1	0	4379		5960	−20	− 2/7	42
43	3392	+12	+ 1/6	628	+ 8	+ 1	4194	+37	+ 4/9	4877	− 8	− 1/9	4319		4892	−23	− 1/3	43
44	3312	+14	+ 1/6	619	+10	+ 1¼	4121	+30	+ 3/8	4812	−14	− 1/5	4257		4882	−24	− 1/3	44
45	3235	+13	+ 1/6	611	+11	+ 1⅜	4047	+24	+ 3/10	4746	−19	− ¼	4194		4751	−24	− 1/3	45
46	3152	+18	+ ¼	603	+12	+ 1½	3973	+18	+ 2/9	4678	−21	− 3/10	4130		4678½	−21½	− 1/3	46
47	3072	+20	+ ¼	594	+13	+ 1½	3897	+14	+ 1/6	4610	−22	− 3/10	4065		4604½	−16½	− ¼	47
48	2991	+23	+ 1/3	586	+13	+ 1½	3821	+10	+ 1/8	4541	−20	− 3/10	3998		4529	− 8	− 1/8	48
49	2911	+25	+ 1/3	577	+13	+ 1½	3744	+ 7	+ 1/12	4471	−13	− 1/5	3931		4452½	+ 5½	+ 1/11	49
50	2831	+26	+ 1/3	569	+12	+ 1⅓	3660	0	0	4400½	− 3½	− 1/17	3862	+57	4375	+22	+ 1/3	50
51	2752	+24	+ 1/3	560	+11	+ 1½	3587	−16	− 1/6	4329	+ 9	+ 1/7	3793	not compared.	4296½	+31½	+ ½	51
52	2672	+22	+ ¼	552	+ 8	+ 1	3508	−32	− 1/3	4256	+20	+ 1/3	3721		4215	+61	+ 1	52
53	2593	+19	+ ¼	543	+ 6	+ ¾	3428	−47	− ½	4182	+29	+ 3/7	3649		4133	+78	1 1/7	53
54	2514	+16	+ 1/5	536	+ 2	+ ¼	3348	−62	− 2/3	4108	+35	+ ½	3575		4050	+93	1 ¼	54
55	2436	+12	+ 1/7	526	+ 0	0				4033	+40	+ 6/11	3501		3966	+107	1 ½	55
56	2358	+ 8	+ 1/10							3957	+43	+ 4/7	3426		3881	+219		56
57	2280	+ 4	+ 1/20							3880	+44	+ ½	3350		3794½	not compared.	not compared.	57
58	2206	− 4	− 1/20							3803	+39	+ 3/7	3273		3706			58
59	2123	− 3	− 1/30							3724	+25	+ ¼	3195		3169			59
60	2052	−14	− 1/5							3646	− 3	0	3116	0	3529			60
	$\lambda(d) = 3{,}9265$; $\lambda^2(g) = \bar{1}{,}11556$; $\lambda(q) = {,}011213$			$\lambda(d) = 3{,}1557$; $\lambda^2(g) = \bar{1}{,}25095$; $\lambda(q) = {,}00704$			$\lambda(d) = 3{,}8703$; $\lambda^2(g) = \bar{2}{,}82861$; $\lambda(q) = {,}0131457$			$\lambda(d) = 3{,}89063$; $\lambda^2(g) = \bar{2}{,}784336$; $\lambda(q) = {,}0120948$			$\lambda(d) = 3{,}813$; $\lambda^2(g) = \bar{2}{,}6051$; $\lambda(q) = {,}0149875$		$\lambda(d) = 3{,}86743$; $\lambda^2(g) = \bar{2}{,}6051$; $\lambda(q) = {,}0149875$			

CHAPTER II.

ARTICLE 1. The near proximity to the geometrical progression of the series expressing the number of persons living at equal small successive intervals of time during short periods, out of a given number of persons living at the commencement of those intervals, affords a very convenient mode of calculating values connected with life contingencies, for short limited periods; by offering a manner of forming general tables, applicable (by means of small auxiliary tables of the particular mortalities) to calculations for any particular mortality; and by easy repetition, to calculate the values for any length of period for any table of mortality we please.

If, for instance, it were required to find the value of an annuity of an unit for p years, on three lives of the age b, c, d, the rate of interest being such that the present value of an unit to be received at the expiration of one year, be equal to r, then the value of the first payment would be $\frac{L_{b+1}}{L_b} \times \frac{L_{c+1}}{L_c} \times \frac{L_{d+1}}{L_d} \times r$; and of the p^{th} payment the present value would be $\frac{L_{b+p}}{L_b} \times \frac{L_{c+p}}{L_c} \times \frac{L_{d+p}}{L_d} \times r^p$; but if $L_{b+p} = L_b \times \left(\frac{L_{b+1}}{L_b}\right)^p$ whether p be 1, 2, 3, &c. which will be the case when L_b, L_{b+1}, L_{b+2}, &c. form a geometrical progression, and similarly, if $L_{c+p} = L_c \times \left(\frac{L_{c+1}}{L_c}\right)^p$, and also, $L_{d+p} = L_d \times \left(\frac{L_{d+1}}{L_d}\right)^p$, the pre-

sent value of the p^{th} payment will be $\left(\frac{L_{1\,:\,b,\,c,\,d}}{L_{b,\,c,\,d}}\,r\right)^p$; hence, if $\frac{L_{1\,:\,b,\,c,\,d}}{L_{b,\,c,\,d}}\,r$ be put $= a$, the value of the annuity will be $a + a^2 + a^3 + a^4 \ldots a^p = \frac{a - a^{p+1}}{1-a} = \frac{1-a^p}{a^{-1}-1}$.

Art. 2. Consequently, let a general table be formed of the logarithm of $\frac{1-a^r}{a^{-1}-1}$ for every value of the log. of a^p; and also let a particular table be formed for every value of the log. of $\frac{L_{x+p}}{L_x}$ according to the particular table of mortality to be adopted; from the last table take the log. of $\frac{L_{b+p}}{L_b}$, $\frac{L_{c+p}}{L_c}$, $\frac{L_{d+p}}{L_d}$; and also from a table constructed for the purpose, take the log. of r^p, add these four logs. together, and the sum will be the log. of $\overline{a}|^p$, which being sought for in the general table, will give the log. of $\left(\frac{1-a^p}{a^{-1}-1}\right)$ which will be the log. of the annuity sought for the term p, on supposition of the geometrical progression being sufficiently near. Here I remark, that were it not for more general questions than the above, it would be preferable to have general tables formed for the values of $\frac{1-a^p}{a^{-1}-1}$, instead of the log. of such values; but from the consideration that for most purposes a table of the logs. of $\frac{1-a^p}{a^{-1}-1}$ will be found most convenient, I have had them calculated in preference.

Art. 3. The shorter the periods are, the nearer does the series of the number of persons living at the equal intervals of successive ages approximate to the geometrical progression; and consequently this mode, by the assumption of sufficiently short periods, and frequent repetitions, will answer

for any degree of accuracy the given table of mortality will admit of, but then the labour will be increased in proportion.

Art. 4. There are different modes of obviating, in a great measure, this inconvenience, by assuming an accommodated ratio for the given age, instead of the real ratio, from amongst which I shall only for the present select a few. The first is as follows: find for every value of a, the log. of $\overset{y}{\underset{p}{\overset{1}{\underset{1}{\Big|}}}}\frac{L_{x+y}}{L_x}$, that is, the log. of $\frac{L_{x+1}}{L_x}+\frac{L_{x+2}}{L_x}+\frac{L_{x+3}}{L_x}\ \ldots\ \frac{L_{x+p}}{L_x}$; seek this value in the general table, which will give the corresponding value of the log. of a^p; and construct a table of such values for every value of x, and adopt these values for log. of a^p, instead of the abovenamed values of the log. of $\frac{L_{x+p}}{L_x}$, for the determination of the values of the limited periods: the preference of this to the first proposed method consists in this; that if the series $\frac{1}{L_b}\times(L_{b+1}+L_{b+2}+L_{b+3}\ldots L_{b+p})=$ $\beta+\beta^2+\beta^3+\&c.\ldots\beta^p$, the series $\frac{L_{b+1}}{L_b},\frac{L_{b+2}}{L_b}$, &c. being nearly in geometrical progression, and $\frac{L_{b+1}}{L_b}-\beta=\varepsilon_1$, $\frac{L_{b+2}}{L_b}-\beta^2=\varepsilon_2$, &c. ε_1, ε_2, &c. will be small, and $\varepsilon_1+\varepsilon_2+\varepsilon_3\ldots\varepsilon_p=o$, and therefore, if the series $\frac{L_{c+1}}{L_c}$, $\frac{L_{c+2}}{L_c}$, $\frac{L_{c+3}}{L_c}$, &c. and $\frac{L_{d+1}}{L_d}$, $\frac{L_{d+2}}{L_d}$, &c. formed accurately geometrical progressions, and the value of $\frac{L_{c+1}\times L_{d+1}}{L_c\times L_d}.r=m$, the value of the annuity for the term, would be accurately equal to $m\beta+m^2\beta^2+m^3\beta^3\ldots\ldots+m^p\beta^p+m\varepsilon_1+m^2\varepsilon_2+m^3\varepsilon_3\ldots\ldots+m^p\varepsilon_p$, but because in

general $\frac{L_{c+1}}{L_c}$, $\frac{L_{d+1}}{L_d}$ and r differ very little from unity, m will not differ much from unity; and therefore if p be not great, m, m^2, m^3, &c. will not differ much from unity; and consequently, as ε_1, ε_2, ε_3, &c. are small, $m\varepsilon_1 + m^2\varepsilon_2 + m^3\varepsilon_3 \ldots\ldots m^p\varepsilon_p$ will not differ much from $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \ldots\ldots + \varepsilon_p$; but this has been shown to be o; consequently $m\varepsilon_1 + m^2\varepsilon_2 + m^3\varepsilon_3 \ldots\ldots + m^p\varepsilon_p$ differs very little from o, or in other words is very small; and consequently, the value of the annuity differs very little from $m\,\mathfrak{C} + m^2\,\mathfrak{C}^2\,m^3\,\mathfrak{C}^3 \ldots\ldots + m^p\,\mathfrak{C}^p$; and the same method of demonstration would apply with any one of the other ages, the remaining ages being supposed to possess the property of the accurate geometrical progression; notwithstanding this, however, as none of them probably will contain that property, but in an approximate degree, a variation in the above approximations may be produced of a small quantity of the second order; that is, if the order of the product of two small quantities; but, as in this approximation, I was only aiming at retaining the quantities of the first order, I do not consider this as affecting the result as far as the approximation is intended to reach: thus far with regard to the first accommodated ratios.

Art. 5. Moreover, on the supposition that L_c, L_{c+1}, L_{c+2}, ... L_{c+p}, and also L_d, L_{d+1}, L_{d+2} ... L_{d+p} are series in geometrical progression, and that $r.\frac{L_{c+1}}{L_c} \times \frac{L_{d+1}}{L_d} = m = n.q$. Since the annuity for p years on the three lives is equal to $\frac{L_{b+1}}{L_b}.m + \frac{L_{b+2}}{L_b}.m^2 + \ldots\ldots \frac{L_{b+p}}{L_b}.m^p$ it follows

that if $\frac{L_{b+1}}{L_b}.n + \frac{L_{b+2}}{L_b}.n^2 + \frac{L_{b+3}}{L_b}.n^3 \ldots\ldots \frac{L_{b+p}}{L_b}.n^p = \varepsilon.n + \varepsilon^2.n^2 + \varepsilon^3.n^3 \ldots\ldots + \varepsilon^p.n^p$ that if n be very nearly equal to m, $\frac{L_{b+1}}{L_b}.n.q + \frac{L_{b+2}}{L_b}.n^2.q^2 +$ &c. $\ldots\ldots \frac{L_{b+p}}{L_b}.n^p q^p$ which will be the value of the annuity on the three lives, will be nearly $= \varepsilon.n.q + \varepsilon^2.n^2.q^2 +$ &c. $\ldots\ldots \varepsilon^p.n^p.q^p$. If q were equal to unity, or, which is the same thing, $m=n$, the equality would be accurate; but it may not be so when m differs from 1; but the nearer n is to m, at least when the difference does not exceed certain limited small quantities, the nearer will be the coincidence. It appears therefore, that if instead of taking the accommodated ratio for ε^p so that $\frac{1}{L_b} \times (L_{b+1} + L_{b+2} + L_{b+3} \ldots L_{b+p}) = \varepsilon + \varepsilon^2 + \varepsilon^3 \ldots\ldots \varepsilon^n$ it will be preferable generally to take it so that $\frac{1}{L_b} \times (n L_{b+1} + n^2 L_{b+2} + n. L_{b+3}$ &c. $\ldots\ldots n^p L_{b+p}) = \varepsilon + \varepsilon^2 + \varepsilon^3$ &c.... ε^p in which n is between m and 1, the nearer m the better generally, though possibly not universally so throughout the whole limit. And the second method I use for increasing the accuracy, is to adopt an accommodated ratio, or ε^p, so that $\frac{1}{L_b} \times \left(1{,}05^{-1} L_{b+1} + 1{,}05^{-2} L_{b+2} + \text{\&c.} \ldots 1{,}05^p L_{b+p}\right) = \overline{1{,}05}|^{-1} \varepsilon + \overline{1{,}05}|^{-2} \varepsilon^2 + \overline{1{,}05}|^{-3} \varepsilon^3 \ldots \overline{1{,}05}|^{-p} \varepsilon^p$. Another method which might have its peculiar advantage, is to assume $\varepsilon^p = \left.\overline{\frac{L_{b+\frac{1}{2}p}}{L_b}}\right|^{\frac{1}{2}p}$ under the idea of using a mean ratio.

The General Tables.*

Art. 6. I have had three general tables calculated for fixed periods, Numbers 1, 2, and 3. Number 1, for pe-

* The chief of the arithmetical operations in the constructions of most of the tables were performed under my direction, by Mr. David Jones, of N°. 10, King-street, Soho; and, as far as my leisure would allow, I have endeavoured to assure myself of their accuracy by different inspections.

riods of ten years; that is, for $\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$, corresponding to a given value of $\lambda\,(a^{10})$. N°. 2, for seven years, or for $\lambda\left(\frac{1-a^{7}}{a^{-1}-1}\right)$, corresponding to $\lambda\,(a^{7})$, and the 3d for five years, or for $\lambda\left(\frac{1-a^{5}}{a^{-1}-1}\right)$, corresponding to $\lambda\,(a^{5})$; calculated (whether $p = 10$, 7 or 5) for every value of $\lambda\,(a^{p})$, answering to $\bar{3}$,00; $\bar{3}$,01; $\bar{3}$,02, &c. . . . . o. The first column containing the aforesaid value of $\lambda.\,(a^{p})$, corresponding to which, in an horizontal line, is placed the log. of $\frac{1-a^{p}}{a^{-1}-p}$, and between each successive value is placed the difference, retaining a decimal figure more; at the head of the other columns for the proportional parts of the differences, are placed a column showing the number of cyphers to be prefixed to the differences entered in the column following, which are headed $\left\{\begin{matrix}1 & 2 & 3 & 4 & \\ 9 & 8 & 7 & 6 & 5\end{matrix}\right\}$ nearest under 1, 2, 3, 4 and 5, and opposite the number; suppose $\bar{2}$,16 of table log. of a^{10}, stands $\left.\begin{matrix}0275, & 0550, & 0826, & 1101, & \\ 2477, & 2202, & 1926, & 1651, & 1376\end{matrix}\right\}$, the upper, with the addition of the two cyphers, give the proportional parts for ,001, 002, 003, ,004, 005: and the under, with the two cyphers, shows the proportional parts for ,009, 008, 007, 006; and the reason of choosing this arrangement, is the advantage which it offers of proof of correctness; thus the sum of the higher an lower numbers of each of the above row with the two cyphers = 002752, which is double ,001376, and equal to the whole difference between the successive terms.

Let it be required to find the logarithm of $\left(\frac{1-a^{10}}{a^{-1}-1}\right)$, corresponding to log. of $a^{10} = \bar{1}.7954$. In the General Table I,

Opposite to $\bar{1}$.79 we have . .	,88868
For ,005 we have proportional part .	256
For ,0004 . ditto	20
The sum .	,89144 is the answer.

If log. of a^p is less than $\bar{3},00$, then it will be necessary to calculate $\lambda\left(\frac{1-a^p}{a^{-1}-1}\right)$ by common methods, as the tables do not go lower. And generally it will be then sufficient, omitting a^p, only to calculate the value of $-\lambda(a^{-1}-1)$; but from this, if more accuracy be required, subtract the number whose common logarithm is $(\bar{1},6378+\lambda(r)^p)$.

If $\lambda\left(\frac{1-a^p}{a^{-1}-1}\right)$ be given, and $\lambda(a)$ be required, proceed thus, $\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$ being $=,89144$ for example. In Table I, the next value of

$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$ is ,88868 to which $\lambda(a^{10})$ corresponding is $\bar{1},79$

Difference ,00256 belonging to $\bar{1}.79$ gives	. .	,005
Difference ,00020 . . .	ditto	,0004

$\therefore$ if $\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)=,89144$ then we have $\lambda(a^{10})=\bar{1},7954$

If $\lambda(a^p)$ is less than $\bar{3}$, proceed thus: put the given value of $\lambda\left(\frac{1-a^p}{a^{-1}-1}\right)=\lambda q$, and we have the common logarithm of $a=-p\times\lambda(1+q^{-1})+$ a small correction if great accuracy be required; which correction is nearly equal to $p\times$ the number whose common log. is $\left\{1.6378-\lambda q-\overline{p+1}.(1+q^{-1})\right\}$

These methods and tables only apply immediately to $\lambda\left(\frac{1-a^p}{a^{-1}-1}\right)$ when a is a proper fraction; but if a be greater than unity, put it equal to b^{-1}, then will b be a proper fraction; but $\frac{1-a^p}{a^{-1}-1}=\frac{a^p-1}{1-a^{-1}}=\frac{b^{-p}-1}{1-b}=b^{-p-1}\times\frac{1-b^p}{b^{-1}-1}=a^{p+1}\times\left(\frac{1-b^p}{b^{-1}-1}\right)$; consequently $\lambda\left(\frac{1-a^p}{a^{-1}-1}\right)=\overline{p+1}.\lambda(p)+\lambda\left(\frac{1-b^p}{b^{-1}-1}\right)$ I have likewise had Table IV. calculated, which is a general table, for the common log. of $\left(\frac{1}{a^{-1}-1}\right)$, corresponding to a given value of λa,

commencing with $\lambda(a) = \bar{1}.7$; $\bar{1},701$; $\bar{1},702$, &c. with the differences between them. I have not, in this table, had the proportional parts inserted, though it would be attended with advantage, as the table is not meant to be of general use; but only given to be applied for rough purposes, or where accuracy is not particularly required for calculating at once the value of a life annuity for the whole term of life, or the whole remaining terms of life, after a given term, by considering the present value of each successive payment to form the successive terms of a geometrical progression whose first term and common ratio are each equal to a. And as $\lambda\left(\frac{1}{a^{-1}-1}\right)$ will represent the log. of the sum of the said geometrical progression, it will likewise express approximatively the logarithm of the value required. For many purposes, a table of $\frac{1}{a^{-1}-1}$, answering to given values of a, would be preferable, but not for general purposes.

Art. 7. I have already, in Art. 4 and 5, Chap. II, introduced the term accommodated ratios, or chances, and endeavoured to explain the methods to be adopted to reap the advantage of the ideas there expressed. Table V, for Carlisle, Deparcieux, and Northampton, are the logarithms of tenth terms of the accommodated ratios, or the logarithms of the accommodated chances for living ten years, calculated according to a mode laid down in Art. 5, Chap. II; that is, it expresses for every age, or value of b, the logarithm of ε^{10}, when $\frac{1}{L_b} \times (1{,}05^{-1} L_{b+1} + 1{,}05^{-2} L_{b+2} + \&c. \dots 1{,}05^{-p} L_{b+p})$

is equal to $1{,}05^{-1}\varepsilon + 1{,}05^{-2}\varepsilon^{2} + \&c. \dots 1{,}05^{-10}\varepsilon^{10}$. and to show, by example, how these are calculated, let it be required to find the logarithm of the accommodated chance for living

ten years, for the age 20, calculated according to the Carlisle table upon the consideration of interest at 5 per cent. According to the Carlisle tables, I find $\lambda\ {}^{1}_{10}\lfloor_{20}^{1,05^{-}}$; that is, the logarithm of the annuity of one pound on a life of 20, for ten years, at 5 per cent $=$,87176, and putting $a = 105^{-1} \cdot \beta$, by hypothesis we shall have $\lambda\ {}^{1}_{10}\rceil^{x}_{1}\, a^{x}$; that is the logarithm of $(a + a^{2} + a^{3} \ldots a^{10}) =$,87176; that is, $\lambda \left(\frac{1 - a^{10}}{a^{-1} - 1}\right) =$,87176; hence proceeding, as shown above, to find from General Table I. $\lambda(a^{10})$

Having given . . ,87176 $= \lambda\left(\frac{1 - a^{10}}{a^{-1} - 1}\right)$			
We have next less $=$,86842	corresponding to	. . .	$\bar{1}$.75
,00334	difference		
,00302	proportional part	. . .	,006
30	. . ditto	. . . .	,0006
2	. . ditto	. . . .	,00004
.87176	corresponds to .	$\lambda(a^{10}) =$	$\bar{1}$.75664
		$\lambda(1,05^{10}) =$	,21189
			$\bar{1}$.96853 for the

log. of the accommodated chance to live 10 years at the Carlisle mortality.

In the same way may the accommodated chance be found for any other term, when general tables for the term are constructed, and from any other base of interest. I may observe, that by using different rates of interest, as a base for determining the accommodated chances, different degrees of accuracy may be obtained. See Art. 5. Chap. II.

Art. 8. Table VI. is the logarithm of the accommodated chances $\mathfrak{C}$ at every age, b for living one year, where $\mathfrak{C}$ is of such value that the sum of the geometrical progression $\frac{\mathfrak{C}}{1,05} + \frac{\mathfrak{C}^{2}}{1,05^{2}} +$ &c. ad infinitum, or, which is the same thing,

$\frac{\frac{1}{1,05}}{\epsilon^{-1}}$ shall be equal to the value of the whole life annuity at five per cent. at such age, namely $\overbrace{{}^{1}_{1}\lfloor\underline{b}}^{1,05^{-1}}$; consequently $\frac{\epsilon}{1,05^{-1}} \times \left(1 + \overbrace{{}^{1}_{1}\lfloor\underline{b}}^{1,05^{-1}}\right) = \overbrace{{}^{1}_{1}\lfloor\underline{b}}^{1,05^{-1}}$; $\therefore \lambda\,\epsilon = \lambda\left(\overbrace{{}^{1}_{1}\lfloor\underline{b}}^{1,05^{-1}}\right) + \lambda(1,05) - \lambda\left(\overbrace{{}^{1}_{0}\lfloor\underline{b}}^{1,05^{-1}}.\right)$ This table is constructed for Carlisle, Deparcieux, and Northampton, and is to be used in conjunction with Table IV., where only a rough value of the contingency is required; and though this table applies as the other tables of accommodated chances, to different rates of interest, still it would be of advantage more particularly *here* for the greater approximation to have similar tables constructed from the formula $\lambda(\epsilon) = \lambda\left(\overbrace{{}^{1}_{1}\lfloor\underline{b}}^{r}\right) + \lambda(r^{-1}) - \lambda\left(\overbrace{{}^{1}_{0}\lfloor\underline{b}}^{r}\right)$ for different values of r.

Art. 9. In calculating the value of life annuities for long periods, by means of adding together the values of portions of those periods, the portions of the distant periods contain factors of the real chance of living to these periods, and likewise of the discounted value of the money of which the payment is not immediate; thus if t be greater than 10, $\overbrace{{}^{1}_{t}\lfloor\underline{a, b, c}}^{r} = \overbrace{{}^{1}_{10}\lfloor\underline{a, b, c}}^{r} + \overbrace{{}^{11}_{t}\lfloor\underline{a, b, c}}^{r} = \overbrace{{}^{1}_{10}\lfloor\underline{a, b, c}}^{r} + \frac{L_{10:a,b,c}}{L_{a,b,c}} \cdot r^{10} \times {}_{(t-10)}\lfloor\underline{a+10, b+10, c+10}$. It will be therefore convenient to have a table of the logarithm of the real chance of living 10, 20, 30 years, &c. and also for other terms; and some of these are given by Tables VII., VIII., IX.

Time will not allow me, for the present, to offer more than a very few examples of the method to be employed in calculating by these tables, which are as follow:

Example 1. Required, according to the Carlisle table, the value of a life annuity, for ten years, on the joint lives 30 and 40, at 3 per cent interest.

In Table VIII. for Carlisle, log. of accommodated chance for 10 years, at the age 30 . .	$= \bar{1}.9552$
Ditto 40	$= \bar{1}.9383$
Ditto λ 1,03 . . .	$= \bar{1}.8716$
Sum . .	$\bar{1}.7651 = \lambda\,(a^{10})$

In Table I, $\bar{1}.76$ corresponds to . . .	.8734
In proportional parts ,005 corresponds to	.253
Ditto . . 0001 corresponds to	. 5
Consequently $\bar{1}$,7651 corresponds to .	.87604

which is the log. of the required value: the number corresponding to this is 7,5169, for the value of the annuity, according to the Carlisle mortality, at 3 per cent. on the joint lives 30 and 40; and by calculation from Mr. Milne's tables, I find the value should be 7,5168; the difference of the two is evidently insignificant. In this way I calculated the log. of the value of the life annuity, at the Carlisle mortality, at 3 per cent. for 10 years, for the joint lives 0 and 10, 10 and 20, 20 and 30, 30 and 40, 40 and 50, 50 and 60, to be ,76580; ,90247; ,89139; ,87604; ,86295; ,81067; and the annuity, or the numbers corresponding to the said logarithms,

5,8318; 7,9874; 7,7874; 7,5169; 7,2937; 6,4665;

and, according to calculation from Mr. Milne's tables, I get

5,8595; 7,992; 7,7906; 7,5168; 7,2916; 6,4679.

The difference between the two sets is insignificant, except

perhaps in the values of $\overset{\overbrace{1,05^{-1}}}{\overset{1}{\tfrac{1}{10}}}\big\lfloor\underline{0,10}$; that is, the value of the annuity on the joint life of a child just born, with one of the age of 10, at 3 per cent. Had we divided the period in portions, the value might have been obtained as near as we pleased; or we should likewise have obtained greater accuracy, had we assumed an accommodated chance deduced at a more appropriate interest than 5 per cent. See Art. 5, Chap. II.

Example 2. Let it be required to find the value of a life annuity at 3 per cent. for 10 years, at the Carlisle mortality, for the five lives of the age 20, 30, 40, 45 and 50.

In Table VIII. log. of accom. chance for 10 years at age	20	$= \bar{1}.9685$
Ditto	30	$= \bar{1}.9552$
Ditto	40	$= \bar{1}.9383$
Ditto	45	$= \bar{1}.9367$
Ditto	50	$= \bar{1}.9292$
	$\lambda\ 1,05^{-10}$	$= \bar{1}.8716$
	$\lambda\ (a^{10})$	$= \bar{1}.5995$

This sought in Table I.; thus,	$\bar{1},59$ giving	,79035
	,009	427
	,0005	23

gives ,79485 the N° to which log. is 6,2352

for the value of $\overset{\overbrace{1,03^{-1}}}{\overset{1}{\tfrac{1}{10}}}\big\lfloor\underline{20, 30, 40, 45, 50}$.

Example 3. Let it be required to find the value of $\overset{\overbrace{1,03^{-1}}}{\overset{1}{\tfrac{1}{1}}}\big\lfloor\underline{b, b+10}$ Carlisle mortality, when $b = 10$, that is, for the whole joint lives of 10 and 20. By dividing the whole in portions of ten years, the operation will stand thus for $\overset{\overbrace{1,03^{-1}}}{\overset{1}{\tfrac{1}{10}}}\big\lfloor\underline{b, b+10}$.

	$b=10$	$b=20$	$b=30$	$b=40$	$b=50$	$b=60$	$b=70$	$b=80$	
Log. of accom. ratio for 10 years =	$\bar{1}.9768$	$\bar{1}.9685$	1.9552	$\bar{1}.9383$	$\bar{1}.9292$	$\bar{1}.8318$	$\bar{1}.6689$	$\bar{1}.3134$	from Tab. VIII. Carlisle.
	$\bar{1}.9685$	$\bar{1}.9552$	$\bar{1}.9383$	$\bar{1}.9292$	$\bar{1}.8318$	$\bar{1}.6689$	$\bar{1}.3134$	$\bar{2}.6695$	
$\lambda\,(1{,}03^{-10})=$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$	1.8716	
sum . . =	$\bar{1}.8169$	$\bar{1}.7953$	$\bar{1}.7651$	$\bar{1}.7391$	$\bar{1}.6326$	$\bar{1}.3723$	$\bar{2}.8539$	$\bar{3}.8545$	
No^s corresponding to sum in Table I.	.90247	.89139	.87604	.86295	.81067	.69156	.48781	.19146	
Log. of ratios for 10 years =		$\bar{1}.97438$	$\bar{1}.94120$	$\bar{1}.89520$	$\bar{1}.83292$	$\bar{1}.75123$	$\bar{1}.57016$	$\bar{1}.16886$	
		$\bar{1}.96681$	$\bar{1}.92082$	$\bar{1}.85854$	$\bar{1}.77684$	$\bar{1}.59577$	$\bar{1}.19448$	$\bar{2}.36767$	
$\lambda\,1{,}03^{-10}$ . . =		$\bar{1}.87163$	$\bar{1}.74325$	$\bar{1}.61488$	$\bar{1}.48651$	$\bar{1}.35814$	$\bar{1}.22977$	$\bar{1}.10139$	
The log. of the present worth of each portion =		.70421	.48131	.23157	$\bar{1}.90694$	$\bar{1}.39670$	$\bar{2}.48222$	$\bar{4}.82938$	

And the present worth of each, or the numbers corresponding to the last logarithms are arranged below.

For first	10 years	7.
2nd	ditto	5.0607
3d	d°	3.0291
4th	d°	1.7044
5th	d°	.8071
6th	d°	.2492
7th	d°	.0303
8th	d°	.0007
	sum	18.8701

As the method by which the logarithms of the present worth of the different portions are found, may not be seen by every reader, I will explain the operation in the third portion; that is, when the logarithm of the portion first found is anticipated for 20 years.

Resume	.87604
Table VII. log. of real chance for age 10 living 20 years . . .	$\bar{1}.94120$
Ditto 20 years living . .	$\bar{1}.92082$
$\lambda\,(1{,}03^{-20})$	

which differs but insignificantly from Mr. MILNE's table, which gives 18.873. In a similar way, I find the value of the joint lives for ages 20 and 30, at 3 per cent. and Carlisle mortality to be 16.745; which, according to Mr. MILNE's table, should be 16.749; which appears to be an insignificant difference.

Example 4. To find, when particular accuracy is not required, according to the formula for the whole of life, the approximate value of $\overset{1{,}03^{-1}}{\overset{1}{1}}\!\rceil\underline{a,\,a+10}$ at the Carlisle mortality, when $a=10$, 20, 30, &c. call the logarithm of accommodated ratios for an unlimited time at the age a, $^{\mathrm{R}}{}_{a}$ standing for the accommodated ratio in Table VI. at the age a.

$a =$	10	20	30	40	50	60	70	80	90
R_a	$\bar{1}.99529$	$\bar{1}.99455$	$\bar{1}.99265$	$\bar{1}.98991$	$\bar{1}.98546$	$\bar{1}.97514$	$\bar{1}.95755$	$\bar{1}.92461$	$\bar{1}.86660$
R_{a+10}	$\bar{1}.99455$	$\bar{1}.99265$	$\bar{1}.98991$	$\bar{1}.98546$	$\bar{1}.97514$	$\bar{1}.95755$	$\bar{1}.92461$	$\bar{1}.86660$	$\bar{1}.81282$
$\lambda\, 1{,}03^{-1}$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$
	$\bar{1}.97700$	$\bar{1}.97436$	$\bar{1}.96972$	$\bar{1}.96253$	$\bar{1}.94776$	$\bar{1}.91985$	$\bar{1}.86932$	$\bar{1}.77837$	$\bar{1}.66658$
Log. which corresponds to	$\bar{1}.26451$ {	1.20975 .00631	1.13083 .01062	1.03886 .00641	.88674 .00667	.68817 .00502	.45337 .00123	.17571 .00093	
		1.21606	1.14145	1.04527	.89341	.69319	.45460	.17664	
Numbers . . Instead of .	18.387 18.573	16.446 16.749	13.850 14.449	11.099 11.954	7.824 8.729	4.9339 5.565	2.8485 3.229	1.5019 1.589	

To find the value corresponding to $\bar{1}.66658$, not in the table, find the number corresponding complement of the log. $\bar{1}.6658$, which number is 2,159; subtract 1, and find the complement of the log. which is $= \bar{1}.9359165$, whose number is ,8628. Mr. Milne's table gives .979. But as it is not always the same rate of interest which gives the best accommodated ratios, in order to try when, for instance, the interest of money is 3 per cent. what rate of interest should be used in determining the ratios, use the following table:*

$$\begin{array}{l|l}
\text{Interest.} & \\
1{,}08 & \lambda\ (1{,}08^{-1} \times 1{,}03) = \bar{1}{,}979 \\
1{,}07 & \lambda\ (1{,}07^{-1} \times 1{,}03) = \bar{1}{,}983 \\
1{,}06 & \lambda\ (1{,}06^{-1} \times 1{,}03) = \bar{1}{,}987 \\
1{,}05 & \lambda\ (1{,}05^{-1} \times 1{,}03) = \bar{1}{,}991 \\
1{,}04 & \lambda\ (1{,}04^{-1} \times 1{,}03) = \bar{1}{,}996
\end{array} \left.\vphantom{\begin{array}{l}1\\1\\1\\1\\1\end{array}}\right\} \text{nearly;}$$

* This is not given as a perfect and unerring rule, but as a method in many cases useful, and which would be perfect for the accommodated ratio of one of the lives, if the other lives followed an exact geometrical ratio throughout; and that the real geometrical ratios were in that case used for them, provided that instead of comparing the said sum with the small table, we take for the base of interest the number whose logarithm is $-\lambda\,(1{,}03)$, when the interest is 3 per cent.; and it is to be recollected that the methods is only given as a rough approximation.

Add the logarithm of accommodated ratios, as given in the Table VI. of all the lives but one in question, together, and see which of those rates of interest it nearest agrees with, and use that to calculate the life left, and proceed so for every life; thus for $\overbrace{{}^{1}_{1}\rceil\underline{30, 40}}^{1,03^{-1}}$; to find the rate of interest for 30, I observe that $R_{40} = \bar{1}.9899$ agrees nearest with 6 per cent. in the little table, and $R_{30} = \bar{1}.99265$ agrees nearest with 5 per cent., I therefore take 6 per cent. for the age 30, and for the other I take 5 per cent.: proceed thus:

Example 5.

R_{30} if calculated at 6 per cent.	$\bar{1}.99316$
R_{40} per table	$\bar{1}.98991$
$\lambda\ 1,05^{-1}$ . . . . =	$\bar{1}.98716$
	$\bar{1}.97023$
	1.14558
Proportionate parts . .	$.00327$
To which logarithm . .	1.14885
The N° corresponding is .	14.088
Instead of	14.449

Example 6.

$\overbrace{{}^{1}_{1}\rceil\underline{40, 50}}^{1,03^{-1}}$

R_{40} at 6 per cent. . .	$\bar{1}.99060$
R_{50} at 6 per cent. . .	$\bar{1}.98632$
	$\bar{1}.98716$
	$\bar{1}.96408$
	$\bar{1}.05336$
	$.00102$
	1.06438
	11.598
Instead of	11.954

Example 7.

$\overbrace{{}^{1}_{1}\rceil\underline{50, 60}}^{1,03^{-1}}$

R_{50} at 8 per cent. . . . . =	$\bar{1}.98759$
R_{60} at 6 per cent. . . . . =	$\bar{1}.97599$
$\lambda\ 1,03^{-1}$ . . . . . =	$\bar{1}.98716$
	$\bar{1}.95074$
	$.91357$
	$.00687$
	$.92044$
which log. corresponds . .	$.8318$
instead of	$.8729$

I observe that I have not given any table of the logarithm of the accommodated ratios for an unlimited term, except that calculated with 5 per cent. as a radix; but by the assistance of a table of life annuities, for single life at different rates per cent., this will enable us, independent of certain exceptions, to derive the quantity for the same rates per cent. for any radix at the per cent. contained in the second table; thus to find R_{50} Carlisle mortality, radix 8 per cent. I look to the Carlisle table of single lives at 8 per cent., and I find the value of the annuity on the life of $50 = 8.987$, I search the age to which this will correspond at 5 per cent. and I find sufficiently nearly 59,82 for the age corresponding, to which from my table (with the radix at 5 per cent.) for the log. of ratios I find $\bar{1}.97536$; to this I add log. of $\frac{1.08}{1.05}$; that is, ,01223, and we get $\bar{1}.98759$, the same as given on the other side. This method is accurately consistent with the definition of accommodated ratios for unlimited periods; and if this description of accommodated ratios at a certain rate per cent. be given for one table, for which at the same rate per cent. we have the value of single lives, we may find the same description of accommodated ratios for any other table of mortality for which, at the same rate per cent. we have a table of the value of single lives: thus, suppose the logarithm of this description of accommodated ratios be given for the Carlisle table at five per cent., and the same be required for the Northampton for the age 60, at the same rate; $\overset{\overbrace{1,05}^{-1}}{\tfrac{1}{1}}\rceil\underline{60}$ Northampton $= 8,392$, this being sought in the Carlisle

table for $\overbrace{{}^{1}_{1}\llcorner^{x}}^{1{,}05^{-1}}$ gives $x = 62{,}41$ for the corresponding age; seek the logarithm of accommodated ratios for an unlimited term, corresponding to this for Carlisle, for the age 6,241, and we have $\bar{1}.9723$, agreeing with the table given.

Previously to concluding this chapter, I shall add a small table, which will be found very useful in the application of the methods here proposed.

n	Log. of $1{,}03^{-n}$	Log. of $1{,}035^{-n}$	Log. of $1{,}04^{-n}$	Log. of $1{,}045^{-n}$	Log. of $1{,}05^{-n}$
1	$\bar{1}.9871628$	$\bar{1}.9850597$	$\bar{1}.9829667$	$\bar{1}.9808837$	$\bar{1}.9788107$
2	$\bar{1}.9743256$	$\bar{1}.9701193$	$\bar{1}.9659333$	$\bar{1}.9617674$	$\bar{1}.9576214$
3	$\bar{1}.9614883$	$\bar{1}.9551790$	$\bar{1}.9489000$	$\bar{1}.9426511$	$\bar{1}.9364321$
4	$\bar{1}.9486511$	$\bar{1}.9402386$	$\bar{1}.9318666$	$\bar{1}.9235348$	$\bar{1}.9152428$
5	$\bar{1}.9358139$	$\bar{1}.9252983$	$\bar{1}.9148333$	$\bar{1}.9044185$	$\bar{1}.8940535$
6	$\bar{1}.9229767$	$\bar{1}.9103579$	$\bar{1}.8978000$	$\bar{1}.8853023$	$\bar{1}.8728642$
7	$\bar{1}.9101394$	$\bar{1}.8954176$	$\bar{1}.8807666$	$\bar{1}.8661860$	$\bar{1}.8516749$
8	$\bar{1}.8973022$	$\bar{1}.8804772$	$\bar{1}.8637333$	$\bar{1}.8470697$	$\bar{1}.8304856$
9	$\bar{1}.8844650$	$\bar{1}.8655369$	$\bar{1}.8466999$	$\bar{1}.8279534$	$\bar{1}.8092963$
10	$\bar{1}.8716278$	$\bar{1}.8505965$	$\bar{1}.8296666$	$\bar{1}.8088371$	$\bar{1}.7881070$

General Table I. $\lambda(a^{10})$, $\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$.

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{3}$.00	,00163	,00	0200	0399	0599	0798	0998
	,00199,6		1796	1597	1397	1198	
$\bar{3}$.01	,0036,2		0200	0401	0601	0801	1002
	,00200,3		1803	1602	1402	1202	
$\bar{3}$.02	,00563		0201	0401	0602	0802	1003
	,00200,6		1805	1605	1404	1204	
$\bar{3}$.03	,00763		0201	0402	0603	0804	1005
	,00201,0		1809	1608	1407	1206	
$\bar{3}$.04	,00964		0202	0403	0605	0806	1008
	,00201,5		1814	1612	1411	1209	
$\bar{3}$.05	,01166		0202	0404	0606	0808	1010
	,00202,0		1818	1616	1414	1212	
$\bar{3}$.06	,01368		0202	0405	0607	0810	1012
	,00202,4		1822	1619	1417	1214	
$\bar{3}$.07	,01570		0203	0406	0608	0811	1014
	,00202,8		1825	1622	1420	1217	
$\bar{3}$.08	,01773		0203	0407	0610	0814	1017
	,00203,4		1831	1627	1424	1220	
$\bar{3}$.09	,01976		0204	0408	0611	0815	1019
	,00203,8		1834	1630	1427	1223	
$\bar{3}$.10	,02180		0204	0409	0613	0817	1022
	,00204,3		1839	1634	1430	1226	
$\bar{3}$.11	,02384		0205	1409	0614	0819	1024
	,00204,7		1842	1638	1433	1228	
$\bar{3}$.12	,02589		0205	0410	0616	0821	1026
	,00205,2		1847	1642	1436	1231	
$\bar{3}$.13	,02794		0206	0411	0617	0823	1029
	,00205,7		1851	1646	1440	1234	
$\bar{3}$.14	,03000		0206	0412	0618	0824	1031
	,00206,1		1855	1649	1443	1237	
$\bar{3}$.15	,03206		0207	0413	0620	0826	1033
	,00206,5		1859	1654	1446	1239	
$\bar{3}$.16	,00207		0207	0415	0622	0829	1037
	,00207,3		1866	1658	1451	1244	
$\bar{3}$.17	,03620		0208	0415	0623	0830	1038
	,00207,6		1868	1661	1453	1246	
$\bar{3}$.18	,03827		0208	0416	0624	0832	1041
	,00208,1		1873	1665	1457	1249	
$\bar{3}$.19	,04036		0209	0417	0626	0834	1043
	,00208,6		1877	1669	1460	1252	
$\bar{3}$.20	,04244		0209	0418	0627	0836	1046
	,00209,1		1882	1673	1464	1255	
$\bar{3}$.21	,04453		0210	0419	0629	0838	1048
	,00209,6		1886	1677	1467	1258	
$\bar{3}$.22	,04663		0210	0420	0630	0840	1051
	,00210,1		1891	1681	1471	1261	
$\bar{3}$.23	,04873		0211	0421	0632	0842	1053
	,00210,6		1895	1685	1474	1264	
$\bar{3}$.24	,05084		0211	0422	0633	0844	1056
	,00211,1		1900	1689	1478	1267	

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{3}$.25	,05295	,00	0212	0423	0635	0847	1059
	,00211,7		1905	1694	1482	1270	
$\bar{3}$.26	,05506		0212	0424	0637	0849	1061
	,00212,2		1910	1698	1485	1273	
$\bar{3}$.27	,05719		0213	0425	0638	0851	1064
	,00212,7		1914	1702	1489	1276	
$\bar{3}$.28	,05931		0213	0426	0640	0853	1066
	,00213,2		1919	1706	1492	1279	
$\bar{3}$.29	,06144		0214	0427	0641	0855	1069
	,00213,7		1923	1710	1496	1282	
$\bar{3}$.30	,06358		0214	0429	0643	0857	1072
	,00214,3		1929	1714	1500	1286	
$\bar{3}$.31	,06572		0215	0430	0644	0859	1074
	,00214,8		1933	1718	1504	1289	
$\bar{3}$.32	,06787		0215	0431	0646	0861	1077
	,00215,3		1938	1722	1507	1292	
$\bar{3}$.33	,07002		0216	0432	0648	0864	1080
	,00216,0		1944	1728	1512	1296	
$\bar{3}$.34	,07218		0216	0433	0649	0866	1082
	,00216,4		1948	1731	1515	1298	
$\bar{3}$.35	,07435		0217	0434	0651	0868	1085
	,00217,0		1953	1736	1519	1302	
$\bar{3}$.36	,07652		0218	0435	0653	0870	1089
	,00217,5		1958	1740	1523	1305	
$\bar{3}$.37	,07869		0218	0436	0654	0872	1090
	,00218,0		1962	1744	1526	1308	
$\bar{3}$.38	,08087		0219	0437	0656	0875	1094
	,00218,7		1968	1750	1531	1312	
$\bar{3}$.39	,08306		0219	0438	0658	0877	1096
	,00219,2		1973	1754	1534	1315	
$\bar{3}$.40	,08525		0220	0439	0659	0879	1099
	,00219,7		1977	1758	1538	1318	
$\bar{3}$.41	,08745		0220	0441	0661	0882	1102
	,00220,4		1984	1763	1543	1322	
$\bar{3}$.42	,08965		0221	0442	0663	0884	1105
	,00221,0		1989	1768	1547	1326	
$\bar{3}$.43	,09186		0221	0443	0664	0886	1107
	,00221,4		1993	1771	1551	1328	
$\bar{3}$.44	,09408		0222	0444	0666	0888	1111
	,00222,1		1999	1777	1555	1333	
$\bar{3}$.45	,09630		0223	0445	0668	0890	1113
	,00222,6		2003	1781	1558	1336	
$\bar{3}$.46	,09852		0223	0446	0670	0893	1116
	,00223,2		2009	1786	1562	1339	
$\bar{3}$.47	,10076		0224	0448	0671	0895	1119
	,00223,8		2014	1790	1567	1343	
$\bar{3}$.48	,10300		0224	0449	0673	0898	1122
	,00224,4		2020	1795	1571	1346	
$\bar{3}$.49	,10524		0225	0450	0675	0900	1125
	,00225,0		2025	1800	1575	135[illegible]	

General Table I. $\lambda\,(a^{10}),\ \lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$.

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5	$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{3}.50$	,10749	,00	0226	0451	0677	0902	1128	$\bar{3}.75$	,16581	,00	0242	0484	0726	0968	1210
	,00225,6		2030	1805	1579	1354			,00242,0		2178	1936	1694	1452	
$\bar{3}.51$	,10975		0226	0452	0679	0905	1131	$\bar{3}.76$	,16823		0243	0485	0728	0971	1214
	,00226,2		2036	1810	1583	1357			,00242,7		2184	1942	1699	1456	
$\bar{3}.52$	,11201		0227	0454	0680	0907	1134	$\bar{3}.77$	,17065		0244	0487	0731	0974	1218
	,00226,8		2041	1814	1588	1361			,00243,5		2192	1948	1705	1461	
$\bar{3}.53$	,11428		0228	0455	0683	0910	1138	$\bar{3}.78$	,17309		0244	0488	0733	0977	1221
	,00227,5		2048	1820	1593	1365			,00244,2		2198	1954	1709	1465	
$\bar{3}.54$	,11655		0228	0456	0684	0912	1141	$\bar{3}.79$	,17553		0245	0490	0735	0980	1225
	,00228,1		2053	1825	1597	1369			,00244,9		2204	1959	1714	1469	
$\bar{3}.55$	,11883		0229	0457	0686	0915	1144	$\bar{3}.80$	,17798		0246	0491	0737	0982	1228
	,00228,7		2058	1830	1601	1372			,00245,6		2210	1965	1719	1474	
$\bar{3}.56$	,12112		0229	0458	0688	0917	1146	$\bar{3}.81$	,18044		0246	0493	0739	0986	1232
	,00229,2		2063	1834	1604	1375			,00246,4		2218	1971	1725	1478	
$\bar{3}.57$	,12341		0230	0460	0690	0920	1151	$\bar{3}.82$	,18290		0247	0494	0741	0988	1236
	,00230,1		2071	1841	1611	1381			,00247,1		2224	1977	1730	1482	
$\bar{3}.58$	,12571		0231	0461	0692	0922	1153	$\bar{3}.83$	,18537		0248	0496	0743	0991	1239
	,00230,6		2075	1845	1614	1384			,00247,8		2230	1982	1735	1487	
$\bar{3}.59$	,12802		0231	0462	0694	0925	1156	$\bar{3}.84$	,18785		0249	0497	0746	0994	1243
	,00231,2		2081	1850	1618	1387			,00248,6		2237	1989	1740	1492	
$\bar{3}.60$	,13033		0232	0464	0696	0928	1160	$\bar{3}.85$	,19034		0249	0499	0748	0997	1247
	,00231,9		2087	1855	1623	1391			,00249,3		2244	1994	1745	1496	
$\bar{3}.61$	,13265		0233	0465	0698	0930	1163	$\bar{3}.86$	,19284		0250	0500	0750	1000	1251
	,00232,5		2093	1860	1628	1395			,00250,1		2251	2001	1751	1501	
$\bar{3}.62$	,13497		0233	0466	0699	0932	1165	$\bar{3}.87$	,19533		0251	0502	0753	1004	1255
	,00233,0		2097	1864	1631	1398			,00250,9		2258	2007	1756	1505	
$\bar{3}.63$	,13730		0234	0468	0701	0935	1169	$\bar{3}.88$	,19784		0252	0503	0755	1006	1258
	,00233,8		2104	1870	1637	1403			,00251,6		2264	2013	1761	1510	
$\bar{3}.64$	,13964		0235	0469	0704	0938	1174	$\bar{3}.89$	,20035		0252	0505	0757	1010	1262
	,00234,5		2111	1876	1642	1407			,00252,4		2272	2019	1767	1514	
$\bar{3}.65$	,14199		0235	0470	0705	0940	1176	$\bar{3}.90$	,20288		0253	0506	0760	1013	1266
	,00235,1		2116	1881	1646	1411			,00253,2		2279	2026	1772	1519	
$\bar{3}.66$	,14434		0236	0472	0707	0943	1179	$\bar{3}.91$	,20541		0254	0508	0762	1016	1271
	,00235,8		2122	1886	1651	1414			,00254,1		2287	2033	1779	1525	
$\bar{3}.67$	,14670		0237	0473	0710	0946	1183	$\bar{3}.92$	,20795		0255	0509	0764	1019	1274
	,00236,6		2129	1893	1656	1420			,00254,7		2292	2038	1783	1528	
$\bar{3}.68$	,14906		0237	0474	0711	0948	1186	$\bar{3}.93$	,21050		0256	0511	0767	1023	1279
	,00237,1		2134	1897	1660	1423			,00255,7		2301	2046	1790	1534	
$\bar{3}.69$	,15143		0238	0476	0714	0952	1190	$\bar{3}.94$	,21306		0256	0513	0769	1025	1282
	,00237,9		2141	1903	1665	1427			,00256,3		2307	2050	1794	1538	
$\bar{3}.70$	,15381		0239	0477	0716	0954	1193	$\bar{3}.95$	,21562		0257	0514	0772	1029	1286
	,00238,5		2147	1908	1670	1431			,00257,2		2315	2058	1800	1543	
$\bar{3}.71$	,15620		0239	0478	0718	0957	1196	$\bar{3}.96$	,21819		0258	0516	0774	1032	1290
	,00239,2		2153	1914	1674	1435			,00258,0		2322	2064	1806	1548	
$\bar{3}.72$	,15859		0240	0480	0720	0960	1200	$\bar{3}.97$	,22077		0259	0518	0776	1035	1294
	,00239,9		2159	1919	1679	1439			,00258,8		2329	2070	1812	1553	
$\bar{3}.73$	,16099		0241	0481	0722	0962	1203	$\bar{3}.98$	,22336		0260	0519	0779	1038	1298
	,00240,6		2165	1925	1684	1444			,00259,6		2336	2077	1817	1558	
$\bar{3}.74$	,16339		0241	0483	0724	0965	1207	$\bar{3}.99$	,22559		0260	0521	0781	1042	1302
	,00241,3		2172	1930	1689	1448			,00260,4		2344	2083	1823	1562	

General Table I. $\lambda\,(a^{10}),\ \lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$.

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{2}.00$	,22856 ,00261,4	,00	0261 2353	0523 2091	0784 1830	1046 1568	1307
$\bar{2}.01$	,23117 ,00262,0		0262 2358	0524 2096	0786 1834	1048 1572	1310
$\bar{2}.02$	,23379 ,00263,0		0263 2367	0526 2104	0789 1841	1052 1578	1315
$\bar{2}.03$	,23642 ,00263,8		0264 2374	0528 2110	0791 1847	1055 1583	1319
$\bar{2}.04$	,23906 ,00264,7		0265 2382	0529 2118	0794 1853	1059 1588	1324
$\bar{2}.05$	,24171 ,00265,6		0266 2390	0531 2125	0797 1859	1062 1594	1328
$\bar{2}.06$	,24036 ,00266,3		0266 2397	0533 2130	0799 1864	1065 1598	1332
2.07	,24703 ,00267,0		0267 2403	0534 2136	0801 1869	1068 1602	1335
$\bar{2}.08$	,24970 ,00268,5		0269 2417	0537 2148	0806 1880	1074 1611	1343
$\bar{2}.09$	,25238 ,00269,0		0269 2421	0538 2152	0807 1883	1076 1614	1345
2.10	,25507 ,00269,9		0270 2429	0540 2159	0810 1889	1080 1619	1350
$\bar{2}.11$	,25777 ,00270,8		0271 2437	0542 2166	0812 1896	1083 1625	1354
$\bar{2}.12$	,26048 ,00271,7		0272 2445	0543 2174	0815 1902	1087 1630	1359
$\bar{2}.13$	,26320 ,00272,6		0273 2453	054[illegible] 2181	0818 1908	1090 1636	1363
$\bar{2}.14$	,26592 ,00273,5		0274 2462	0547 2188	0821 1915	1094 1641	1368
$\bar{2}.15$	,26866 ,00274,6		0275 2471	0549 2197	0824 1922	1098 1648	1373
$\bar{2}.16$	,27140 ,00275,2		0275 2477	0550 2202	0826 1926	1101 1651	1376
$\bar{2}.17$	,27415 ,00276,3		0276 2487	0553 2210	0829 1934	1105 1658	1382
$\bar{2}.18$	,27692 ,00277,2		0277 2495	0554 2218	0832 1940	1109 1663	1386
$\bar{2}.19$	,27969 ,00278,1		0278 2503	0556 2225	0834 1947	1112 1669	1391
$\bar{2}.20$	,28247 ,00279,1		0279 2512	0558 2233	0837 1954	1116 1675	1396
$\bar{2}.21$	,28526 ,00280,1		0280 2521	0560 2241	0840 1961	1120 1681	1401
$\bar{2}.22$	,28806 ,00281,1		0281 2530	0562 2249	0843 1968	1124 1687	1406
$\bar{2}.23$	,29087 ,00281,9		0282 2537	0564 2255	0846 1973	1128 1691	1410
$\bar{2}.24$	,29369 ,00282,9		0283 2546	0566 2263	0849 1980	1132 1697	1415
$\bar{2}.25$	,29652 ,00283,9	,00	0284 2555	0568 2271	0852 1987	1136 1703	1420
$\bar{2}.26$	,29936 ,00284,9		0285 2564	0570 2279	0855 1994	1140 1709	1425
$\bar{2}.27$	,30221 ,00285,9		0286 2573	0572 2287	0858 2001	1144 1715	1430
$\bar{2}.28$	,30507 ,00286,8		0287 2581	0574 2294	0860 2008	1147 1721	1434
$\bar{2}.29$	,30794 ,00287,9		0288 2591	0576 2303	0864 2015	1152 1727	1440
$\bar{2}.30$	,31081 ,00288,9		0289 2600	0578 2311	0867 2022	1156 1733	1445
$\bar{2}.31$	,31370 ,00289,9		0290 2609	0580 2319	0870 2029	1160 1739	1450
$\bar{2}.32$	,31660 ,00290,9		0291 2618	0582 2327	0873 2036	1164 1745	1455
$\bar{2}.33$	,31951 ,00291,9		0292 2627	0584 2335	0876 2043	1168 1751	1460
$\bar{2}.34$	,32243 ,00293,0		0293 2637	0586 2344	0879 2051	1172 1758	1465
$\bar{2}.35$	,32536 ,00294,0		0294 2646	0588 2352	0882 2058	1176 1764	1470
$\bar{2}.36$	,32830 ,00295,0		0295 2655	0590 2360	0885 2065	1180 1770	1475
$\bar{2}.37$	,33125 ,00296,1		0296 2665	0592 2369	0888 2073	1184 1777	1481
$\bar{2}.38$	,33421 ,00297,1		0297 2674	0594 2377	0891 2080	1188 1783	1485
2.39	,33718 ,00298,2		0298 2684	0596 2386	0895 2087	1193 1789	1491
$\bar{2}.40$	,34016 ,00299,3		0299 2694	0599 2394	0898 2095	1197 1796	1497
$\bar{2}.41$	,34316 ,00300,3		0300 2703	0601 2402	0901 2102	1201 1802	1502
$\bar{2}.42$	,34316 ,00301,4		0301 2713	0603 2411	0904 2110	1206 1808	1507
$\bar{2}.43$	,34917 ,00302,5		0303 2723	0605 2420	0908 2118	1210 1815	1513
$\bar{2}.44$	,35220 ,00303,6		0304 2732	0607 2429	0911 2125	1214 1822	1518
$\bar{2}.45$	,35523 ,00304,7		0305 2742	0609 2438	0914 2133	1219 1828	1524
$\bar{2}.46$	,35828 ,00305,8		0306 2752	0612 2446	0917 2141	1223 1835	1529
$\bar{2}.47$	,36134 ,00306,9		0307 2762	0614 2455	0921 2148	1228 1841	1535
$\bar{2}.48$	,36441 ,00308,1		0308 2773	0616 2465	0924 2157	1232 1849	1541
$\bar{2}.49$	,36749 ,00309,2		0309 2783	0618 2474	0928 2164	1237 1855	1546

General Table I. $\lambda(a^{10})$, $\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$.

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{2}$.50	,37058 ,00310,3	,00	0310 2794	0621 2482	0931 2172	1241 1862	1552
$\bar{2}$.51	,37368 ,00311,5		0312 2804	0623 2492	0935 2181	1246 1869	1558
$\bar{2}$.52	,37680 ,00312,6		0313 2813	0625 2501	0938 2188	1250 1876	1563
$\bar{2}$.53	,37993 ,00313,8		0314 2824	0628 2510	0941 2197	1255 1883	1569
$\bar{2}$.54	,38306 ,00314,9		0315 2834	0630 2519	0945 2204	1260 1889	1575
$\bar{2}$.55	,38621 ,00316,1		0316 2845	0632 2529	0948 2213	1265 1897	1581
$\bar{2}$.56	,38937 ,00317,3		0317 2856	0635 2538	0952 2221	1269 1904	1587
$\bar{2}$.57	,39255 ,00318,5		0319 2867	0637 2548	0956 2230	1274 1911	1593
$\bar{2}$.58	,39573 ,00319,6		0320 2876	0639 2557	0959 2237	1278 1918	1598
$\bar{2}$.59	,39893 ,00320,8		0321 2887	0642 2566	0962 2246	1283 1925	1604
$\bar{2}$.60	,40213 ,00322,0		0322 2898	0644 2576	0966 2254	1288 1932	1610
$\bar{2}$.61	,40536 ,00323,3		0323 2910	0647 2586	0970 2263	1293 1940	1617
$\bar{2}$.62	,40859 ,00324,5		0325 2921	0649 2596	0974 2272	1298 1947	1623
$\bar{2}$.63	,41183 ,00325,6		0326 2930	0651 2605	0977 2279	1302 1954	1628
$\bar{2}$.64	,41509 ,00326,8		0327 2941	0654 2614	0980 2288	1307 1961	1634
$\bar{2}$.65	,41836 ,00328,3		0328 2955	0657 2626	0985 2298	1313 1970	1642
$\bar{2}$.66	,42164 ,00329,4		0329 2965	0659 2635	0988 2306	1318 1976	1647
$\bar{2}$.67	,42493 ,00330,6		0331 2975	0661 2645	0992 2314	1322 1984	1653
$\bar{2}$.68	,42833 ,00331,9		0332 2987	0664 2655	0996 2323	1328 1991	1660
$\bar{2}$.69	,43156 ,00333,2		0333 2999	0666 2666	1000 2332	1333 1999	1666
$\bar{2}$.70	,43490 ,00334,4		0334 3010	0669 2675	1003 2341	1338 2006	1672
$\bar{2}$.71	,43824 ,00335,7		0336 3021	0671 2686	1007 2350	1343 2014	1679
$\bar{2}$.72	,44159 ,00337,1		0337 3034	0674 2697	1011 2360	1348 2023	1686
$\bar{2}$.73	,44496 ,00338,2		0338 3044	0676 2706	1015 2367	1353 2029	1691
$\bar{2}$.74	,44835 ,00339,6		0340 3056	0679 2717	1019 2377	1358 2038	1698

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{2}$.75	,45174 ,00340,8	,00	0341 3067	0682 2726	1022 2386	1363 2045	1704
$\bar{2}$.76	,45515 ,00342,3		0342 3081	0685 2738	1027 2396	1369 2054	1712
$\bar{2}$.77	,45857 ,00343,5		0343 3092	0687 2748	1031 2405	1374 2061	1718
2.78	,46201 ,00345,0		0345 3105	0690 2760	1035 2415	1380 2070	1725
$\bar{2}$.79	,46546 ,00346,1		0346 3115	0692 2769	1038 2423	1384 2077	1731
$\bar{2}$.80	,46892 ,00347,5		0348 3128	0695 2780	1043 2433	1390 2085	1738
$\bar{2}$.81	,47239 ,00348,9		0349 3140	0698 2791	1047 2442	1396 2093	1745
$\bar{2}$.82	,47588 ,00350,2		0350 3152	0700 2802	1051 2451	1410 2101	1751
$\bar{2}$.83	,47938 ,00351,6		0352 3164	0703 2813	1055 2461	1406 2110	1758
$\bar{2}$.84	,48290 ,00353,0		0353 3177	0706 2824	1059 2471	1412 2118	1765
$\bar{2}$.85	,48643 ,00354,3		0354 3189	0709 2834	1063 2486	1417 2126	1772
$\bar{2}$.86	,48997 ,00355,8		0356 3202	0712 2846	1067 2491	1423 2135	1779
$\bar{2}$.87	,49353 ,00357,1		0357 3214	0714 2857	1071 2500	1428 2143	1786
$\bar{2}$.88	,49710 ,00358,5		0359 3227	0717 2868	1076 2510	1434 2151	1793
$\bar{2}$.89	,50069 ,00360,0		0360 3240	0720 2880	1080 2520	1440 2160	1800
$\bar{2}$.90	,50429 ,00361,4		0361 3253	0723 2891	1084 2530	1446 2168	1807
$\bar{2}$.91	,50790 ,00362,7		0363 3264	0725 2902	1088 2539	1451 2176	1814
$\bar{2}$.92	,51153 ,00364,2		0364 3278	0728 2914	1093 2549	1457 2185	1821
$\bar{2}$.93	,51517 ,00365,6		0366 3290	0731 2925	1097 2559	1462 2194	1828
$\bar{2}$.94	,51883 ,00367,1		0367 3304	0734 2937	1101 2570	1468 2203	1836
$\bar{2}$.95	,52250 ,00368,5		0369 3317	0737 2948	1106 2580	1474 2211	1843
$\bar{2}$.96	,52618 ,00369,9		0370 3329	0740 2959	1110 2589	1480 2219	1850
$\bar{2}$.97	,52988 ,00371,4		0371 3343	0743 2971	1114 2600	1486 2228	1857
$\bar{2}$.98	,53360 ,00372,9		0373 3356	0746 2983	1119 2610	1492 2237	1865
$\bar{2}$.99	,53732 ,00374,4		0374 3370	0749 2995	1123 2620	1498 2246	1872

General Table I. $\lambda\,(a^{10}),\ \lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$.

$\lambda\,(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
1.00	,54107	,00	0376	0752	1127	1503	1879
	,00375,8		3382	3006	2631	2255	
1.01	,54483		0377	0755	1132	1510	1887
	,00377,4		3397	3019	2642	2264	
1.02	,54860		0379	0757	1136	1515	1894
	,00378,7		3408	3030	2651	2273	
1.03	,55239		0380	0761	1141	1521	1902
	,00380,3		3423	3042	2662	2282	
1.04	,55619		0382	0764	1146	1528	1910
	,00381,9		3437	3055	2673	2291	
1.05	,56001		0383	0767	1150	1533	1917
	,00383,3		3450	3066	2683	2300	
1.06	,56384		0385	0770	1155	1540	1925
	,00384,9		3464	3079	2694	2309	
1.07	,56769		0386	0773	1159	1546	1932
	,00386,4		3478	3091	2705	2318	
1.08	,57156		0388	0776	1164	1552	1940
	,00388,0		3492	3104	2716	2328	
$\bar{1}$.09	,57544		0389	0779	1168	1558	1947
	,00389,4		3505	3115	2726	2336	
$\bar{1}$.10	,57933		0391	0782	1173	1564	1955
	,00391,0		3519	3128	2737	2346	
$\bar{1}$.11	,58324		0393	0785	1178	1570	1963
	,00392,5		3533	3140	2748	2355	
1.12	,58717		0394	0788	1183	1577	1971
	,00394,2		3548	3154	2759	2365	
1.13	,59111		0396	0792	1188	1584	1980
	,00395,9		3563	3167	2771	2375	
1.14	,59507		0397	0794	1191	1588	1986
	,00397,1		3574	3177	2780	2383	
$\bar{1}$.15	,59904		0399	0798	1196	1595	1994
	,00398,8		3589	3190	2792	2393	
$\bar{1}$.16	,60303		0401	0801	1202	1602	2003
	,00400,5		3605	3204	2804	2403	
$\bar{1}$.17	,60703		0402	0804	1206	1608	2011
	,00402,1		3619	3217	2815	2413	
$\bar{1}$.18	,61105		0404	0827	1211	1615	2019
	,00403,7		3633	3230	2826	2422	
$\bar{1}$.19	,61509		0405	0811	1216	1621	2027
	,00405,3		3648	3242	2837	2432	
$\bar{1}$.20	,61914		0407	0814	1221	1628	2035
	,00406,9		3662	3255	2848	2441	
$\bar{1}$.21	,62321		0409	0817	1226	1634	2043
	,00408,5		3677	3268	2860	2451	
$\bar{1}$.22	,62729		0410	0820	1230	1640	2051
	,00410,1		3691	3281	2871	2461	
$\bar{1}$.23	,63140		0412	0824	1235	1647	2059
	,00411,8		3706	3294	2883	2471	
$\bar{1}$.24	,63551		0413	0826	1239	1652	2066
	,00413,1		3718	3305	2892	2489	

$\lambda\,(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{1}$.25	,63964	,00	0415	0831	1246	1661	2077
	,00415,3		3748	3322	2907	2492	
$\bar{1}$.26	,64380		0417	0833	1250	1667	2084
	,00416,7		3750	3334	2917	2500	
$\bar{1}$.27	,64796		0418	0837	1255	1674	2092
	,00418,4		3766	3347	2929	2510	
$\bar{1}$.28	,65215		0420	0840	1260	1680	2100
	,00420,0		3780	3360	2940	2520	
$\bar{1}$.29	,65685		0422	0843	1265	1687	2109
	,00421,7		3795	3374	2952	2530	
$\bar{1}$.30	,66056		0423	0847	1270	1693	2117
	,00423,3		3810	3386	2963	2540	
$\bar{1}$.31	,66480		0425	0850	1275	1700	2125
	,00425,0		3825	3400	2975	2550	
$\bar{1}$.32	,66905		0427	0853	1280	1707	2134
	,00426,7		3840	3414	2987	2560	
$\bar{1}$.33	,67331		0428	0857	1285	1714	2143
	,00428,4		3856	3427	2999	2570	
$\bar{1}$.34	,67760		0430	0860	1290	1720	2151
	,00430,1		3871	3441	3011	2581	
$\bar{1}$.35	,68190		0432	0863	1295	1727	2159
	,00431,7		3885	3454	3022	2590	
$\bar{1}$.36	,68622		0434	0868	1302	1736	2170
	,00434,0		3906	3472	3038	2604	
$\bar{1}$.37	,69056		0435	0869	1304	1739	2174
	,00434,7		3912	3478	3043	2608	
$\bar{1}$.38	,69490		0437	0874	1311	1748	2185
	,00437,0		3933	3496	3059	2622	
$\bar{1}$.39	,69927		0439	0877	1316	1755	2194
	,00438,7		3948	3510	3071	2632	
$\bar{1}$.40	,70366		0440	0881	1321	1762	2202
	,00440,4		3964	3523	3083	2642	
$\bar{1}$.41	,70806		0442	0884	1326	1768	2211
	,00442,1		3979	3537	3095	2653	
$\bar{1}$.42	,71249		0444	0888	1331	1775	2219
	,00443,8		3994	3550	3107	2663	
$\bar{1}$.43	,71692		0445	0890	1335	1780	2225
	,00445,0		4005	3560	3115	2670	
$\bar{1}$.44	,72137		0448	0896	1344	1792	2240
	,00448,0		4032	3584	3136	2688	
$\bar{1}$.45	,72585		0449	0898	1347	1796	2246
	,00449,1		4042	3593	3144	2695	
$\bar{1}$.46	,73035		0451	0902	1352	1803	2254
	,00450,8		4057	3606	3156	2705	
$\bar{1}$.47	,73485		0453	0905	1358	1811	2264
	,00452,7		4074	3622	3169	2716	
$\bar{1}$.48	,73938		0454	0909	1363	1818	2272
	,00454,4		4090	3635	3181	2726	
$\bar{1}$.49	,74393		0456	0912	1368	1824	2281
	,00456,1		4105	3649	3193	2737	

General Table I. $\lambda(a^{10})$; $\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$.

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5	$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{1}.50$	,74849	,00	0458	0916	1374	1832	2290	$\bar{1}.75$	,86842	,00	0504	1007	1511	2015	2519
	,00458,0		4122	3664	3206	2748			,00503,7		4533	4030	3526	3022	
$\bar{1}.51$	,75307		0460	0920	1379	1839	2299	$\bar{1}.76$	,87346		0506	1011	1517	2022	2528
	,00459,8		4138	3678	3219	2759			,00505,6		4550	4045	3539	3034	
$\bar{1}.52$	,75766		0462	0923	1385	1846	2308	$\bar{1}.77$	,87851		0507	1015	1522	2030	2537
	,00461,6		4154	3693	3231	2770			,00507,4		4567	4059	3552	3044	
$\bar{1}.53$	,76228		0463	0927	1390	1853	2317	$\bar{1}.78$	,88359		0509	1019	1528	2038	2547
	,00463,3		4170	3706	3243	2780			,00509,4		4585	4075	3566	3056	
$\bar{1}.54$	,76691		0465	0930	1396	1861	2326	$\bar{1}.79$	,88868		0511	1022	1534	2045	2556
	,00465,2		4187	3722	3256	2791			,00511,2		4611	4090	3578	3067	
$\bar{1}.55$	,77156		0467	0933	1400	1867	2334	$\bar{1}.80$	,89379		0513	1026	1539	2052	2566
	,00466,7		4200	3734	3267	2800			,00513,1		4618	4105	3592	3079	
$\bar{1}.56$	,77623		0469	0938	1407	1876	2345	$\bar{1}.81$	,89892		0515	1030	1545	2060	2575
	,00469,0		4221	3752	3283	2814			,00515,0		4635	4120	3605	3090	
$\bar{1}.57$	,78092		0470	0941	1411	1882	2352	$\bar{1}.82$	,90407		0517	1034	1550	2067	2584
	,00470,4		4234	3763	3293	2822			,00516,8		4651	4134	3618	3101	
$\bar{1}.58$	,78562		0472	0945	1417	1890	2362	$\bar{1}.83$	,90924		0519	1038	1556	2075	2594
	,00472,4		4252	3779	3307	2834			,00518,8		4669	4150	3632	3113	
$\bar{1}.59$	,79035		0474	0948	1422	1896	2371	$\bar{1}.84$	,91443		0521	1041	1562	2082	2603
	,00474,1		4267	3793	3319	2845			,00520,6		4685	4165	3644	3124	
$\bar{1}.60$	,79509		0476	0952	1428	1904	2380	$\bar{1}.85$	,91964		0523	1045	1568	2090	2613
	,00476,0		4284	3808	3332	2856			,00522,6		4703	4181	3658	3136	
$\bar{1}.61$	,79985		0478	0956	1433	1911	2389	$\bar{1}.86$	,92486		0524	1049	1573	2098	2622
	,00477,8		4300	3832	3345	2867			,00524,4		4720	4195	3671	3146	
$\bar{1}.62$	,80463		0480	0959	1439	1918	2398	$\bar{1}.87$	,93011		0526	1052	1578	2104	2631
	,00479,6		4316	3837	3357	2878			,00526,1		4735	4209	3683	3157	
$\bar{1}.63$	,80942		0482	0963	1445	1926	2408	$\bar{1}.88$	,93537		0528	1057	1585	2113	2642
	,00481,5		4334	3852	3371	2889			,00528,3		4755	4226	3698	3170	
$\bar{1}.64$	,81424		0483	0967	1450	1933	2417	$\bar{1}.89$	,94065		0530	1060	1590	2120	2650
	,00483,3		4350	3866	3383	2900			,00530,0		4770	4240	3710	3180	
$\bar{1}.65$	,81907		0486	0971	1457	1943	2429	$\bar{1}.90$	,94595		0532	1064	1596	2128	2660
	,00485,7		4371	3886	3400	2914			,00532,0		4788	4256	3724	3192	
$\bar{1}.66$	,82393		0486	0973	1459	1946	2432	$\bar{1}.91$	,95127		0534	1068	1601	2135	2669
	,00486,4		4378	3891	3405	2918			,00533,8		4804	4270	3737	3203	
$\bar{1}.67$	,82879		0489	0978	1466	1955	2444	$\bar{1}.92$	,95661		0536	1072	1607	2143	2679
	,00488,8		4399	3910	3422	2933			,00535,8		4822	4286	3751	3215	
$\bar{1}.68$	,83368		0492	0983	1475	1967	2459	$\bar{1}.93$	,96197		0538	1075	1613	2150	2683
	,00491,7		4425	3934	3442	2950			,00537,6		4838	4301	3763	3226	
$\bar{1}.69$	,83859		0493	0985	1478	1970	2463	$\bar{1}.94$	,96734		0540	1079	1619	2158	2698
	,00492,5		4433	3940	3448	2955			,00539,6		4826	4317	3777	3238	
$\bar{1}.70$	,84351		0494	0989	1483	1978	2472	$\bar{1}.95$	,97274		0541	1083	1624	2166	2707
	,00494,4		4450	3955	3461	2966			,00541,4		4873	4331	3790	3248	
$\bar{1}.71$	,84846		0496	0993	1489	1985	2482	$\bar{1}.96$	,97815		0543	1087	1630	2173	2717
	,00496,3		4467	3970	3474	2978			,00543,3		4890	4346	3703	3260	
$\bar{1}.72$	,85342		0498	0996	1494	1992	2491	$\bar{1}.97$	,98359		0545	1090	1636	2181	2726
	,00498,1		4483	3985	3487	2989			,00545,2		4907	4362	3716	3271	
$\bar{1}.73$	,85840		0500	1000	1500	2000	2500	$\bar{1}.98$	,98904		0547	1094	1642	2189	2736
	,00499,9		4499	3999	3499	2999			,00547,2		4925	4378	3830	3283	
$\bar{1}.74$	,86340		0502	1004	1506	2008	2510	$\bar{1}.99$	,99451						
	,00501,9		4517	4005	3513	3011									

General Table II. $\lambda(a^7), \lambda\left(\frac{1-a^7}{a^{-1}-1}\right)$.

$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{3}.00$	$\bar{1},77356$	,00	0227	0454	0681	0908	1135
	,00227,0		2043	1816	1589	1362	
$\bar{3}.01$	$\bar{1},77583$		0227	0455	0682	0910	1137
	,00227,4		2047	1819	1592	1364	
$\bar{3}.02$	$\bar{1},77810$		0228	0456	0683	0911	1139
	,00227,8		2050	1822	1595	1367	
$\bar{3}.03$	$\bar{1},78038$		0228	0457	0685	0913	1142
	,00228,3		2055	1826	1598	1370	
$\bar{3}.04$	$\bar{1},78266$		0229	0457	0686	0914	1143
	,00228,6		2057	1829	1600	1372	
$\bar{3}.05$	$\bar{1},78495$		0229	0458	0688	0917	1146
	,00229,2		2063	1834	1604	1375	
$\bar{3}.06$	$\bar{1},78724$		0230	0460	0689	0918	1148
	,00229,6		2066	1837	1607	1378	
$\bar{3}.07$	$\bar{1},78954$		0230	0460	0690	0920	1150
	,00230,0		2070	1840	1610	1380	
$\bar{3}.08$	$\bar{1},79184$		0231	0461	0692	0922	1153
	,00230,5		2075	1844	1614	1383	
$\bar{3}.09$	$\bar{1}.79414$		0231	0462	0692	0923	1154
	,00230,8		2077	1846	1616	1385	
$\bar{3}.10$	$\bar{1},79645$		0231	0463	0694	0926	1157
	,00231,4		2083	1851	1620	1388	
$\bar{3}.11$	$\bar{1},79877$		0232	0464	0695	0927	1159
	,00231,8		2086	1854	1623	1391	
$\bar{1}.12$	$\bar{1},80108$		0232	0464	0697	0929	1161
	,00232,2		2090	1858	1625	1393	
$\bar{3}.13$	$\bar{1},80341$		0233	0465	0698	0931	1164
	,00232,7		2094	1862	1629	1396	
$\bar{3}.14$	$\bar{1},80573$		0233	0466	0700	0933	1166
	,00233,2		2099	1866	1632	1399	
$\bar{3}.15$	$\bar{1},80806$		0234	0467	0701	0934	1168
	,00233,6		2102	1869	1635	1402	
$\bar{3}.16$	$\bar{1},81040$		0234	0468	0703	0937	1171
	,00234,2		2108	1874	1639	1405	
$\bar{3}.17$	$\bar{1},81274$		0235	0469	0704	0938	1173
	,00234,5		2111	1876	1642	1407	
$\bar{3}.18$	$\bar{1},81509$		0235	0470	0705	0940	1175
	,00235,0		2115	1880	1645	1410	
$\bar{3}.19$	$\bar{1},81744$		0236	0471	0707	0942	1178
	00,235,5		2120	1884	1649	1413	
$\bar{3}.20$	$\bar{1},81979$		0236	0472	0708	0944	1180
	,00236,0		2124	1888	1652	1416	
$\bar{3}.21$	$\bar{1},82215$		0237	0473	0710	0946	1183
	,00236,5		2129	1892	1656	1419	
$\bar{3}.22$	$\bar{1},82452$		0237	0474	0711	0948	1185
	,00237,0		2133	1896	1659	1422	
$\bar{3}.23$	$\bar{1},82689$		0237	0475	0712	0950	1187
	,00237,4		2137	1899	1662	1424	
$\bar{3}.24$	$\bar{1},82926$		0238	0476	0714	0952	1190
	,00238,0		2142	1904	1666	1428	

$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{3}.25$	$\bar{1},83164$	,00	0239	0477	0716	0954	1193
	,00238,5		2147	1908	1670	1431	
$\bar{3}.26$	$\bar{1},83403$		0239	0478	0717	0956	1195
	,00239,0		2151	1912	1673	1434	
$\bar{3}.27$	$\bar{1},83641$		0240	0479	0719	0958	1198
	,00239,5		2156	1916	1677	1437	
$\bar{3}.28$	$\bar{1},83881$		0240	0480	0720	0960	1200
	,00240,0		2160	1920	1680	1440	
$\bar{3}.29$	$\bar{1},84121$		0241	0481	0722	0962	1203
	,00240,5		2165	1924	1684	1443	
$\bar{3}.30$	$\bar{1},84361$		0241	0482	0723	0964	1205
	,00240,9		2168	1927	1686	1445	
$\bar{3}.31$	$\bar{1},84602$		0242	0483	0725	0966	1208
	,00241,5		2174	1932	1691	1449	
$\bar{3}.32$	$\bar{1},84844$		0242	0484	0726	0968	1210
	,00241,9		2177	1935	1693	1451	
$\bar{3}.33$	$\bar{1},85086$		0243	0485	0728	0970	1213
	,00242,5		2183	1940	1698	1455	
$\bar{3}.34$	$\bar{1},85328$		0243	0486	0730	0973	1216
	,00243,2		2189	1946	1702	1459	
$\bar{3}.35$	$\bar{1},85571$		0244	0487	0731	0974	1218
	,00243,6		2192	1949	1705	1462	
$\bar{3}.36$	$\bar{1},85815$		0244	0488	0732	0976	1221
	,00244,1		2197	1953	1709	1463	
$\bar{3}.37$	$\bar{1},86059$		0245	0489	0734	0979	1224
	,00244,7		2202	1958	1713	1468	
$\bar{3}.38$	$\bar{1},86304$		0245	0490	0736	0981	1226
	,00245,2		2207	1962	1716	1471	
$\bar{3}.39$	$\bar{1},86549$		0246	0492	0737	0983	1229
	,00245,8		2212	1966	1721	1475	
$\bar{3}.40$	$\bar{1},86795$		0246	0492	0739	0985	1231
	,00246,2		2216	1970	1723	1477	
$\bar{3}.41$	$\bar{1},87041$		0247	0494	0741	0988	1235
	,00246,9		2222	1975	1728	1481	
$\bar{3}.42$	$\bar{1},87288$		0247	0495	0742	0990	1237
	,00247,4		2227	1979	1732	1484	
$\bar{3}.43$	$\bar{1},87535$		0248	0496	0744	0992	1240
	,00247,9		2231	1983	1735	1487	
$\bar{3}.44$	$\bar{1},87783$		0249	0497	0746	0994	1243
	,00248,5		2237	1988	1740	1491	
$\bar{3}.45$	$\bar{1},88032$		0249	0498	0747	0996	1246
	,00249,1		2242	1993	1744	1495	
$\bar{3}.46$	$\bar{1},88281$		0250	0499	0749	0999	1248
	,00249,7		2247	1998	1748	1498	
$\bar{3}.47$	$\bar{1},88530$		0250	0500	0751	1001	1251
	,00250,2		2252	2002	1751	1501	
$\bar{3}.48$	$\bar{1},88780$		0251	0502	0752	1003	1254
	,00250,8		2257	2006	1756	1505	
$\bar{3}.49$	$\bar{1},89031$		0251	0503	0754	1006	1257
	,00251,4		2263	2011	1760	1508	

General Table II. $\lambda\,(a^7),\ \lambda\left\{\frac{1-a^7}{a^{-1}-1}\right\}$.

$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{3}.50$	$\bar{1}$,89283	,00	0252	0504	0756	1008	1260
	,00252,0		2268	2016	1764	1512	
$\bar{3}.51$	$\bar{1}$,89535		0253	0505	0758	1010	1263
	,00252,6		2273	2021	1768	1516	
$\bar{3}.52$	$\bar{1}$,89787		0253	0506	0759	1012	1266
	,00253,1		2278	2025	1772	1519	
$\bar{3}.53$	$\bar{1}$,90040		0254	0507	0761	1015	1269
	,00253,7		2283	2030	1776	1522	
$\bar{3}.54$	$\bar{1}$,90294		0254	0509	0763	1017	1272
	,00254,3		2289	2034	1780	1526	
$\bar{3}.55$	$\bar{1}$,90548		0255	0510	0765	1020	1275
	,00255,0		2295	2040	1785	1530	
$\bar{3}.56$	$\bar{1}$,90803		0256	0511	0767	1022	1278
	,00255,5		2300	2044	1789	1533	
$\bar{3}.57$	$\bar{1}$,91059		0256	0512	0769	1025	1281
	,00256,2		2306	2050	1793	1537	
$\bar{3}.58$	$\bar{1}$,91315		0257	0514	0770	1027	1284
	,00256,8		2311	2054	1798	1541	
$\bar{3}.59$	$\bar{1}$,91572		0257	0515	0772	1030	1287
	,00257,4		2317	2059	1802	1544	
$\bar{3}.60$	$\bar{1}$,91829		0258	0516	0774	1032	1290
	,00258,0		2322	2064	1806	1548	
$\bar{3}.61$	$\bar{1}$,92087		0259	0517	0776	1035	1294
	,00258,7		2328	2070	1811	1552	
$\bar{3}.62$	$\bar{1}$,92346		0259	0518	0778	1037	1296
	,00259,2		2333	2074	1814	1555	
$\bar{3}.63$	$\bar{1}$,92605		0260	0520	0780	1040	1300
	,00259,9		2339	2079	1819	1559	
$\bar{3}.64$	$\bar{1}$,92865		0261	0521	0782	1042	1303
	,00260,6		2345	2085	1824	1564	
$\bar{3}.65$	$\bar{1}$,93125		0261	0522	0784	1045	1306
	,00261,2		2351	2090	1828	1567	
$\bar{3}.66$	$\bar{1}$,93387		0262	0524	0785	1047	1309
	,00261,8		2356	2094	1833	1571	
$\bar{3}.67$	$\bar{1}$,93648		0263	0525	0788	1050	1313
	,00262,5		2363	2100	1838	1575	
$\bar{3}.68$	$\bar{1}$,93911		0263	0526	0790	1053	1316
	,00263,2		2369	2106	1842	1579	
$\bar{3}.69$	$\bar{1}$,94174		0264	0528	0791	1055	1319
	,00263,8		2374	2110	1847	1583	
$\bar{3}.70$	$\bar{1}$,94438		0264	0529	0793	1058	1322
	,00264,4		2380	2115	1851	1586	
$\bar{3}.71$	$\bar{1}$,94702		0265	0530	0795	1060	1326
	,00265,1		2386	2121	1856	1591	
$\bar{3}.72$	$\bar{1}$,94967		0266	0532	0797	1063	1329
	,00265,8		2392	2126	1861	1595	
$\bar{3}.73$	$\bar{1}$,95233		0267	0533	0800	1066	1333
	,00266,5		2399	2132	1866	1599	
$\bar{3}.74$	$\bar{1}$,95500		0267	0534	0802	1069	1336
	,00267,2		2405	2138	1870	1603	

$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{3}.75$	$\bar{1}$,95767	,00	0268	0536	0804	1072	1340
	,00268,0		2412	2144	1876	1608	
$\bar{3}.76$	$\bar{1}$,96035		0269	0537	0806	1074	1343
	,00268,5		2417	2148	1880	1611	
$\bar{3}.77$	$\bar{1}$,96303		0269	0539	0808	1077	1347
	,00269,3		2424	2154	1885	1616	
$\bar{3}.78$	$\bar{1}$,96573		0270	0540	0810	1080	1350
	,00270,0		2430	2160	1890	1620	
$\bar{3}.79$	$\bar{1}$,96843		0271	0541	0812	1083	1354
	,00270,7		2436	2166	1895	1624	
$\bar{3}.80$	$\bar{1}$,97113		0271	0543	0814	1085	1357
	,00271,3		2442	2170	1899	1628	
$\bar{3}.81$	$\bar{1}$,97385		0272	0544	0816	1088	1361
	,00272,1		2449	2177	1905	1633	
$\bar{3}.82$	$\bar{1}$,97657		0273	0546	0819	1092	1365
	,00272,9		2456	2183	1910	1637	
$\bar{3}.83$	$\bar{1}$,97930		0273	0547	0820	1094	1367
	,00273,4		2461	2187	1914	1640	
$\bar{3}.84$	$\bar{1}$,98203		0274	0549	0823	1097	1372
	,00274,3		2469	2194	1920	1646	
$\bar{3}.85$	$\bar{1}$,98477		0275	0550	0825	1100	1375
	,00275,0		2475	2200	1925	1650	
$\bar{3}.86$	$\bar{1}$,98752		0276	0552	0827	1103	1379
	,00275,8		2482	2206	1931	1655	
$\bar{3}.87$	$\bar{1}$,99028		0277	0553	0830	1106	1383
	,00276,5		2489	2212	1936	1659	
$\bar{3}.88$	$\bar{1}$,99305		0277	0555	0832	1109	1387
	,00277,3		2496	2218	1941	1664	
$\bar{3}.89$	$\bar{1}$,99582		0278	0556	0834	1112	1390
	,00278,0		2502	2224	1946	1668	
$\bar{3}.90$	$\bar{1}$,99860		0279	0558	0836	1115	1394
	,00278,8		2509	2230	1952	1673	
$\bar{3}.91$	,00139		0280	0559	0839	1118	1398
	,00279,5		2516	2236	1957	1677	
$\bar{3}.92$	,00418		0280	0561	0841	1121	1402
	,00280,3		2523	2242	1962	1682	
$\bar{3}.93$	,00699		0281	0562	0843	1124	1406
	,00281,1		2530	2249	1968	1687	
$\bar{3}.94$	,00980		0282	0564	0846	1128	1410
	,00281,9		2537	2255	1973	1691	
$\bar{3}.95$	,01262		0283	0565	0848	1131	1414
	,00282,7		2544	2262	1979	1696	
$\bar{3}.96$	,01544		0283	0567	0850	1134	1417
	,00283,4		2551	2267	1984	1700	
$\bar{3}.97$	,01828		0284	0568	0853	1137	1421
	,00284,2		2558	2274	1989	1705	
$\bar{3}.98$	,02112		0285	0570	0855	1140	1425
	,00285,0		2565	2280	1995	1710	
$\bar{3}.99$	,02397		0286	0572	0858	1144	1430
	,00285,9		2573	2287	2001	1715	

General Table II. $\lambda(a^7)$, $\lambda\left(\frac{1-a^7}{a^{-1}-1}\right)$.

$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{2}$.00	,02683	,00	0287	0573	0860	1147	1434
	,00286,7		2580	2294	2007	1720	
$\bar{2}$.01	,02969		0288	0575	0863	1150	1438
	,00287,5		2588	2300	2013	1725	
$\bar{2}$.02	,03257		0288	0577	0865	1153	1442
	,00288,3		2595	2306	2018	1730	
$\bar{2}$.03	,03545		0289	0578	0867	1156	1446
	,00289,1		2602	2313	2024	1735	
$\bar{2}$.04	,03834		0290	0580	0870	1160	1450
	,00290,0		2610	2320	2030	1740	
$\bar{2}$.05	,04124		0291	0582	0872	1163	1454
	,00290,8		2617	2326	2036	1745	
$\bar{2}$.06	,04415		0292	0583	0875	1167	1459
	,00291,7		2625	2334	2043	1750	
$\bar{2}$.07	,04707		0293	0585	0878	1170	1463
	,00292,5		2633	2340	2048	1755	
$\bar{2}$.08	,04999		0293	0587	0880	1174	1467
	,00293,4		2641	2347	2054	1760	
$\bar{2}$.09	,05293		0294	0588	0883	1177	1471
	,00294,2		2648	2354	2059	1765	
$\bar{2}$.10	,05587		0295	0590	0885	1180	1476
	,00295,1		2656	2361	2066	1771	
$\bar{2}$.11	,05882		0296	0592	0888	1184	1480
	,00296,0		2664	2368	2072	1776	
$\bar{2}$.12	,06178		0297	0594	0890	1187	1484
	,00296,8		2671	2374	2078	1781	
$\bar{2}$.13	,06475		0298	0596	0893	1191	1489
	,00297,8		2680	2382	2085	1787	
$\bar{2}$.14	,06772		0299	0597	0896	1194	1493
	,00298,6		2687	2389	2090	1792	
$\bar{2}$.15	,07071		0300	0599	0899	1198	1498
	,00299,5		2696	2396	2097	1797	
$\bar{2}$.16	,07370		0300	0601	0901	1202	1502
	,00300,4		2704	2403	2103	1802	
$\bar{2}$.17	,07671		0301	0603	0904	1205	1507
	,00301,3		2712	2410	2109	1808	
$\bar{2}$.18	,07972		0302	0604	0907	1209	1511
	,00302,2		2720	2418	2115	1813	
$\bar{2}$.19	,08274		0303	0606	0902	1212	1516
	,00303,1		2728	2425	2122	1819	
$\bar{2}$.20	,08577		0304	0608	0912	1216	1521
	,00304,1		2737	2433	2129	1825	
$\bar{2}$.21	,08882		0305	0610	0915	1220	1525
	,00305,0		2745	2440	2135	1830	
$\bar{2}$.22	,09187		0306	0612	0918	1224	1530
	,00305,9		2753	2447	2141	1835	
$\bar{2}$.23	,09493		0307	0614	0921	1228	1535
	,00306,9		2762	2455	2148	1841	
$\bar{2}$.24	,09799		0308	0616	0923	1231	1539
	,00307,8		2770	2462	2155	1847	

$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{2}$.25	,10107	,00	0309	0618	0926	1235	1544
	,00308,8		2779	2470	2162	1853	
$\bar{2}$.26	,10416		0310	0620	0929	1239	1549
	,00309,8		2788	2478	2169	1859	
$\bar{2}$.27	,10726		0311	0621	0932	1243	1554
	,00310,7		2796	2486	2175	1864	
$\bar{2}$.28	,11037		0312	0623	0935	1247	1559
	,00311,7		2805	2494	2182	1870	
$\bar{2}$.29	,11348		0313	0625	0938	1251	1564
	,00312,7		2814	2502	2189	1876	
$\bar{2}$.30	,11661		0314	0627	0941	1255	1569
	,00313,7		2823	2510	2196	1882	
$\bar{2}$.31	,11975		0315	0629	0944	1258	1573
	,00314,6		2831	2517	2202	1888	
$\bar{2}$.32	,12289		0316	0631	0947	1262	1578
	,00315,6		2840	2525	2209	1894	
$\bar{2}$.33	,12605		0317	0633	0950	1267	1583
	,00316,7		2850	2534	2216	1900	
$\bar{2}$.34	,12921		0318	0635	0953	1271	1589
	,00317,7		2859	2542	2224	1906	
$\bar{2}$.35	,13239		0319	0637	0956	1275	1594
	,00318,7		2868	2550	2231	1912	
$\bar{2}$.36	,13558		0320	0639	0959	1279	1599
	,00319,7		2877	2558	2238	1918	
$\bar{2}$.37	,13898		0321	0641	0962	1283	1604
	,00320,7		2886	2566	2245	1924	
$\bar{2}$.38	,14198		0322	0644	0965	1287	1609
	,00321,8		2896	2574	2253	1931	
$\bar{2}$.39	,14520		0323	0646	0968	1291	1614
	,00322,8		2905	2582	2260	1937	
$\bar{2}$.40	,14843		0324	0648	0972	1296	1620
	,00323,9		2915	2591	2267	1943	
$\bar{2}$.41	,15167		0325	0650	0975	1300	1625
	,00324,9		2924	2599	2274	1949	
$\bar{2}$.42	,15492		0326	0652	0978	1304	1630
	,00325,9		2933	2607	2281	1955	
$\bar{2}$.43	,15818		0327	0654	0981	1308	1636
	,00327,1		2944	2617	2290	1963	
$\bar{2}$.44	,16145		0328	0656	0984	1312	1641
	,00328,1		2953	2625	2297	1969	
$\bar{2}$.45	,16473		0329	0658	0988	1317	1646
	,00329,2		2963	2634	2304	1975	
$\bar{2}$.46	,16802		0330	0661	0991	1321	1652
	,00330,3		2973	2642	2312	1982	
$\bar{2}$.47	,17132		0331	0663	0994	1326	1657
	,00331,4		2983	2651	2320	1988	
$\bar{2}$.48	,17464		0333	0665	0998	1330	1663
	,00332,5		2993	2660	2328	1995	
$\bar{2}$.49	,17796		0334	0667	1001	1334	1668
	,00333,6		3002	2669	2335	2002	

General Table II. $\lambda(a^\gamma)$, $\lambda\left(\frac{1-a^\gamma}{a^{-1}-1}\right)$.

$\lambda(a^\gamma)$	$\lambda\left(\frac{1-a^\gamma}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{2}.50$	,18130	,00	0335	0669	1004	1339	1674
	,00334,7		3012	2678	2343	2008	
$\bar{2}.51$	,18464		0336	0672	1007	1343	1679
	,00335,8		3022	2686	2351	2015	
$\bar{2}.52$	,18880		0337	0674	1011	1348	1685
	,00336,9		3032	2695	2358	2021	
$\bar{2}.53$	,19137		0338	0676	1014	1352	1691
	,00338,1		3043	2705	2367	2029	
$\bar{2}.54$	,19475		0339	0678	1018	1357	1696
	,00339,2		3053	2714	2374	2035	
$\bar{2}.55$	,19815		0340	0681	1021	1362	1702
	,00340,4		3064	2723	2383	2042	
$\bar{2}.56$	,20155		0342	0683	1025	1366	1708
	,00341,5		3074	2732	2391	2049	
$\bar{2}.57$	,20496		0343	0685	1028	1371	1714
	,00342,7		3084	2742	2399	2056	
$\bar{2}.58$	,20839		0344	0688	1032	1376	1720
	,00343,9		3095	2751	2407	2063	
$\bar{2}.59$	,21183		0345	0690	1035	1380	1725
	,00345,0		3105	2760	2415	2070	
$\bar{2}.60$	,21528		0346	0692	1039	1385	1731
	,00346,2		3116	2770	2423	2077	
$\bar{2}.61$	,21874		0347	0695	1042	1390	1737
	,00347,4		3127	2779	2432	2084	
$\bar{2}.62$	,22222		0349	0697	1046	1395	1744
	,00348,7		3138	2790	2441	2092	
$\bar{2}.63$	,22570		0350	0699	1049	1399	1749
	,00349,7		3147	2798	2448	2098	
$\bar{2}.64$	,22920		0351	0702	1053	1404	1756
	,00351,1		3160	2809	2458	2107	
$\bar{2}.65$	,23271		0352	0705	1057	1409	1762
	,00352,3		3171	2818	2466	2113	
$\bar{2}.66$	,23623		0354	0707	1061	1414	1768
	,00353,5		3182	2828	2475	2121	
$\bar{2}.67$	,23977		0355	0709	1064	1419	1774
	,00354,7		3192	2838	2483	2128	
$\bar{2}.68$	,24332		0356	0712	1068	1424	1780
	,00356,0		3204	2848	2492	2136	
$\bar{2}.69$	,24688		0357	0714	1072	1429	1786
	,00357,2		3215	2858	2500	2143	
$\bar{2}.70$	,25045		0358	0717	1075	1434	1792
	,00358,4		3226	2867	2509	2150	
$\bar{2}.71$	,25403		0360	0719	1079	1439	1799
	,00359,7		3237	2878	2518	2158	
$\bar{2}.72$	,25763		0361	0722	1083	1444	1805
	,00361,0		3249	2888	2527	2166	
$\bar{2}.73$	,26124		0362	0724	1087	1449	1811
	,00362,2		3260	2898	2535	2173	
$\bar{2}.74$	,26486		0364	0727	1091	1454	1817
	,00363,5		3272	2908	2545	2181	

$\lambda(a^\gamma)$	$\lambda\left(\frac{1-a^\gamma}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{2}.75$	,26850	,00	0365	0729	1094	1459	1824
	,00364,7		3282	2918	2553	2188	
$\bar{2}.76$	,27214		0366	0732	1099	1465	1831
	,00366,2		3296	2930	2563	2197	
$\bar{2}.77$	,27581		0367	0735	1102	1470	1837
	,00367,4		3307	2939	2572	2204	
$\bar{2}.78$	,27948		0369	0737	1106	1475	1844
	,00368,7		3318	2950	2581	2212	
$\bar{2}.79$	,28317		0370	0740	1110	1480	1851
	,00370,1		3331	2961	2591	2221	
$\bar{2}.80$	,28687		0370	0743	1114	1485	1857
	,00371,3		3343	2970	2599	2228	
$\bar{2}.81$	,29058		0373	0746	1118	1491	1864
	,00372,8		3355	2982	2610	2237	
$\bar{2}.82$	,29431		0374	0748	1122	1496	1870
	,00373,9		3365	2991	2617	2243	
$\bar{2}.83$	,29805		0375	0751	1126	1502	1877
	,00375,4		3379	3003	2628	2252	
$\bar{2}.84$	,30180		0377	0753	1130	1507	1884
	,00376,7		3390	3014	2637	2260	
$\bar{2}.85$	,30557		0378	0756	1134	1512	1891
	,00378,1		3403	3025	2647	2269	
$\bar{2}.86$	,30935		0379	0759	1138	1518	1897
	,00379,4		3415	3035	2656	2276	
$\bar{2}.87$	,31314		0381	0762	1142	1523	1904
	,00380,8		3427	3046	2666	2285	
$\bar{2}.88$	,31695		0382	0764	1146	1529	1911
	,00382,2		3440	3058	2675	2293	
$\bar{2}.89$	,32077		0384	0767	1151	1534	1918
	,00383,6		3452	3069	2685	2302	
$\bar{2}.90$	,32461		0385	0770	1155	1540	1925
	,00385,0		3465	3080	2695	2310	
$\bar{2}.91$	,32846		0386	0773	1159	1545	1932
	,00386,3		3477	3090	2704	2318	
$\bar{2}.92$	,33232		0388	0776	1163	1551	1939
	,00387,8		3490	3102	2715	2327	
$\bar{2}.93$	,33620		0389	0778	1168	1557	1946
	,00389,2		3503	3114	2724	2335	
$\bar{2}.94$	,34009		0391	0781	1172	1562	1953
	,00390,6		3515	3125	2734	2344	
$\bar{2}.95$	,34400		0392	0784	1176	1568	1960
	,00392,0		3528	3136	2744	2352	
$\bar{2}.96$	,34792		0394	0787	1181	1574	1968
	,00393,5		3542	3148	2755	2361	
$\bar{2}.97$	,35185		0395	0790	1185	1580	1975
	,00394,9		3554	3159	2764	2369	
$\bar{2}.98$	,35580		0396	0793	1189	1585	1982
	,00396,3		3567	3170	2774	2378	
$\bar{2}.99$	,35976		0398	0796	1193	1591	1989
	,00397,8		3580	3182	2785	2387	

General Table II. $\lambda(a^x)$, $\lambda\left(\frac{1-a^x}{a^{-1}-1}\right)$.

$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{1}.00$	,36374	,00	0399	0799	1198	1597	1997
	,00399,5		3594	3194	2795	2396	
$\bar{1}.01$	,36773		0401	0801	1202	1603	2004
	,00400,7		3606	3206	2805	2404	
$\bar{1}.02$	,37174		0402	0804	1207	1609	2011
	,00402,2		3620	3218	2815	2413	
$\bar{1}.03$	,37576		0404	0807	1211	1615	2019
	,00403,7		3633	3230	2826	2422	
$\bar{1}.04$	,37980		0405	0811	1216	1622	2027
	,00405,4		3649	3243	2838	2432	
$\bar{1}.05$	,38385		0406	0813	1219	1626	2032
	,00406,4		3658	3251	2845	2438	
$\bar{1}.06$	,38792		0408	0816	1225	1633	2041
	,00408,2		3674	3266	2857	2449	
$\bar{1}.07$	,39200		0410	0819	1229	1639	2050
	,00409,7		3687	3278	2868	2458	
$\bar{1}.08$	,39610		0411	0822	1234	1645	2056
	,00411,2		3701	3290	2878	2467	
$\bar{1}.09$	,40021		0413	0826	1238	1651	2064
	,00412,8		3715	3302	2890	2477	
$\bar{1}.10$	,40434		0414	0828	1243	1657	2071
	,00414,2		3728	3314	2899	2485	
$\bar{1}.11$	,40848		0416	0832	1247	1663	2079
	,00415,8		3742	3326	2911	2495	
$\bar{1}.12$	,41264		0417	0835	1252	1669	2087
	,00417,3		3756	3338	2921	2504	
$\bar{1}.13$	,41681		0419	0838	1257	1676	2096
	,00419,1		3772	3353	2934	2515	
$\bar{1}.14$	,42100		0420	0840	1261	1681	2101
	,00420,2		3782	3362	2941	2521	
$\bar{1}.15$	,42520		0422	0844	1266	1688	2110
	,00421,9		3797	3375	2953	2531	
$\bar{1}.16$	,42942		0424	0847	1271	1694	2118
	,00423,6		3812	3389	2965	2542	
$\bar{1}.17$	,43366		0425	0850	1275	1700	2126
	,00425,1		3826	3401	2976	2551	
$\bar{1}.18$	,43791		0427	0854	1280	1707	2134
	,00426,8		3841	3414	2988	2561	
$\bar{1}.19$	,44218		0428	0856	1284	1712	2141
	,00428,1		3853	3425	2997	2569	
$\bar{1}.20$	,44646		0430	0860	1290	1720	2150
	,00429,9		3869	3439	3009	2579	
$\bar{1}.21$	,45076		0432	0863	1295	1726	2158
	,00431,5		3884	3452	3021	2589	
$\bar{1}.22$	,45507		0433	0866	1299	1732	2166
	,00433,1		3898	3465	3032	2599	
$\bar{1}.23$	,45940		0435	0869	1304	1739	2174
	,00434,7		3912	3478	3043	2608	
$\bar{1}.24$	,46375		0436	0873	1309	1745	2182
	,00436,3		3927	3490	3054	2618	

$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{1}.25$	,46811	,00	0438	0876	1314	1752	2190
	,00438,0		3942	3504	3066	2628	
$\bar{1}.26$	,47249		0439	0879	1318	1758	2197
	,00439,4		3955	3515	3076	2636	
$\bar{1}.27$	,47689		0441	0882	1324	1765	2206
	,00441,2		3971	3530	3088	2647	
$\bar{1}.28$	,48130		0443	0886	1329	1772	2215
	,00442,9		3986	3543	3100	2657	
$\bar{1}.29$	,48573		0445	0889	1334	1778	2223
	,00444,5		4001	3556	3112	2667	
$\bar{1}.30$	,49017		0446	0892	1339	1785	2231
	,00446,2		4016	3570	3123	2677	
$\bar{1}.31$	,49463		0448	0896	1343	1791	2239
	,00447,8		4030	3582	3135	2687	
$\bar{1}.32$	,49911		0450	0899	1349	1798	2248
	,00449,5		4046	3596	3147	2697	
$\bar{1}.33$	,50361		0451	0902	1353	1804	2256
	,00451,1		4060	3609	3158	2707	
$\bar{1}.34$	,50812		0453	0909	1359	1812	2265
	,00452,9		4076	3623	3170	2717	
$\bar{1}.35$	,51265		0455	0909	1364	1818	2273
	,00454,5		4090	3636	3182	2727	
$\bar{1}.36$	,51719		0456	0912	1369	1825	2281
	,00456,2		4106	3650	3193	2737	
$\bar{1}.37$	,52175		0458	0916	1373	1831	2289
	,00457,8		4120	3662	3205	2747	
$\bar{1}.38$	,52633		0460	0919	1379	1839	2299
	,00459,7		4137	3678	3218	2758	
$\bar{1}.39$	,53093		0461	0923	1384	1846	2307
	,00461,4		4153	3691	3230	2768	
$\bar{1}.40$	,53554		0463	0926	1389	1852	2315
	,00462,9		4166	3703	3240	2777	
$\bar{1}.41$	,54017		0465	0930	1394	1859	2324
	,00464,8		4183	3718	3254	2789	
$\bar{1}.42$	,54482		0467	0933	1400	1866	2333
	,00466,5		4199	3732	3266	2799	
$\bar{1}.43$	,54948		0468	0936	1404	1872	2341
	,00468,1		4213	3745	3277	2809	
$\bar{1}.44$	,55417		0470	0940	1410	1880	2350
	,00469,9		4229	3759	3289	2819	
$\bar{1}.45$	,55886		0472	0943	1415	1887	2359
	,00471,7		4245	3774	3302	2830	
$\bar{1}.46$	,56358		0474	0947	1421	1894	2368
	,00473,6		4262	3789	3315	2842	
$\bar{1}.47$	,56832		0475	0950	1425	1900	2376
	,00475,1		4276	3801	3326	2851	
$\bar{1}.48$	,57307		0477	0954	1431	1901	2385
	,00476,9		4292	3815	3338	2861	
$\bar{1}.49$	,57784		0479	0957	1436	1915	2394
	,00478,7		4308	3830	3351	2872	

General Table II. $\lambda(a^r)$, $\lambda\left(\frac{1-a^r}{a^{-1}-1}\right)$.

$\lambda(a^r)$	$\lambda\left(\frac{1-a^r}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{1}.50$	,58262	,00	0480	0960	1441	1921	2401
	,00480,2		4322	3842	3361	2881	
$\bar{1}.51$	,58742		0482	0964	1447	1929	2411
	,00482,2		4340	3858	3375	2893	
$\bar{1}.52$	,59225		0484	0968	1452	1936	2420
	,00483,9		4355	3871	3387	2903	
$\bar{1}.53$	,59709		0486	0971	1457	1943	2429
	,00485,7		4371	3886	3400	2914	
$\bar{1}.54$	,60194		0487	0975	1462	1950	2437
	,00487,4		4387	3899	3412	2924	
$\bar{1}.55$	,60682		0489	0979	1468	1957	2447
	,00489,3		4404	3914	3425	2936	
$\bar{1}.56$	,61171		0491	0982	1473	1964	2456
	,00491,1		4420	3929	3438	2947	
$\bar{1}.57$	,61662		0493	0985	1478	1971	2464
	,00492,7		4434	3942	3449	2956	
$\bar{1}.58$	,62155		0495	0989	1484	1978	2473
	,00494,6		4451	3957	3462	2968	
$\bar{1}.59$	,62649		0496	0993	1489	1986	2482
	,00496,4		4468	3971	3475	2978	
$\bar{1}.60$	,63146		0498	0997	1495	1993	2492
	,00498,3		4485	3986	3488	2990	
$\bar{1}.61$	,63644		0500	1000	1500	2000	2500
	,00500,0		4500	4000	3500	3000	
$\bar{1}.62$	,64144		0502	1004	1505	2007	2509
	,00501,8		4516	4014	3513	3011	
$\bar{1}.63$	,64646		0504	1007	1511	2015	2519
	,00503,7		4533	4030	3526	3022	
$\bar{1}.64$	,65150		0505	1011	1516	2021	2527
	,00505,3		4548	4042	3537	3032	
$\bar{1}.65$	,65655		0507	1015	1522	2030	2537
	,00507,4		4567	4059	3552	3044	
$\bar{1}.66$	,66162		0509	1018	1527	2036	2546
	,00509,1		4582	4073	3564	3055	
$\bar{1}.67$	,66671		0511	1022	1533	2044	2555
	,00511,0		4599	4088	3577	3066	
$\bar{1}.68$	,67182		0513	1025	1538	2051	2564
	,00512,7		4614	4102	3589	3076	
$\bar{1}.69$	,67695		0515	1029	1544	2058	2573
	,00514,6		4631	4117	3602	3088	
$\bar{1}.70$	,68209		0517	1033	1550	2066	2583
	,00516,5		4649	4132	3616	3099	
$\bar{1}.71$	,68726		0518	1036	1554	2072	2591
	,00518,1		4663	4145	3627	3089	
$\bar{1}.72$	,69244		0520	1040	1561	2081	2601
	,00520,2		4682	4162	3641	3121	
$\bar{1}.73$	,69764		0522	1044	1566	2088	2610
	,00522,0		4698	4176	3654	3132	
$\bar{1}.74$	,70286		0524	1047	1571	2095	2619
	,00523,7		4713	4190	3666	3142	

$\lambda(a^r)$	$\lambda\left(\frac{1-a^r}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{1}.75$	,70810	,00	0526	1052	1578	2104	2630
	,00526,0		4734	4208	3682	3156	
$\bar{1}.76$	,71336		0527	1055	1582	2109	2637
	,00527,3		4746	4218	3691	3164	
$\bar{1}.77$	,71863		0530	1059	1589	2118	2648
	,00529,6		4766	4237	3707	3178	
$\bar{1}.78$	,72392		0531	1062	1593	2124	2655
	,00531,0		4779	4248	3717	3186	
$\bar{1}.79$	,72924		0533	1067	1600	2133	2667
	,00533,3		4800	4266	3733	3200	
$\bar{1}.80$	,73457		0535	1069	1604	2139	2674
	,00534,7		4812	4278	3743	3208	
$\bar{1}.81$	,73992		0536	1072	1608	2144	2681
	,00536,1		4825	4289	3753	3217	
$\bar{1}.82$	,74528		0539	1077	1616	2154	2693
	,00538,5		4847	4308	3770	3231	
$\bar{1}.83$	,75067		0541	1081	1622	2163	2704
	,00540,7		4866	4326	3785	3244	
$\bar{1}.84$	,75608		0542	1085	1627	2169	2712
	,00542,3		4881	4338	3796	3254	
$\bar{1}.85$	,76150		0544	1088	1632	2176	2721
	,00544,1		4897	4353	3809	3265	
$\bar{1}.86$	,76694		0546	1092	1638	2184	2731
	,00546,1		4915	4369	3823	3277	
$\bar{1}.87$	,77240		0548	1096	1644	2192	2741
	,00548,1		4933	4385	3837	3289	
$\bar{1}.88$	,77788		0550	1100	1650	2200	2750
	,00550,0		4950	4400	3850	3300	
$\bar{1}.89$	,78338		0551	1103	1654	2206	2757
	,00551,4		4963	4411	3860	3308	
$\bar{1}.90$	,78889		0554	1108	1661	2215	2769
	,00553,8		4984	4430	3877	3323	
$\bar{1}.91$	,79443		0556	1111	1667	2222	2778
	,00555,5		5000	4444	3889	3333	
$\bar{1}.92$	,79999		0557	1115	1672	2229	2787
	,00557,3		5016	4458	3901	3344	
$\bar{1}.93$	,80556		0560	1119	1679	2238	2798
	,00559,5		5036	4476	3917	3357	
$\bar{1}.94$	,81116		0561	1122	1683	2244	2805
	,00561,0		5049	4488	3927	3366	
$\bar{1}.95$	,81677		0563	1126	1690	2253	2816
	,00563,2		5069	4506	3942	3379	
$\bar{1}.96$	,82240		0565	1129	1694	2258	2823
	,00564,6		5081	4517	3952	3388	
$\bar{1}.97$	,82804		0568	1137	1705	2274	2842
	,00568,4		5116	4547	3979	3419	
$\bar{1}.98$	,83373		0568	1137	1705	2273	2842
	,00568,3		5115	4546	3978	3410	
$\bar{1}.99$	,83941						

General Table III. $\lambda(a^x)$, $\lambda\left(\frac{1-a^x}{a^{-1}-1}\right)$.

$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{3}$.00	$\bar{1}$,52519	,00	0266	0533	0799	1065	1332
	,00266,3		2397	2130	1864	1598	
$\bar{3}$.01	$\bar{1}$,52786		0267	0533	0800	1066	1333
	,00266,5		2399	2132	1866	1599	
$\bar{3}$.02	$\bar{1}$,53052		0267	0534	0802	1069	1336
	,00267,2		2405	2138	1870	1603	
$\bar{3}$.03	$\bar{1}$,53319		0268	0535	0803	1070	1338
	,00267,5		2408	2140	1873	1605	
$\bar{3}$.04	$\bar{1}$,53587		0268	0536	0804	1072	1340
	,00267,9		2411	2143	1875	1607	
$\bar{3}$.05	$\bar{1}$,53855		0268	0537	0805	1073	1342
	,00268,3		2415	2146	1878	1610	
$\bar{3}$.06	$\bar{1}$,54123		0269	0537	0806	1075	1344
	,00268,7		2418	2150	1881	1612	
$\bar{3}$.07	$\bar{1}$,54392		0269	0538	0807	1076	1346
	,00269,1		2422	2153	1884	1615	
$\bar{3}$.08	$\bar{1}$,54661		0270	0539	0809	1078	1348
	,00269,5		2426	2156	1887	1617	
$\bar{3}$.09	$\bar{1}$,54930		0270	0540	0810	1080	1350
	,00269,9		2429	2159	1889	1619	
$\bar{3}$.10	$\bar{1}$,55200		0270	0541	0811	1081	1352
	,00270,3		2433	2162	1892	1622	
$\bar{3}$.11	$\bar{1}$,55470		0271	0541	0812	1083	1354
	,00270,7		2436	2166	1895	1624	
$\bar{3}$.12	$\bar{1}$,55741		0271	0542	0814	1085	1356
	,00271,2		2441	2170	1898	1627	
$\bar{3}$.13	$\bar{1}$,56012		0272	0543	0815	1086	1358
	,00271,6		2444	2173	1901	1630	
$\bar{3}$.14	$\bar{1}$,56284		0272	0544	0816	1088	1360
	,00272,0		2448	2176	1904	1632	
$\bar{3}$.15	$\bar{1}$,56556		0273	0545	0818	1090	1363
	,00272,5		2453	2180	1908	1635	
$\bar{3}$.16	$\bar{1}$,56828		0273	0546	0819	1092	1365
	,00272,9		2456	2183	1910	1637	
$\bar{3}$.17	$\bar{1}$,57101		0273	0547	0820	1093	1367
	,00273,3		2460	2186	1913	1640	
$\bar{3}$.18	$\bar{1}$,57375		0274	0548	0821	1095	1369
	,00273,8		2464	2190	1916	1643	
$\bar{3}$.19	$\bar{1}$,57648		0274	0548	0823	1097	1371
	,00274,2		2468	2194	1919	1645	
$\bar{3}$.20	$\bar{1}$,57923		0275	0549	0824	1099	1374
	,00274,7		2472	2198	1923	1648	
$\bar{3}$.21	$\bar{1}$,58197		0275	0550	0825	1100	1376
	,00275,1		2476	2201	1926	1651	
$\bar{3}$.22	$\bar{1}$,58472		0276	0551	0827	1102	1378
	,00275,6		2480	2205	1929	1654	
$\bar{3}$.23	$\bar{1}$,58748		0276	0552	0828	1104	1380
	,00276,0		2484	2208	1932	1656	
$\bar{3}$.24	$\bar{1}$,59024		0277	0553	0830	1106	1383
	,00276,5		2489	2212	1936	1659	

$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{3}$.25	$\bar{1}$,59300	,00	0277	0554	0831	1108	1385
	,00276,9		2492	2215	1938	1661	
$\bar{3}$.26	$\bar{1}$,59577		0277	0555	0832	1110	1387
	,00277,4		2497	2219	1942	1664	
$\bar{3}$.27	$\bar{1}$,59855		0278	0556	0834	1112	1390
	,00277,9		2491	2223	1945	1667	
$\bar{3}$.28	$\bar{1}$,60133		0278	0557	0835	1114	1392
	,00278,4		2507	2227	1949	1670	
$\bar{3}$.29	$\bar{1}$,60411		0279	0558	0836	1115	1394
	,00278,8		2509	2230	1952	1673	
$\bar{3}$.30	$\bar{1}$,60690		0279	0559	0838	1117	1397
	,00279,3		2514	2234	1955	1676	
$\bar{3}$.31	$\bar{1}$,60969		0280	0560	0839	1119	1399
	,00279,8		2518	2238	1959	1679	
$\bar{3}$.32	$\bar{1}$,61249		0280	0560	0841	1121	1401
	,00280,2		2522	2242	1961	1681	
$\bar{3}$.33	$\bar{1}$,61529		0281	0562	0842	1123	1404
	,00280,8		2527	2244	1966	1685	
$\bar{3}$.34	$\bar{1}$,61810		0281	0563	0844	1125	1407
	,00281,3		2532	2250	1969	1688	
$\bar{3}$.35	$\bar{1}$,62091		0282	0564	0845	1127	1409
	,00281,8		2536	2254	1973	1691	
$\bar{3}$.36	$\bar{1}$,62373		0282	0564	0847	1129	1411
	,00282,2		2540	2258	1975	1693	
$\bar{3}$.37	$\bar{1}$,62655		0283	0566	0848	1131	1414
	,00282,8		2545	2262	1979	1697	
$\bar{3}$.38	$\bar{1}$,62938		0283	0567	0850	1133	1417
	,00283,3		2550	2266	1983	1700	
$\bar{3}$.39	$\bar{1}$,63221		0284	0568	0851	1135	1419
	,00283,8		2554	2270	1987	1703	
$\bar{3}$.40	$\bar{1}$,63505		0284	0569	0853	1137	1422
	,00284,3		2559	2274	1990	1706	
$\bar{3}$.41	$\bar{1}$,63789		0285	0570	0854	1139	1424
	,00284,8		2561	2278	1994	1709	
$\bar{3}$.42	$\bar{1}$,64074		0285	0571	0856	1141	1427
	,00285,3		2568	2282	1997	1712	
$\bar{3}$.43	$\bar{1}$,64359		0286	0572	0857	1143	1429
	,00285,8		2572	2286	2001	1715	
$\bar{3}$.44	$\bar{1}$,64645		0286	0573	0859	1145	1432
	,00286,3		2577	2290	2004	1718	
$\bar{3}$.45	$\bar{1}$,64931		0287	0574	0861	1148	1435
	,00286,9		2582	2295	2008	1721	
$\bar{3}$.46	$\bar{1}$,65218		0287	0575	0862	1150	1437
	,00287,4		2587	2299	2012	1724	
$\bar{3}$.47	$\bar{1}$,65506		0288	0576	0864	1152	1440
	,00288,0		2592	2304	2016	1728	
$\bar{3}$.48	$\bar{1}$,65794		0289	0577	0866	1154	1443
	,00288,5		2597	2308	2020	1731	
$\bar{3}$.49	$\bar{1}$,66082		0289	0578	0867	1156	1445
	,00289,0		2601	2312	2023	1734	

General Table III. $\lambda(a^s);\ \lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$.

$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{3}$.50	$\bar{1}$,66371	,00	0290	0579	0869	1158	1448
	,00289,6		2606	2317	2027	1738	
$\bar{3}$.51	$\bar{1}$,66661		0290	0580	0870	1160	1451
	,00290,1		2611	2321	2031	1741	
$\bar{3}$.52	$\bar{1}$,66951		0291	0581	0872	1163	1454
	,00290,7		2616	2326	2035	1744	
$\bar{3}$.53	$\bar{1}$,67242		0291	0583	0874	1166	1457
	,00291,4		2625	2331	2040	1748	
$\bar{3}$.54	$\bar{1}$,67533		0292	0584	0875	1167	1459
	,00291,8		2626	2334	2043	1751	
$\bar{3}$.55	$\bar{1}$,67825		0292	0585	0877	1170	1462
	,00292,4		2632	2339	2047	1754	
$\bar{3}$.56	$\bar{1}$,68117		0293	0586	0879	1172	1465
	,00292,9		2636	2343	2050	1757	
$\bar{3}$.57	$\bar{1}$,68410		0294	0587	0881	1175	1469
	,00293,7		2643	2350	2056	1762	
$\bar{3}$.58	$\bar{1}$,68704		0294	0588	0882	1176	1471
	,00294,1		2647	2333	2059	1765	
$\bar{3}$.59	$\bar{1}$,68998		0295	0589	0884	1179	1474
	,00294,7		2652	2358	2063	1768	
$\bar{3}$.60	$\bar{1}$,69293		0295	0591	0886	1181	1477
	,00295,3		2658	2362	2067	1772	
$\bar{3}$.61	$\bar{1}$,69588		0296	0592	0888	1184	1480
	,00295,9		2663	2367	2071	1775	
$\bar{3}$.62	$\bar{1}$,69884		0297	0593	0890	1186	1483
	,00296,5		2669	2372	2076	1779	
$\bar{3}$.63	$\bar{1}$,70180		0297	0594	0891	1188	1486
	,00297,1		2674	2377	2080	1783	
$\bar{3}$.64	$\bar{1}$,70477		0298	0595	0893	1191	1489
	,00297,7		2679	2382	2084	1786	
$\bar{3}$.65	$\bar{1}$,70775		0298	0597	0895	1194	1492
	,00298,4		2686	2387	2089	1790	
$\bar{3}$.66	$\bar{1}$,71074		0299	0598	0896	1195	1494
	,0029,88		2689	2390	2092	1793	
$\bar{3}$.67	$\bar{1}$,71372		0300	0599	0899	1198	1498
	,00299,6		2696	2397	2097	1798	
$\bar{3}$.68	$\bar{1}$,71672		0300	0600	0901	1201	1501
	,00300,2		2702	2402	2101	1801	
$\bar{3}$.69	$\bar{1}$,71972		0301	0602	0902	1203	1504
	,00300,8		2707	2406	2106	1805	
$\bar{3}$.70	$\bar{1}$,72273		0301	0603	0904	1206	1507
	,00301,4		2713	2411	2110	1808	
$\bar{3}$.71	$\bar{1}$,72574		0302	0604	0906	1208	1510
	,00302,0		2718	2416	2114	1812	
$\bar{3}$.72	$\bar{1}$,72876		0303	0605	0908	1211	1514
	,00302,7		2724	2422	2119	1816	
$\bar{3}$.73	$\bar{1}$,73179		0303	0607	0910	1214	1517
	,00303,4		2731	2427	2124	1820	
$\bar{3}$.74	$\bar{1}$,73483		0304	0608	0912	1216	1520
	,00304,0		2736	2432	2128	1824	
$\bar{3}$.75	$\bar{1}$,73787	,00	0305	0609	0914	1219	1524
	,00304,7		2742	2438	2133	1828	
$\bar{3}$.76	$\bar{1}$,74091		0305	0611	0916	1221	1527
	,00305,3		2748	2442	2137	1832	
$\bar{3}$.77	$\bar{1}$,74393		0306	0612	0918	1224	1530
	,00306,0		2754	2448	2142	1836	
$\bar{3}$.78	$\bar{1}$,74702		0307	0613	0920	1226	1533
	,00306,6		2759	2453	2146	1840	
$\bar{3}$.79	$\bar{1}$,75009		0307	0615	0922	1230	1537
	,00307,4		2767	2459	2152	1844	
$\bar{3}$.80	$\bar{1}$,75316		0308	0616	0924	1232	1540
	,00308,0		2772	2464	2156	1848	
$\bar{3}$.81	$\bar{1}$,75624		0309	0618	0926	1235	1544
	,00308,8		2779	2470	2162	1853	
$\bar{3}$.82	$\bar{1}$,75933		0309	0609	0928	1237	1547
	,00309,3		2784	2474	2165	1856	
$\bar{3}$.83	$\bar{1}$,76243		0310	0620	0930	1240	1551
	,00310,1		2771	2481	2171	1861	
$\bar{3}$.84	$\bar{1}$,76553		0311	0622	0932	1243	1554
	,00310,8		2797	2486	2176	1865	
$\bar{3}$.85	$\bar{1}$,76863		0311	0623	0934	1246	1557
	,00311,4		2803	2491	2180	1868	
$\bar{3}$.86	$\bar{1}$,77175		0312	0624	0937	1249	1561
	,00312,2		2810	2498	2185	1873	
$\bar{3}$.87	$\bar{1}$,77487		0313	0626	0939	1252	1565
	,00313,0		2817	2504	2191	1878	
$\bar{3}$.88	$\bar{1}$,77800		0314	0627	0941	1254	1568
	,00313,6		2822	2509	2195	1882	
$\bar{3}$.89	$\bar{1}$,78113		0314	0629	0943	1257	1572
	,00314,3		2829	2514	2200	1886	
$\bar{3}$.90	$\bar{1}$,78428		0315	0630	0945	1260	1575
	,00315,0		2835	2520	2205	1890	
$\bar{3}$.91	$\bar{1}$,78743		0316	0632	0947	1263	1579
	,00315,8		2842	2526	2211	1895	
$\bar{3}$.92	$\bar{1}$,79059		0316	0633	0949	1266	1582
	,00316,4		2848	2531	2215	1898	
$\bar{3}$.93	$\bar{1}$,79375		0317	0635	0952	1269	1587
	,00317,3		2856	2538	2221	1904	
$\bar{3}$.94	$\bar{1}$,79692		0318	0636	0954	1272	1590
	,00318,0		2862	2544	2226	1908	
$\bar{3}$.95	$\bar{1}$,80010		0319	0637	0956	1275	1594
	,00318,7		2868	2550	2231	1912	
$\bar{3}$.96	$\bar{1}$,80329		0320	0639	0959	1278	1598
	,00319,5		2876	2556	2237	1917	
$\bar{3}$.97	$\bar{1}$,80648		0320	0641	0961	1281	1602
	,00320,3		2883	2562	2242	1922	
$\bar{3}$.98	$\bar{1}$,80969		0321	0642	0963	1284	1605
	,00321,0		2889	2568	2247	1926	
$\bar{3}$.99	$\bar{1}$,81290		0322	0644	0965	1287	1609
	,00321,8		2896	2574	2253	1931	

General Table III. $\lambda(a^x)$; $\lambda\left(\frac{1-a^x}{a^{-1}-1}\right)$.

$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{2}$,00	$\bar{1}$,81612	,00	0323	0645	0968	1290	1613
	,00322,5		2903	2580	2258	1935	
$\bar{2}$.01	$\bar{1}$,81934		0323	0647	0970	1294	1617
	,00323,4		2911	2587	2264	1940	
$\bar{2}$.02	$\bar{1}$,82257		0324	0648	0973	1297	1621
	,00324,2		2918	2594	2269	1945	
$\bar{2}$.03	$\bar{1}$,82582		0325	0650	0975	1300	1625
	,00324,9		2924	2599	2274	1949	
$\bar{2}$.04	$\bar{1}$,82907		0326	0652	0977	1303	1629
	,00325,8		2932	2606	2281	1955	
$\bar{2}$.05	$\bar{1}$,83232		0327	0653	0980	1306	1633
	,00326,5		2939	2612	2286	1959	
$\bar{2}$,06	$\bar{1}$,83559		0327	0655	0982	1309	1637
	,00327,3		2946	2618	2291	1964	
$\bar{2}$,07	$\bar{1}$,83886		0328	0656	0985	1313	1641
	,00328,2		2954	2626	2297	1969	
$\bar{2}$.08	$\bar{1}$,84214		0329	0658	0987	1316	1645
	,00329,0		2961	2632	2303	1974	
$\bar{2}$.09	$\bar{1}$,84543		0330	0660	0989	1319	1649
	,00329,8		2968	2638	2309	1979	
$\bar{2}$.10	$\bar{1}$,84873		0331	0661	0992	1322	1653
	,00330,6		2975	2645	2314	1984	
$\bar{2}$.11	$\bar{1}$,85204		0332	0663	0995	1326	1658
	,00331,5		2983	2652	2320	1989	
$\bar{2}$.12	$\bar{1}$,85553		0332	0665	0997	1330	1662
	,00332,4		2992	2659	2327	1994	
$\bar{2}$.13	$\bar{1}$,85868		0333	0666	1000	1333	1666
	,00333,2		2999	2666	2332	1999	
$\bar{2}$.14	$\bar{1}$,86201		0334	0668	1002	1336	1670
	,00334,0		3006	2672	2338	2004	
$\bar{2}$.15	$\bar{1}$,86535		0335	0670	1005	1340	1675
	,00334,9		3014	2679	2344	2009	
$\bar{2}$.16	$\bar{1}$,86870		0336	0671	1007	1343	1679
	,00335,7		3021	2686	2350	2014	
$\bar{2}$.17	$\bar{1}$,87205		0337	0673	1010	1346	1683
	,00336,6		3049	2693	2356	2020	
$\bar{2}$.18	$\bar{1}$,87542		0338	0675	1013	1350	1688
	,00337,5		3038	2700	2363	2025	
$\bar{2}$.19	$\bar{1}$,87880		0338	0677	1015	1354	1692
	,00338,4		3046	2707	2369	2030	
$\bar{2}$.20	$\bar{1}$,88218		0339	0679	1018	1357	1697
	,00339,3		3054	2714	2375	2036	
$\bar{2}$.21	$\bar{1}$,88557		0340	0680	1021	1361	1701
	,00340,2		3062	2722	2381	2041	
$\bar{2}$.22	$\bar{1}$,88897		0341	0682	1023	1364	1705
	,00341,0		3069	2728	2387	2046	
$\bar{2}$.23	$\bar{1}$,89238		0342	0684	1026	1368	1711
	,00342,1		3079	2737	2395	2053	
$\bar{2}$.24	$\bar{1}$,89581		0343	0686	1028	1371	1714
	,00342,8		3085	2742	2400	2057	

$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{2}$.25	$\bar{1}$,89923	,00	0344	0688	1031	1375	1719
	,00343,8		3094	2750	2407	2063	
$\bar{2}$.26	$\bar{1}$,90267		0345	0689	1034	1379	1724
	,00344,7		3102	2758	2413	2068	
$\bar{2}$.27	$\bar{1}$,90612		0346	0691	1037	1383	1729
	,00345,7		3111	2766	2420	2074	
$\bar{2}$.28	$\bar{1}$,90958		0347	0693	1040	1386	1733
	,00346,6		3119	2773	2426	2080	
$\bar{2}$.29	$\bar{1}$,91304		0348	0695	1043	1390	1738
	,00347,6		3128	2781	2433	2086	
$\bar{2}$.30	$\bar{1}$,91652		0349	0697	1046	1394	1743
	,00348,5		3137	2788	2440	2091	
$\bar{2}$.31	$\bar{1}$,92000		0349	0699	1048	1397	1747
	,00349,3		3144	2794	2445	2096	
$\bar{2}$.32	$\bar{1}$,92349		0351	0701	1052	1403	1754
	,00350,7		3156	2806	2455	2104	
$\bar{2}$.33	$\bar{1}$,92700		0351	0703	1054	1406	1757
	,00351,4		3163	2811	2460	2108	
$\bar{2}$.34	$\bar{1}$,93052		0352	0705	1057	1410	1762
	,00352,4		3172	2819	2467	2114	
$\bar{2}$.35	$\bar{1}$,93404		0353	0707	1060	1413	1767
	,00353,3		3180	2826	2473	2120	
$\bar{2}$.36	$\bar{1}$,93757		0354	0709	1063	1417	1771
	,00354,3		3189	2834	2480	2126	
$\bar{2}$.37	$\bar{1}$,94112		0355	0711	1066	1422	1777
	,00355,4		3199	2843	2488	2132	
$\bar{2}$.38	$\bar{1}$,94467		0356	0713	1069	1426	1782
	,00356,4		3208	2851	2495	2138	
2.39	$\bar{1}$,94823		0357	0715	1072	1429	1787
	,00357,3		3216	2858	2501	2144	
$\bar{2}$.40	$\bar{1}$,95181		0358	0717	1075	1434	1792
	,00358,4		3226	2867	2509	2150	
$\bar{2}$.41	$\bar{1}$,95539		0359	0719	1078	1438	1797
	,00359,4		3235	2875	2516	2156	
$\bar{2}$.42	$\bar{1}$,95898		0360	0721	1081	1442	1802
	,00360,4		3244	2883	2523	2162	
$\bar{2}$.43	$\bar{1}$,96259		0361	0723	1084	1446	1807
	,00361,4		3253	2891	2530	2168	
$\bar{2}$.44	$\bar{1}$,96620		0362	0725	1087	1450	1812
	,00362,4		3262	2899	2537	2174	
$\bar{2}$.45	$\bar{1}$,96983		0363	0727	1090	1454	1817
	,00363,4		3271	2907	2544	2180	
$\bar{2}$.46	$\bar{1}$,97346		0365	0729	1094	1458	1823
	,00364,6		3281	2917	2552	2188	
$\bar{2}$.47	$\bar{1}$,97711		0366	0731	1097	1462	1828
	,00365,6		3290	2925	2559	2194	
$\bar{2}$.48	$\bar{1}$,98076		0367	0733	1100	1467	1834
	,00366,7		3300	2934	2567	2200	
$\bar{2}$.49	$\bar{1}$,98443		0368	0735	1103	1471	1839
	,00367,8		3310	2942	2575	2207	

General Table III. $\lambda(a^s)$, $\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$.

$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{2}.50$	$\bar{1}$,98811	,00	0369	0738	1106	1475	1844	$\bar{2}.75$	,08373	,00	0398	0796	1194	1592	1990
	,00368,8		3319	2950	2582	2213			,00397,9		3581	3183	2785	2387	
$\bar{2}.51$	$\bar{1}$,99179		0370	0740	1109	1480	1850	$\bar{2}.76$	,08771		0399	0799	1198	1598	1997
	,00369,9		3329	2959	2589	2219			,00399,4		3595	3195	2796	2396	
$\bar{2}.52$	$\bar{1}$,99549		0371	0742	1113	1484	1855	$\bar{2}.77$	,09170		0401	0801	1202	1602	2003
	,00371,0		3339	2968	2597	2226			,00400,6		3605	3205	2804	2404	
$\bar{2}.53$	$\bar{1}$,99920		0372	0744	1116	1488	1861	$\bar{2}.78$	,09571		0402	0803	1205	1607	2009
	,00372,1		3349	2977	2605	2233			,00401,7		3615	3214	2812	2410	
$\bar{2}.54$	,00293		0373	0747	1120	1493	1867	$\bar{2}.79$	,09972		0403	0806	1209	1612	2016
	,00373,3		3360	2986	2613	2240			,00403,1		3628	3225	2822	2419	
$\bar{2}.55$	,00666		0374	0749	1123	1497	1872	$\bar{2}.80$	,10376		0404	0809	1213	1618	2022
	,00374,3		3369	2994	2620	2246			,00404,4		3640	3235	2831	2426	
$\bar{2}.56$	,01040		0376	0751	1127	1502	1878	$\bar{2}.81$	,10780		0406	0811	1217	1623	2029
	,00375,5		3380	3004	2629	2253			,00405,7		3651	3246	2840	2434	
$\bar{2}.57$	,01416		0376	0753	1129	1506	1882	$\bar{2}.82$	,11186		0407	0814	1221	1628	2035
	,00376,4		3388	3011	2635	2258			,00407,0		3663	3256	2849	2442	
$\bar{2}.58$	,00792		0378	0756	1134	1512	1890	$\bar{2}.83$	,11593		0408	0817	1225	1633	2042
	,00377,9		3401	3023	2645	2267			,00408,3		3675	3266	2858	2450	
$\bar{2}.59$	,02170		0379	0758	1136	1515	1894	$\bar{2}.84$	,12001		0410	0819	1229	1638	2048
	,00378,3		3409	3030	2652	2273			,00409,5		3686	3276	2867	2457	
$\bar{2}.60$	,02549		0380	0760	1140	1520	1900	$\bar{2}.85$	,12410		0411	0822	1233	1644	2055
	,00380,0		3420	3040	2660	2280			,00410,9		3698	3287	2876	2465	
$\bar{2}.61$	,02929		0381	0762	1144	1525	1906	$\bar{2}.86$	,12821		0412	0824	1237	1649	2061
	,00381,2		3431	3050	2668	2287			,00412,2		3710	3298	2885	2473	
$\bar{2}.62$	,03310		0382	0765	1147	1530	1912	$\bar{2}.87$	,13234		0414	0827	1241	1654	2068
	,00382,4		3442	3059	2677	2294			,00413,6		3722	3309	2895	2482	
$\bar{2}.63$	,03692		0383	0767	1150	1534	1917	$\bar{2}.88$	,13647		0415	0830	1245	1660	2075
	,00383,4		3451	3067	2684	2300			,00414,9		3734	3319	2904	2489	
2.64	,04076		0385	0769	1154	1539	1924	$\bar{2}.89$	,14062		0416	0833	1249	1665	2082
	,00384,7		3462	3078	2693	2308			,00416,3		3747	3330	2914	2498	
$\bar{2}.65$	,04460		0386	0772	1157	1543	1929	$\bar{2}.90$	,14478		0418	0835	1253	1670	2088
	,00385,8		3472	3086	2701	2315			,00417,6		3758	3341	2923	2506	
$\bar{2}.66$	,04846		0387	0774	1161	1548	1936	$\bar{2}.91$	,14896		0419	0838	1257	1676	2095
*	,00387,1		3484	3097	2710	2323			,00419,0		3771	3352	2933	2514	
2.67	,05233		0388	0776	1165	1553	1941	$\bar{2}.92$	,15315		0420	0841	1261	1682	2102
	,00388,2		3494	3106	2717	2329			,00420,4		3784	3363	2943	2522	
$\bar{2}.68$	,05621		0389	0779	1168	1558	1947	$\bar{2}.93$	,15735		0422	0843	1265	1687	2109
	,00389,4		3505	3115	2726	2336			,00421,7		3795	3344	2952	2530	
$\bar{2}.69$	,06010		0391	0781	1172	1562	1953	$\bar{2}.94$	,16157		0423	0846	1269	1692	2116
	,00390,6		3515	3125	2734	2344			,00423,1		3808	3385	2962	2539	
$\bar{2}.70$	,06401		0392	0784	1176	1568	1960	$\bar{2}.95$	,16580		0425	0849	1274	1698	2123
	,00391,9		3527	3135	2743	2351			,00424,6		3821	3397	2972	2548	
$\bar{2}.71$	,06793		0393	0786	1179	1572	1965	$\bar{2}.96$	,17005		0426	0852	1277	1703	2129
	,00393,0		3537	3144	2751	2358			,00425,8		3832	3406	2981	2555	
$\bar{2}.72$	,07186		0394	0789	1183	1577	1972	$\bar{2}.97$	,17430		0427	0855	1282	1719	2137
	,00394,3		3549	3154	2760	2366			,00427,3		3846	3418	2991	2564	
$\bar{2}.73$	,07581		0396	0791	1187	1582	1978	$\bar{2}.98$	,17858		0429	0857	1286	1715	2144
	00395,5		3560	3164	2769	2373			,00428,7		3858	3430	3001	2572	
$\bar{2}.74$	,07976		0397	0794	1190	1587	1984	$\bar{2}.99$	,18286		0430	0860	1290	1720	2151
	,00396,8		3571	3174	2778	2381			,00430,1		3871	3441	3011	2581	

General Table III. $\lambda(a^5)$, $\lambda\left(\frac{1-a^5}{a^{-1}-1}\right)$.

$\lambda(a^5)$	$\lambda\left(\frac{1-a^5}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{1}.00$	,18717	,00	0432	0863	1295	1726	2158
	,00431,6		3884	3453	3021	2590	
$\bar{1}.01$	,19148		0433	0866	1299	1732	2165
	,00433,0		3897	3464	3031	2598	
$\bar{1}.02$	,19581		0434	0869	1304	1738	2175
	,00434,5		3911	3476	3042	2607	
$\bar{1}.03$	,20016		0435	0871	1306	1742	2177
	,00435,4		3919	3483	3048	2612	
$\bar{1}.04$	,20451		0438	0875	1313	1750	2188
	,00437,6		3938	3401	3063	2626	
$\bar{1}.05$	,20889		0439	0877	1316	1755	2194
	,00438,7		3948	3510	3071	2632	
$\bar{1}.06$	,21328		0440	0880	1321	1761	2201
	,00440,2		3962	3522	3081	2641	
$\bar{1}.07$	,21768		0442	0883	1325	1767	2209
	,00441,7		3975	3534	3092	2650	
$\bar{1}.08$	,22209		0443	0886	1330	1773	2216
	,00443,2		3989	3546	3102	2659	
$\bar{1}.09$	,22653		0445	0889	1334	1779	2224
	,00444,7		4002	3558	3113	2668	
$\bar{1}.10$	,23097		0446	0892	1338	1784	2231
	,00446,1		4015	3569	3123	2677	
$\bar{1}.11$	,23543		0448	0895	1343	1791	2240
	,00447,7		4029	3582	3134	2686	
$\bar{1}.12$	,23991		0449	0898	1348	1797	2246
	,00449,2		4043	3594	3144	2695	
$\bar{1}.13$	,24440		0451	0992	1353	1804	2255
	,00450,9		4058	3607	3156	2705	
$\bar{1}.14$	,24891		0452	0904	1356	1808	2260
	,00452,0		4068	3616	3164	2712	
$\bar{1}.15$	,25343		0454	0907	1361	1815	2269
	,00453,7		4083	3630	3176	2722	
$\bar{1}.16$	,25797		0455	09 1	1366	1821	2277
	,00455,3		4098	3642	3187	2732	
$\bar{1}.17$	,26252		0457	0914	1370	1827	2284
	,00456,8		4111	3654	3198	2741	
$\bar{1}.18$	,26709		0458	0917	1375	1833	2292
	,00458,3		4125	3666	3208	2750	
$\bar{1}.19$	,27167		0460	0920	1380	1840	2300
	,00459,9		4139	3679	3219	2759	
$\bar{1}.20$	,27627		0461	0923	1384	1846	2307
	,00461,4		4153	3691	3230	2768	
$\bar{1}.21$	,28089		0463	0926	1389	1852	2315
	,00463,0		4167	3704	3241	2778	
$\bar{1}.22$	,28551		0465	0929	1394	1858	2323
	,00464,6		4181	3717	3252	2788	
$\bar{1}.23$	,29016		0466	0932	1398	1864	2331
	,00466,1		4194	3729	3263	2797	
$\bar{1}.24$	,29482		0468	0935	1403	1871	2339
	,00467,7		4209	3742	3274	2806	

$\lambda(a^5)$	$\lambda\left(\frac{1-a^5}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{1}.25$	,29950	,00	0469	0939	1408	1877	2347
	,00469,3		4224	3754	3285	2816	
$\bar{1}.26$	,30419		0471	0942	1413	1884	2355
	,00470,9		4238	3767	3206	2825	
$\bar{1}.27$	,30890		0473	0945	1418	1890	2363
	,00472,5		4253	3780	3308	2835	
$\bar{1}.28$	,31363		0474	0948	1422	1896	2371
	,00474,1		4267	3793	3319	2845	
$\bar{1}.29$	,31837		0476	0951	1427	1903	2379
	,00475,7		4281	3806	3330	2854	
$\bar{1}.30$	,32312		0477	0955	1432	1909	2387
	,00477,3		4296	3818	3341	2864	
$\bar{1}.31$	,32790		0479	0958	1437	1916	2395
	,00479,0		4310	3831	3352	2873	
$\bar{1}.32$	,33269		0481	0961	1442	1922	2403
	,00480,6		4325	3845	3364	2884	
$\bar{1}.33$	,33749		0482	0964	1447	1929	2411
	,00482,2		4340	3858	3375	2893	
$\bar{1}.34$	,34232		0484	0968	1452	1936	2420
	,00483,9		4355	3871	3387	2903	
$\bar{1}.35$	,34715		0486	0971	1457	1942	2428
	,00485,5		4370	3884	3399	2913	
$\bar{1}.36$	,35201		0487	0974	1462	1949	2436
	,00487,2		4385	3898	3410	2923	
$\bar{1}.37$	,35688		0489	0978	1466	1955	2444
	,00488,8		4399	3910	3422	2933	
$\bar{1}.38$	,36177		0490	0981	1471	1962	2452
	,00490,4		4414	3923	3433	2942	
$\bar{1}.39$	,36676		0492	0984	1476	1968	2461
	,00492,1		4429	3937	3445	2953	
$\bar{1}.40$	,37159		0494	0988	1481	1975	2469
	,00493,8		4444	3950	3457	2963	
$\bar{1}.41$	,37653		0496	0991	1487	1982	2477
	,00495,5		4460	3964	3469	2973	
$\bar{1}.42$	,38149		0497	0994	1492	1989	2486
	,00497,2		4475	3988	3480	2983	
$\bar{1}.43$	,38646		0499	0998	1497	1996	2495
	,00498,9		4490	3991	3492	2993	
$\bar{1}.44$	,39145		0501	1001	1502	2002	2503
	,00500,6		4505	4005	3504	3004	
$\bar{1}.45$	,39645		0502	1004	1507	2009	2511
	,00502,2		4520	4018	3515	3013	
$\bar{1}.46$	,40148		0504	1008	1512	2016	2520
	,00594,0		4536	4032	3528	3024	
$\bar{1}.47$	,40652		0506	1011	1517	2023	2529
	,00505,7		4551	4046	3540	3034	
$\bar{1}.48$	,41157		0507	1015	1522	2030	2537
	,00507,4		4567	4059	3552	3044	
$\bar{1}.49$	,41665		0509	1018	1527	2036	2546
	,00509,0		4582	4073	3564	3055	

General Table III. $\lambda(a^s)$, $\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$.

$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$		1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{1}$.50	,42174	,00	0511	1022	1532	2043	2554
	,00510,8		4597	4086	3576	3065	
$\bar{1}$.51	,42684		0513	1025	1538	2050	2563
	,00512,5		4613	4100	3588	3075	
$\bar{1}$.52	,43197		0514	1028	1543	2057	2571
	,00514,2		4628	4114	3599	3085	
$\bar{1}$.53	,43711		0516	1032	1548	2064	2581
	,00516,1		4645	4129	3613	3097	
$\bar{1}$.54	,44227		0518	1036	1553	2071	2589
	,00517,8		4660	4142	3625	3107	
$\bar{1}$.55	,44745		0520	1039	1559	2078	2598
	,00519,5		4676	4156	3637	3117	
$\bar{1}$.56	,45264		0521	1042	1564	2085	2606
	,00521,2		4691	4170	3648	3127	
$\bar{1}$.57	,45786		0523	1046	1569	2092	2615
	,00523,0		4707	4184	3661	3138	
$\bar{1}$.58	,46309		0525	1049	1574	2099	2624
	,00524,7		4722	4198	3673	3148	
$\bar{1}$.59	,46833		0527	1053	1580	2106	2633
	,00526,5		4739	4212	3686	3159	
$\bar{1}$.60	,47360		0528	1057	1585	2113	2642
	,00528,3		4755	4226	3698	3170	
$\bar{1}$.61	,47888		0530	1060	1590	2120	2650
	,00530,0		4770	4240	3710	3180	
$\bar{1}$.62	,48418		0532	1064	1595	2127	2659
	,00531,8		4786	4254	3723	3191	
$\bar{1}$.63	,48950		0534	1067	1601	2134	2668
	,00533,6		4802	4269	3735	3202	
$\bar{1}$.64	,49484		0535	1071	1606	2141	2677
	,00535,3		4818	4282	3747	3212	
$\bar{1}$.65	,50019		0537	1074	1611	2148	2688
	,00537,1		4834	4297	3760	3223	
$\bar{1}$.66	,50556		0539	1078	1617	2156	2695
	,00538,9		4850	4311	3772	3233	
$\bar{1}$.67	,51095		0541	1081	1622	2163	2704
	,00540,7		4866	4326	3785	3244	
$\bar{1}$.68	,51636		0543	1085	1628	2170	2713
	,00542,5		4883	4340	3798	3255	
$\bar{1}$.69	,52178		0544	1089	1633	2177	2722
	,00544,3		4899	4354	3810	3266	
$\bar{1}$.70	,52722		0546	1092	1638	2184	2731
	,00546,1		4915	4369	3823	3277	
$\bar{1}$.71	,53269		0548	1096	1643	2191	2739
	,00547,8		4920	4382	3835	3287	
$\bar{1}$.72	,53816		0550	1099	1649	2199	2749
	,00549,7		4947	4398	3848	3298	
$\bar{1}$.73	,54366		0552	1103	1655	2206	2758
	,00551,5		4964	4412	3861	3309	
$\bar{1}$.74	,54918		0553	1107	1660	2213	2767
	,00553,3		4980	4426	3873	3320	

$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$	$\lambda(a^s)$	1 / 9	2 / 8	3 / 7	4 / 6	5
$\bar{1}$.75	,55471	,00	0555	1110	1666	2221	2776
	,00555,2		4997	4442	3886	3331	
$\bar{1}$.76	,56026		0557	1114	1671	2228	2785
	,00556,9		5012	4455	3898	3341	
$\bar{1}$.77	,56583		0559	1117	1676	2234	2793
	,00558,6		5027	4469	3910	3352	
$\bar{1}$.78	,57142		0561	1121	1682	2243	2804
	,00560,7		5046	4486	3925	3364	
$\bar{1}$.79	,57702		0562	1125	1687	2250	2812
	,00562,4		5062	4499	3937	3374	
$\bar{1}$.80	,58265		0564	1128	1693	2257	2821
	,00564,2		5078	4514	3949	3385	
$\bar{1}$.81	,58829		0566	1132	1698	2264	2830
	,00565,9		5093	4527	3961	3395	
$\bar{1}$.82	,59395		0568	1136	1703	2271	2839
	,00567,8		5110	4542	3975	3407	
$\bar{1}$.83	,59963		0570	1139	1709	2279	2849
	,00569,7		5127	4558	3988	3418	
$\bar{1}$.84	,60532		0572	1143	1715	2286	2858
	,00571,5		5144	4572	4001	3429	
$\bar{1}$.85	,61104		0573	1147	1720	2293	2867
	,00573,3		5160	4586	4013	3440	
$\bar{1}$.86	,61677		0575	1150	1725	2300	2876
	,00575,1		4176	4601	4026	3451	
$\bar{1}$.87	,62252		0577	1154	1731	2308	2885
	,00577,0		5193	4616	4039	3462	
$\bar{1}$.88	,62829		0579	1158	1736	2315	2894
	,00578,8		5210	4630	4052	3473	
$\bar{1}$.89	,63408		0581	1162	1742	2323	2904
	,00580,8		5227	4646	4066	3485	
$\bar{1}$.90	,63989		0583	1165	1748	2330	2913
	,00582,5		5243	4660	4078	3495	
$\bar{1}$.91	,64572		0584	1169	1753	2338	2922
	,00584,4		5260	4675	4091	3506	
$\bar{1}$.92	,65156		0586	1172	1759	2345	2931
	,00586,2		5276	4690	4103	3517	
$\bar{1}$.93	,65742		0588	1176	1764	2352	2940
	,00587,9		5291	4713	4115	3527	
$\bar{1}$.94	,66330		0590	1180	1770	2360	2950
	,00590,0		5310	4720	4130	3540	
$\bar{1}$.95	,66920		0592	1183	1775	2367	2959
	,00591,7		5325	4734	4142	3550	
$\bar{1}$.96	,67512		0594	1187	1781	2374	2968
	,00593,5		5342	4748	4155	3561	
$\bar{1}$.97	,68105		0595	1191	1786	2382	2977
	,00595,4		5359	4763	4168	3572	
$\bar{1}$.98	,68701		0597	1195	1791	2389	2987
	,00597,3		5376	4778	4181	3584	
$\bar{1}$.99	,69298						

General Table IV. For the whole of life. $-\lambda(a^{-1}1)$.

$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$
$\bar{1}$.700	,00206	$\bar{1}$.725	,05372	$\bar{1}$.750	,10886	$\bar{1}$.775	,16826	$\bar{1}$.800	,23292	$\bar{1}$.825	,30431
	,00201		,00213		,00229		,00247		,00271		,00302
$\bar{1}$.701	,00407	$\bar{1}$.726	,05585	$\bar{1}$.751	,11115	$\bar{1}$.776	,17073	$\bar{1}$.801	,23564	$\bar{1}$.826	,30733
	,00201		,00214		,00230		,00248		,00272		,00304
$\bar{1}$.702	,00608	$\bar{1}$.727	,05799	$\bar{1}$.752	,11345	$\bar{1}$.777	,17322	$\bar{1}$.802	,23836	$\bar{1}$.827	,31037
	,00202		,00215		,00230		,00249		,00274		,00305
$\bar{1}$.703	,00810	$\bar{1}$.728	,06014	$\bar{1}$.753	,11575	$\bar{1}$.778	,17571	$\bar{1}$.803	,24110	$\bar{1}$.828	,31342
	,00202		,00215		,00231		,00250		,00275		,00307
$\bar{1}$.704	,01012	$\bar{1}$.729	,06229	$\bar{1}$.754	,11806	$\bar{1}$.779	,17822	$\bar{1}$.804	,24385	$\bar{1}$.829	,31649
	,00203		,00216		,00232		,00251		,00276		,00308
$\bar{1}$.705	,01214	$\bar{1}$.730	,06445	$\bar{1}$.755	,12037	$\bar{1}$.780	,18073	$\bar{1}$.805	,24661	$\bar{1}$.830	,31957
	,00203		,00216		,00233		,00252		,00277		,00309
$\bar{1}$.706	,01418	$\bar{1}$.731	,06661	$\bar{1}$.756	,12270	$\bar{1}$.781	,18325	$\bar{1}$.806	,24938	$\bar{1}$.831	,32266
	,00203		,00217		,00233		,00253		,00278		,00311
$\bar{1}$.707	,01621	$\bar{1}$.732	,06878	$\bar{1}$.757	,12503	$\bar{1}$.782	,18578	$\bar{1}$.807	,25216	$\bar{1}$.832	,32577
	,00204		,00217		,00234		,00254		,00279		,00313
$\bar{1}$.708	,01825	$\bar{1}$.733	,07095	$\bar{1}$.758	,12736	$\bar{1}$.783	,18832	$\bar{1}$.808	,25495	$\bar{1}$.833	,32890
	,00205		,00219		,00235		,00254		,00281		,00314
$\bar{1}$.709	,02030	$\bar{1}$.734	,07314	$\bar{1}$.759	,12971	$\bar{1}$.784	,19086	$\bar{1}$.809	,25776	$\bar{1}$.834	,33204
	,00205		,00218		,00235		,00256		,00281		,00315
$\bar{1}$.710	,02235	$\bar{1}$.735	,07532	$\bar{1}$.760	,13206	$\bar{1}$.785	,19342	$\bar{1}$.810	,26057	$\bar{1}$.835	,33519
	,00205		,00219		,00236		,00257		,00283		,00317
$\bar{1}$.711	,02440	$\bar{1}$.736	,07751	$\bar{1}$.761	,13442	$\bar{1}$.786	,19599	$\bar{1}$.811	,26340	$\bar{1}$.836	,33836
	,00206		,00220		,00237		,00258		,00284		,00319
$\bar{1}$.712	,02646	$\bar{1}$.737	,07971	$\bar{1}$.762	,13679	$\bar{1}$.787	,19856	$\bar{1}$.812	,26624	$\bar{1}$.837	,34155
	,00207		,00221		,00237		,02259		,00285		,00320
$\bar{1}$.713	,02853	$\bar{1}$.738	,08192	$\bar{1}$.763	,13916	$\bar{1}$.788	,20115	$\bar{1}$.813	,26909	$\bar{1}$.838	,34475
	,00207		,00221		,00238		,00259		,00287		,00322
$\bar{1}$.714	,03060	$\bar{1}$.739	,08413	$\bar{1}$.764	,14154	$\bar{1}$.789	,20374	$\bar{1}$.814	,27196	$\bar{1}$.839	,34797
	,00207		,00221		,00239		,00260		,00287		,00324
$\bar{1}$.715	,03267	$\bar{1}$.740	,08634	$\bar{1}$.765	,14393	$\bar{1}$.790	,20634	$\bar{1}$.815	,27483	$\bar{1}$.840	,35121
	,00208		,00223		,00240		,00261		,00289		,00325
$\bar{1}$.716	,03476	$\bar{1}$.741	,08857	$\bar{1}$.766	,14633	$\bar{1}$.791	,20896	$\bar{1}$.816	,27772	$\bar{1}$.841	,35446
	,00208		,00223		,00240		,00262		,00291		,00327
$\bar{1}$.717	,03684	$\bar{1}$.742	,09080	$\bar{1}$.767	,14873	$\bar{1}$.792	,21158	$\bar{1}$.817	,28063	$\bar{1}$.842	,35773
	,00209		,00223		,00241		,00263		,00291		,00329
$\bar{1}$.718	,03893	$\bar{1}$.743	,09303	$\bar{1}$.768	,15114	$\bar{1}$.793	,21421	$\bar{1}$.818	,28354	$\bar{1}$.843	,36102
	,00210		,00224		,00242		,00264		,00293		,00331
$\bar{1}$.719	,04103	$\bar{1}$.744	,09527	$\bar{1}$.769	,15356	$\bar{1}$.794	,21685	$\bar{1}$.819	,28647	$\bar{1}$.844	,36433
	,00210		,00225		,00243		,00265		,00294		,00332
$\bar{1}$.720	,04313	$\bar{1}$.745	,09752	$\bar{1}$.770	,15599	$\bar{1}$.795	,21951	$\bar{1}$.820	,28941	$\bar{1}$.845	,36765
	,00211		,00226		,00244		,00266		,00295		,00334
$\bar{1}$.721	,04524	$\bar{1}$.746	,09978	$\bar{1}$.771	,15843	$\bar{1}$.796	,22217	$\bar{1}$.821	,29236	$\bar{1}$.846	,37099
	,00211		,00226		,00244		,00267		,00297		,00336
$\bar{1}$.722	,04735	$\bar{1}$.747	,10204	$\bar{1}$.772	,16087	$\bar{1}$.797	,22484	$\bar{1}$.822	,29533	$\bar{1}$.847	,37435
	,00212		,00227		,00245		,00268		,00298		,00338
$\bar{1}$.723	,04947	$\bar{1}$.748	,10431	$\bar{1}$.773	,16333	$\bar{1}$.798	,22753	$\bar{1}$.823	,29831	$\bar{1}$.848	,37773
	,00212		,00227		,00246		,00269		,00299		,00339
$\bar{1}$.724	,05159	$\bar{1}$.749	,10658	$\bar{1}$.774	,16579	$\bar{1}$.799	,23022	$\bar{1}$.824	,30130	$\bar{1}$.849	,38112
	,00213		,00228		,00247		,00270		,00301		,00342

General Table IV. For the whole of life. $-\lambda(a^{-1}-1)$.

$\lambda(a)$	$-\lambda(a^{-1}-1)$	$\lambda(a)$	$-\lambda(a^{-1}-1)$	$\lambda(a)$	$-\lambda(a^{-1}-1)$	$\lambda(a)$	$-\lambda(a^{-1}-1)$	$\lambda(a)$	$-\lambda(a^{-1}-1)$	$\lambda(a)$	$-\lambda(a^{-1}-1)$
$\bar{1}.850$	,38454	$\bar{1}.875$	,47688	$\bar{1}.900$	,58683	$\bar{1}.925$	,72468	$\bar{1}.950$	,91357	$\bar{1}.975$	1,22728
	,00343		,00401		,00488		,00635		,00929		,01824
$\bar{1}.851$	,38797	$\bar{1}.876$	,48088	$\bar{1}.901$	,59171	$\bar{1}.926$	,73103	$\bar{1}.951$	,92286	$\bar{1}.976$	1,24552
	,00345		,00405		,00493		,00642		,00946		,01899
$\bar{1}.852$	,39142	$\bar{1}.877$	,48493	$\bar{1}.902$	,59664	$\bar{1}.927$	,73745	$\bar{1}.952$	,93232	$\bar{1}.977$	1,26451
	,00348		,00407		,00497		,00650		,00966		,01980
$\bar{1}.853$	,39490	$\bar{1}.878$	,48900	$\bar{1}.903$	,60161	$\bar{1}.928$	,74395	$\bar{1}.953$	,94198	$\bar{1}.978$	1,28431
	,00349		,00410		,00502		,00659		,00984		,02071
$\bar{1}.854$	,39839	$\bar{1}.879$	,49310	$\bar{1}.904$	,60663	$\bar{1}.929$	,75054	$\bar{1}.954$	,95182	$\bar{1}.979$	1,30502
	,00351		,00412		,00507		,00668		,01006		,02170
$\bar{1}.855$	,40190	$\bar{1}.880$	,49722	$\bar{1}.905$	,61170	$\bar{1}.930$	,75722	$\bar{1}.955$	,96188	$\bar{1}.980$	1,32672
	,00353		,00416		,00511		,00676		,01027		,02278
$\bar{1}.856$	,40543	$\bar{1}.881$	,50138	$\bar{1}.906$	,61681	$\bar{1}.931$	,76398	$\bar{1}.956$	,97215	$\bar{1}.981$	1,34950
	,00355		,00419		,00516		,00685		,01049		,02398
$\bar{1}.857$	,40899	$\bar{1}.882$	,50557	$\bar{1}.907$	,62197	$\bar{1}.932$	,77083	$\bar{1}.957$	,98264	$\bar{1}.982$	1,37348
	,00357		,00422		,00521		,00695		,01073		,02583
$\bar{1}.858$	,41256	$\bar{1}.883$	,50979	$\bar{1}.908$	,62718	$\bar{1}.933$	,77778	$\bar{1}.958$	,99337	$\bar{1}.983$	1,39881
	,00360		,00425		,00527		,00704		,01097		,02683
$\bar{1}.859$	,41616	$\bar{1}.884$	,51404	$\bar{1}.909$	,63245	$\bar{1}.934$	,78482	$\bar{1}.959$	1,00434	$\bar{1}.984$	1,42564
	,00362		,00428		,00531		,00715		,01123		,02853
$\bar{1}.860$	,41978	$\bar{1}.885$	,51832	$\bar{1}.910$	,63776	$\bar{1}.935$	,79197	$\bar{1}.960$	1,01557	$\bar{1}.985$	1,45417
	,00364		,00431		,00537		,00724		,01150		,03047
$\bar{1}.861$	,42342	$\bar{1}.886$	,52263	$\bar{1}.911$	,64313	$\bar{1}.936$	,79921	$\bar{1}.961$	1,02707	$\bar{1}.986$	1,48464
	,00366		,00435		,00543		,00735		,01179		,03269
$\bar{1}.862$	,42708	$\bar{1}.887$	,52698	$\bar{1}.912$	,64856	$\bar{1}.937$	,80656	$\bar{1}.962$	1,03886	$\bar{1}.987$	1,51733
	,00368		,00438		,00548		,00746		,01209		,03526
$\bar{1}.863$	,44076	$\bar{1}.888$	,53136	$\bar{1}.913$	,65404	$\bar{1}.938$	,81402	$\bar{1}.963$	1,05095	$\bar{1}.988$	1,55259
	,00371		,00442		,00554		,00758		0,01241		,03829
$\bar{1}.864$	,43447	$\bar{1}.889$	,53578	$\bar{1}.914$	,65958	$\bar{1}.939$	,82160	$\bar{1}.964$	1,06336	$\bar{1}.989$	1,59088
	,00373		,00445		,00559		,00769		,01274		,04189
$\bar{1}.865$	,43820	$\bar{1}.890$	,54023	$\bar{1}.915$	,66517	$\bar{1}.940$	,82929	$\bar{1}.965$	1,07610	$\bar{1}.990$	1,63277
	,00376		,00449		,00566		,00781		,01309		,04626
$\bar{1}.866$	44196	$\bar{1}.891$	,54472	$\bar{1}.916$	,67083	$\bar{1}.941$	,83710	$\bar{1}.966$	1,08919	$\bar{1}.991$	1,67903
	,00378		,00452		,00572		,00793		,01348		,05163
$\bar{1}.867$	,44574	$\bar{1}.892$	,54924	$\bar{1}.917$	,67655	$\bar{1}.942$	,84503	$\bar{1}.967$	1,10267	$\bar{1}.992$	1,73069
	,00380		,00456		,00578		,00807		,01387		,05849
$\bar{1}.868$	,44954	$\bar{1}.893$	,55380	$\bar{1}.918$	,68233	$\bar{1}.943$	,85310	$\bar{1}.968$	1,11654	$\bar{1}.993$	1,78918
	,00383		,00460		,00584		,00820		,01429		,06745
$\bar{1}.869$	,45337	$\bar{1}.894$	,55840	$\bar{1}.919$	,68817	$\bar{1}.944$	,86130	$\bar{1}.969$	1,13083	$\bar{1}.994$	1,85663
	,00385		,00464		,00591		,00833		,01475		,07968
$\bar{1}.870$	,45722	$\bar{1}.895$	,56304	$\bar{1}.920$	,69408	$\bar{1}.945$	,86963	$\bar{1}.970$	1,14558	$\bar{1}.995$	1,93631
	,00388		,00467		,00598		,00848		,01523		,09741
$\bar{1}.871$	,46110	$\bar{1}.896$	,56771	$\bar{1}.921$	,70006	$\bar{1}.946$	,87811	$\bar{1}.971$	1,16081	$\bar{1}.996$	2,03372
	,00390		,00472		,00695		,00863		,01574		,12544
$\bar{1}.872$	,46500	$\bar{1}.897$	,57243	$\bar{1}.922$	,70611	$\bar{1}.947$	,88674	$\bar{1}.972$	1,17655	$\bar{1}.997$	2,15916
	,00393		,00476		,00611		,00878		,01530		,17459
$\bar{1}.873$	,46893	$\bar{1}.898$	,57719	$\bar{1}.923$	,71222	$\bar{1}.948$	,89552	$\bar{1}.973$	1,19285	$\bar{1}.998$	2,33575
	,00396		,00479		,00620		,00894		,01690		,30153
$\bar{1}.874$	,47289	$\bar{1}.899$	,58198	$\bar{1}.924$	,71842	$\bar{1}.949$	,90446	$\bar{1}.974$	1,20975	$\bar{1}.999$	2,63728
	,00399		,00485		,00626		,00911		,01753		

TABLE V.—Logarithms of the accommodated chances of living 10 years, deduced from the value of an annuity for 10 years, at 5 per cent. from the actual tables of mortality, and considered equal to a geometrical series of ten terms, of which the common ratio is the same as the first term, and the tenth term the accommodated chance; and to find the accommodated chance for 5, 7 years, &c. without a table calculated for the purpose, it may be considered sufficient to multiply by ,5; ,7, &c. the accommodated ratio in this table when extreme accuracy be not required.

Age.	Carlisle.	Deparcieux.	Northampton.	Age.	Carlisle.	Deparcieux.	Northampton.
0	$\bar{1}$,6892	—	—	52	$\bar{1}$,9172	$\bar{1}$,9006	$\bar{1}$,8528
1	$\bar{1}$,6763	—	$\bar{1}$,7044	53	$\bar{1}$,9098	$\bar{1}$,8957	$\bar{1}$,8471
2	$\bar{1}$,8699	—	$\bar{1}$,8356	54	$\bar{1}$,9013	$\bar{1}$,8901	$\bar{1}$,8417
3	$\bar{1}$,9159	$\bar{1}$,9166	$\bar{1}$,8790	55	$\bar{1}$,8915	$\bar{1}$,8853	$\bar{1}$,8357
4	$\bar{1}$,9401	$\bar{1}$,9315	$\bar{1}$,9081	56	$\bar{1}$,8803	$\bar{1}$,8799	$\bar{1}$,8294
5	$\bar{1}$,9586	$\bar{1}$,9411	$\bar{1}$,9220	57	$\bar{1}$,8680	$\bar{1}$,8732	$\bar{1}$,8228
6	$\bar{1}$,9686	$\bar{1}$,9486	$\bar{1}$,9369	58	$\bar{1}$,8513	$\bar{1}$,8673	$\bar{1}$,8156
7	$\bar{1}$,9737	$\bar{1}$,9544	$\bar{1}$,9476	59	$\bar{1}$,8435	$\bar{1}$,8601	$\bar{1}$,8081
8	$\bar{1}$,9764	$\bar{1}$,9592	$\bar{1}$,9550	60	$\bar{1}$,8318	$\bar{1}$,8511	$\bar{1}$,7998
9	$\bar{1}$,9773	$\bar{1}$,9637	$\bar{1}$,9586	61	$\bar{1}$,8243	$\bar{1}$,8398	$\bar{1}$,7908
10	$\bar{1}$,9768	$\bar{1}$,9669	$\bar{1}$,9592	62	$\bar{1}$,8171	$\bar{1}$,8264	$\bar{1}$,7811
11	$\bar{1}$,9754	$\bar{1}$,9679	$\bar{1}$,9582	63	$\bar{1}$,8090	$\bar{1}$,8120	$\bar{1}$,7699
12	$\bar{1}$,9742	$\bar{1}$,9669	$\bar{1}$,9566	64	$\bar{1}$,7974	$\bar{1}$,7946	$\bar{1}$,7576
13	$\bar{1}$,9729	$\bar{1}$,9658	$\bar{1}$,9546	65	$\bar{1}$,7860	$\bar{1}$,7735	$\bar{1}$,7431
14	$\bar{1}$,9716	$\bar{1}$,9704	$\bar{1}$,9521	66	$\bar{1}$,7703	$\bar{1}$,7510	$\bar{1}$,7267
15	$\bar{1}$,9704	$\bar{1}$,9628	$\bar{1}$,9490	67	$\bar{1}$,7506	$\bar{1}$,7270	$\bar{1}$,7083
16	$\bar{1}$,9698	$\bar{1}$,9609	$\bar{1}$,9455	68	$\bar{1}$,7107	$\bar{1}$,7017	$\bar{1}$,6879
17	$\bar{1}$,9694	$\bar{1}$,9600	$\bar{1}$,9419	69	$\bar{1}$,7005	$\bar{1}$,6754	$\bar{1}$,6651
18	$\bar{1}$,9693	$\bar{1}$,9586	$\bar{1}$,9388	70	$\bar{1}$,6689	$\bar{1}$,6480	$\bar{1}$,6402
19	$\bar{1}$,9690	$\bar{1}$,9574	$\bar{1}$,9358	71	$\bar{1}$,6319	$\bar{1}$,6167	$\bar{1}$,6126
20	$\bar{1}$,9685	$\bar{1}$,9559	$\bar{1}$,9337	72	$\bar{1}$,5936	$\bar{1}$,5841	$\bar{1}$,5823
21	$\bar{1}$,9679	$\bar{1}$,9554	$\bar{1}$,9321	73	$\bar{1}$,5563	$\bar{1}$,5500	$\bar{1}$,5487
22	$\bar{1}$,9670	$\bar{1}$,9549	$\bar{1}$,9311	74	$\bar{1}$,5269	$\bar{1}$,5119	$\bar{1}$,5117
23	$\bar{1}$,9659	$\bar{1}$,9544	$\bar{1}$,9298	75	$\bar{1}$,4940	$\bar{1}$,4711	$\bar{1}$,4723
24	$\bar{1}$,9644	$\bar{1}$,9540	$\bar{1}$,9289	76	$\bar{1}$,4642	$\bar{1}$,4218	$\bar{1}$,4308
25	$\bar{1}$,9628	$\bar{1}$,9534	$\bar{1}$,9277	77	$\bar{1}$,4344	$\bar{1}$,3684	$\bar{1}$,3846
26	$\bar{1}$,9573	$\bar{1}$,9531	$\bar{1}$,9264	78	$\bar{1}$,4007	$\bar{1}$,3134	$\bar{1}$,3307
27	$\bar{1}$,9591	$\bar{1}$,9524	$\bar{1}$,9257	79	$\bar{1}$,3538	$\bar{1}$,2497	$\bar{1}$,2644
28	$\bar{1}$,9570	$\bar{1}$,9521	$\bar{1}$,9238	80	$\bar{1}$,3134	$\bar{1}$,1876	$\bar{1}$,1900
29	$\bar{1}$,9556	$\bar{1}$,9518	$\bar{1}$,9226	81	$\bar{1}$,2582	$\bar{1}$,1214	$\bar{1}$,1101
30	$\bar{1}$,9552	$\bar{1}$,9514	$\bar{1}$,9211	82	$\bar{1}$,2043	$\bar{1}$,0609	$\bar{1}$,0234
31	$\bar{1}$,9548	$\bar{1}$,9514	$\bar{1}$,9196	83	$\bar{1}$,1765	$\bar{2}$,9688	$\bar{2}$,9341
32	$\bar{1}$,9540	$\bar{1}$,9514	$\bar{1}$,9180	84	$\bar{1}$,0727	$\bar{2}$,8536	$\bar{2}$,8592
33	$\bar{1}$,9528	$\bar{1}$,9515	$\bar{1}$,9164	85	$\bar{2}$,9939	$\bar{2}$,7199	$\bar{2}$,7813
34	$\bar{1}$,9513	$\bar{1}$,9517	$\bar{1}$,9146	86	$\bar{2}$,9166	$\bar{2}$,5736	$\bar{2}$,7003
35	$\bar{1}$,9485	$\bar{1}$,9522	$\bar{1}$,9126	87	$\bar{2}$,8490	$\bar{2}$,4254	$\bar{2}$,6149
36	$\bar{1}$,9477	$\bar{1}$,9528	$\bar{1}$,9104	88	$\bar{2}$,8055	$\bar{2}$,1943	$\bar{2}$,5369
37	$\bar{1}$,9452	$\bar{1}$,9534	$\bar{1}$,9083	89	$\bar{2}$,7537	$\bar{3}$,9129	$\bar{2}$,4179
38	$\bar{1}$,9437	$\bar{1}$,9527	$\bar{1}$,8057	90	$\bar{2}$,6695	$\bar{3}$,5265	$\bar{2}$,2414
39	$\bar{1}$,9406	$\bar{1}$,9517	$\bar{1}$,9031	91	$\bar{2}$,6658	$\bar{3}$,0266	$\bar{3}$,9356
40	$\bar{1}$,9383	$\bar{1}$,9506	$\bar{1}$,9001	92	$\bar{2}$,7323	$\bar{4}$,3694	$\bar{3}$,5037
41	$\bar{1}$,9372	$\bar{1}$,9488	$\bar{1}$,8973	93	$\bar{2}$,8031	$\bar{5}$,2971	$\bar{4}$,7375
42	$\bar{1}$,9365	$\bar{1}$,9466	$\bar{1}$,8943	94	$\bar{2}$,8355	—	$\bar{5}$,5769
43	$\bar{1}$,9365	$\bar{1}$,9438	$\bar{1}$,8915	95	$\bar{2}$,8107	—	$\bar{7}$,0496
44	$\bar{1}$,9366	$\bar{1}$,9403	$\bar{1}$,8882	96	$\bar{2}$,8279	—	—
45	$\bar{1}$,9367	$\bar{1}$,9361	$\bar{1}$,8848	97	$\bar{2}$,7589	—	—
46	$\bar{1}$,9366	$\bar{1}$,9308	$\bar{1}$,8810	98	$\bar{2}$,6695	—	—
47	$\bar{1}$,9358	$\bar{1}$,9263	$\bar{1}$,8767	99	$\bar{2}$,5111	—	—
48	$\bar{1}$,9351	$\bar{1}$,9200	$\bar{1}$,8740	100	$\bar{2}$,1629	—	—
49	$\bar{1}$,9328	$\bar{1}$,9158	$\bar{1}$,8631	101	$\bar{3}$,5689	—	—
50	$\bar{1}$,9292	$\bar{1}$,9098	$\bar{1}$,8621	102	$\bar{4}$,3245	—	—
51	$\bar{1}$,9233	$\bar{1}$,9027	$\bar{1}$,8571	103	6,0595	—	—

TABLE VI.—Accommodated annual ratio for an unlimited period for every age a

$$\lambda r = \lambda \overset{1,05^{-1}}{\tfrac{\lambda}{1}} \rceil \underline{a} - \lambda \overset{1,05^{-1}}{\tfrac{1}{0}} \rceil \underline{a} + \lambda\, 1,05.$$

a	λr Carlisle.	λr Deparcieux.	λr Northampton.
0	$\bar{1}$,98665	—	—
1	$\bar{1}$,99121	—	$\bar{1}$,98517
2	$\bar{1}$,99313	—	$\bar{1}$,98997
3	$\bar{1}$,99458	$\bar{1}$,99399	$\bar{1}$,99151
4	$\bar{1}$,99528	$\bar{1}$,99446	$\bar{1}$,99244
5	$\bar{1}$,99577	$\bar{1}$,99473	$\bar{1}$,99284
6	$\bar{1}$,99599	$\bar{1}$,99493	$\bar{1}$,99324
7	$\bar{1}$,99606	$\bar{1}$,99505	$\bar{1}$,99346
8	$\bar{1}$,99606	$\bar{1}$,99513	$\bar{1}$,99357
9	$\bar{1}$,99600	$\bar{1}$,99519	$\bar{1}$,99354
10	$\bar{1}$,99529	$\bar{1}$,99519	$\bar{1}$,99341
11	$\bar{1}$,99576	$\bar{1}$,99514	$\bar{1}$,99323
12	$\bar{1}$,99563	$\bar{1}$,99503	$\bar{1}$,99304
13	$\bar{1}$,99549	$\bar{1}$,99491	$\bar{1}$,99284
14	$\bar{1}$,99535	$\bar{1}$,99478	$\bar{1}$,99262
15	$\bar{1}$,99522	$\bar{1}$,99464	$\bar{1}$,99240
16	$\bar{1}$,99509	$\bar{1}$,99450	$\bar{1}$,99213
17	$\bar{1}$,99487	$\bar{1}$,99438	$\bar{1}$,99190
18	$\bar{1}$,99484	$\bar{1}$,99425	$\bar{1}$,99167
19	$\bar{1}$,99471	$\bar{1}$,99413	$\bar{1}$,99145
20	$\bar{1}$,99455	$\bar{1}$,99398	$\bar{1}$,99124
21	$\bar{1}$,99442	$\bar{1}$,99388	$\bar{1}$.99106
22	$\bar{1}$.99425	$\bar{1}$,99375	$\bar{1}$,99088
23	$\bar{1}$,99408	$\bar{1}$,99364	$\bar{1}$,99070
24	$\bar{1}$,99390	$\bar{1}$,99350	$\bar{1}$,99051
25	$\bar{1}$,99370	$\bar{1}$,99338	$\bar{1}$,99030
26	$\bar{1}$,99349	$\bar{1}$,99323	$\bar{1}$,99009
27	$\bar{1}$,99328	$\bar{1}$,99308	$\bar{1}$,98988
28	$\bar{1}$,99306	$\bar{1}$,99293	$\bar{1}$,98965
29	$\bar{1}$,99290	$\bar{1}$,99277	$\bar{1}$,98943
30	$\bar{1}$,99265	$\bar{1}$,99259	$\bar{1}$,98917
31	$\bar{1}$,99245	$\bar{1}$,99241	$\bar{1}$,98892
32	$\bar{1}$,99224	$\bar{1}$,99223	$\bar{1}$,98865
33	$\bar{1}$,99201	$\bar{1}$,99202	$\bar{1}$,98837
34	$\bar{1}$,99176	$\bar{1}$,99181	$\bar{1}$,98808
35	$\bar{1}$,99149	$\bar{1}$,99158	$\bar{1}$,98777
36	$\bar{1}$,99110	$\bar{1}$,99134	$\bar{1}$,98745
37	$\bar{1}$,99090	$\bar{1}$,99109	$\bar{1}$,98811
38	$\bar{1}$,99058	$\bar{1}$,99077	$\bar{1}$,98675
39	$\bar{1}$,99025	$\bar{1}$,99043	$\bar{1}$,98637
40	$\bar{1}$,98991	$\bar{1}$,99006	$\bar{1}$,98597
41	$\bar{1}$,98958	$\bar{1}$,98967	$\bar{1}$,98556
42	$\bar{1}$,98924	$\bar{1}$,98924	$\bar{1}$,98513
43	$\bar{1}$,98890	$\bar{1}$,98878	$\bar{1}$,98469
44	$\bar{1}$,98853	$\bar{1}$,98828	$\bar{1}$,98423
45	$\bar{1}$,98814	$\bar{1}$,98771	$\bar{1}$,98375
46	$\bar{1}$,98771	$\bar{1}$,98714	$\bar{1}$,98323
47	$\bar{1}$,98725	$\bar{1}$,98655	$\bar{1}$,98270
48	$\bar{1}$,98673	$\bar{1}$,98590	$\bar{1}$,98209
49	$\bar{1}$,98612	$\bar{1}$,98526	$\bar{1}$,98148
50	$\bar{1}$,98546	$\bar{1}$,98456	$\bar{1}$,98083
51	$\bar{1}$,98471	$\bar{1}$,98386	$\bar{1}$,98017
52	$\bar{1}$,98390	$\bar{1}$,98216	$\bar{1}$,97950
53	$\bar{1}$,98305	$\bar{1}$,98240	$\bar{1}$,97878
54	$\bar{1}$,98212	$\bar{1}$,98156	$\bar{1}$,97802
55	$\bar{1}$,98112	$\bar{1}$,98073	$\bar{1}$,97721
56	$\bar{1}$,98005	$\bar{1}$,97982	$\bar{1}$,97635
57	$\bar{1}$,97887	$\bar{1}$,97882	$\bar{1}$,97542
58	$\bar{1}$,97763	$\bar{1}$,97780	$\bar{1}$,97445
59	$\bar{1}$,97637	$\bar{1}$,97668	$\bar{1}$,97341
60	$\bar{1}$,97514	$\bar{1}$,97545	$\bar{1}$,97230
61	$\bar{1}$,97400	$\bar{1}$,97408	$\bar{1}$,97111
62	$\bar{1}$,97281	$\bar{1}$,97254	$\bar{1}$,96983
63	$\bar{1}$,97154	$\bar{1}$,97093	$\bar{1}$,96843
64	$\bar{1}$,97014	$\bar{1}$,96912	$\bar{1}$,96693
65	$\bar{1}$,96858	$\bar{1}$,96707	$\bar{1}$,96526
66	$\bar{1}$,96685	$\bar{1}$,96489	$\bar{1}$,96346
67	$\bar{1}$,96491	$\bar{1}$,96250	$\bar{1}$,96150
68	$\bar{1}$,96273	$\bar{1}$,96009	$\bar{1}$,95936
69	$\bar{1}$,96029	$\bar{1}$,95750	$\bar{1}$,95703
70	$\bar{1}$,95755	$\bar{1}$,95480	$\bar{1}$,95448
71	$\bar{1}$,95440	$\bar{1}$,95181	$\bar{1}$,95171
72	$\bar{1}$,95111	$\bar{1}$,94870	$\bar{1}$,94869
73	$\bar{1}$,94784	$\bar{1}$,94542	$\bar{1}$,94541
74	$\bar{1}$,94467	$\bar{1}$,94180	$\bar{1}$,94186
75	$\bar{1}$,94185	$\bar{1}$,93790	$\bar{1}$,93812
76	$\bar{1}$,93888	$\bar{1}$,93330	$\bar{1}$,93423
77	$\bar{1}$,93591	$\bar{1}$,92833	$\bar{1}$,92994
78	$\bar{1}$,93265	$\bar{1}$,92314	$\bar{1}$,92503
79	$\bar{1}$,92863	$\bar{1}$,91715	$\bar{1}$,91916
80	$\bar{1}$,92461	$\bar{1}$,91124	$\bar{1}$,91246
81	$\bar{1}$,91891	$\bar{1}$,90493	$\bar{1}$,90509
82	$\bar{1}$,91491	$\bar{1}$,89856	$\bar{1}$,89697
83	$\bar{1}$,90939	$\bar{1}$,89069	$\bar{1}$,88844
84	$\bar{1}$,90344	$\bar{1}$,88005	$\bar{1}$,88110
85	$\bar{1}$,89657	$\bar{1}$,86782	$\bar{1}$,87361
86	$\bar{1}$,88978	$\bar{1}$,85416	$\bar{1}$,86599
87	$\bar{1}$,88369	$\bar{1}$,84011	$\bar{1}$,85803
88	$\bar{1}$,87972	$\bar{1}$,81788	$\bar{1}$,85069
89	$\bar{1}$,87481	$\bar{1}$,78941	$\bar{1}$,85454
90	$\bar{1}$,86660	$\bar{1}$,75223	$\bar{1}$,82243
91	$\bar{1}$,86560	$\bar{1}$,70265	$\bar{1}$,79303
92	$\bar{1}$,87056	$\bar{1}$,63694	$\bar{1}$,74992
93	$\bar{1}$,87595	$\bar{1}$,52971	$\bar{1}$,67376
94	$\bar{1}$,87840	—	$\bar{1}$,55753
95	$\bar{1}$,87967	—	$\bar{1}$,30505
96	$\bar{1}$,77774	—	—
97	$\bar{1}$,77140	—	—
98	$\bar{1}$,76333	—	—
99	$\bar{1}$,84829	—	—
100	$\bar{1}$,81282	—	—
101	$\bar{1}$,75663	—	—
102	$\bar{1}$,65421	—	—
103	$\bar{1}$,40266	—	—

TABLE VII.—Logarithm of Carlisle chance of living 5 years at every age *a*.

a	λ chance.	*a*	λ chance.	*a*	λ chance.	*a*	λ chance.	*a*	λ chance.
0	$\bar{1}$,83232	20	$\bar{1}$,98469	40	$\bar{1}$,96915	60	$\bar{1}$,91826	80	$\bar{1}$,66927
1	$\bar{1}$,89709	21	$\bar{1}$,98457	41	$\bar{1}$,96836	61	$\bar{1}$,91483	81	$\bar{1}$,64194
2	$\bar{1}$,92823	22	$\bar{1}$,98439	42	$\bar{1}$,96790	62	$\bar{1}$,91180	82	$\bar{1}$,61095
3	$\bar{1}$,95354	23	$\bar{1}$,98405	43	$\bar{1}$,96780	63	$\bar{1}$,90864	83	$\bar{1}$,57100
4	$\bar{1}$,96747	24	$\bar{1}$,98333	44	$\bar{1}$,96808	64	$\bar{1}$,90492	84	$\bar{1}$,53422
5	$\bar{1}$,97792	25	$\bar{1}$,98213	45	$\bar{1}$,96857	65	$\bar{1}$,90067	85	$\bar{1}$,50393
6	$\bar{1}$,98376	26	$\bar{1}$,98091	46	$\bar{1}$,96918	66	$\bar{1}$,89586	86	$\bar{1}$,45652
7	$\bar{1}$,98703	27	$\bar{1}$,97967	47	$\bar{1}$,96941	67	$\bar{1}$,88838	87	$\bar{1}$,40377
8	$\bar{1}$,98869	28	$\bar{1}$,97863	48	$\bar{1}$,96915	68	$\bar{1}$,87746	88	$\bar{1}$,36691
9	$\bar{1}$,98930	29	$\bar{1}$,97804	49	$\bar{1}$,96818	69	$\bar{1}$,86279	89	$\bar{1}$,34438
10	$\bar{1}$,98911	30	$\bar{1}$,97789	50	$\bar{1}$,96676	70	$\bar{1}$,84362	90	$\bar{1}$,32483
11	$\bar{1}$,98836	31	$\bar{1}$,97783	51	$\bar{1}$,96477	71	$\bar{1}$,82305	91	$\bar{1}$,34054
12	$\bar{1}$,98754	32	$\bar{1}$,97767	52	$\bar{1}$,96269	72	$\bar{1}$,80220	92	$\bar{1}$,38021
13	$\bar{1}$,98670	33	$\bar{1}$,97736	53	$\bar{1}$,96017	73	$\bar{1}$,78348	93	$\bar{1}$,41373
14	$\bar{1}$,98593	34	$\bar{1}$,97687	54	$\bar{1}$,95660	74	$\bar{1}$,76877	94	$\bar{1}$,43933
15	$\bar{1}$,98528	35	$\bar{1}$,97611	55	$\bar{1}$,95155	75	$\bar{1}$,75508	95	$\bar{1}$,47712
16	$\bar{1}$,98490	36	$\bar{1}$,97490	56	$\bar{1}$,94461	76	$\bar{1}$,74231	96	$\bar{1}$,48337
17	$\bar{1}$,98479	37	$\bar{1}$,97349	57	$\bar{1}$,93711	77	$\bar{1}$,72712	97	$\bar{1}$,44370
18	$\bar{1}$,98476	38	$\bar{1}$,97194	58	$\bar{1}$,92973	78	$\bar{1}$,71062	98	$\bar{1}$,33099
19	$\bar{1}$,98472	39	$\bar{1}$,97044	59	$\bar{1}$,92343	79	$\bar{1}$,68963		

Logarithm of the Carlisle chance of living 10 years at every age *a*.

0	$\bar{1}$,81023	19	$\bar{1}$,96805	38	$\bar{1}$,93973	57	$\bar{1}$,84891	76	$\bar{1}$,38425
1	$\bar{1}$,88086	20	$\bar{1}$,96682	39	$\bar{1}$,93851	58	$\bar{1}$,83836	77	$\bar{1}$,33807
2	$\bar{1}$,91526	21	$\bar{1}$,96548	40	$\bar{1}$,93772	59	$\bar{1}$,82835	78	$\bar{1}$,28163
3	$\bar{1}$,94223	22	$\bar{1}$,96406	41	$\bar{1}$,93754	60	$\bar{1}$,81893	79	$\bar{1}$,22385
4	$\bar{1}$,95677	23	$\bar{1}$,96268	42	$\bar{1}$,93731	61	$\bar{1}$,81070	80	$\bar{1}$,17320
5	$\bar{1}$,96702	24	$\bar{1}$,96136	43	$\bar{1}$,93694	62	$\bar{1}$,80018	81	$\bar{1}$,09846
6	$\bar{1}$.97213	25	$\bar{1}$,96002	44	$\bar{1}$,93626	63	$\bar{1}$,78610	82	$\bar{1}$,01472
7	$\bar{1}$,97457	26	$\bar{1}$,95873	45	$\bar{1}$,93533	64	$\bar{1}$,76771	83	$\bar{2}$,93791
8	$\bar{1}$,97540	27	$\bar{1}$,95734	46	$\bar{1}$,93395	65	$\bar{1}$,74430	84	$\bar{2}$,87860
	$\bar{1}$,97523	28	$\bar{1}$,95598	47	$\bar{1}$,93211	66	$\bar{1}$,71891	85	$\bar{2}$,82876
10	$\bar{1}$,97438	29	$\bar{1}$,95490	48	$\bar{1}$,92932	67	$\bar{1}$,69058	86	$\bar{2}$,79706
11	$\bar{1}$,97326	30	$\bar{1}$,95400	49	$\bar{1}$,92478	68	$\bar{1}$,66094	87	$\bar{2}$,78398
12	$\bar{1}$,97233	31	$\bar{1}$,95273	50	$\bar{1}$,91830	69	$\bar{1}$,63157	88	$\bar{2}$,78064
13	$\bar{1}$,97146	32	$\bar{1}$,95116	51	$\bar{1}$,90938	70	$\bar{1}$,59870	89	$\bar{2}$,78371
14	$\bar{1}$,07065	33	$\bar{1}$,94929	52	$\bar{1}$,89980	71	$\bar{1}$,56536	90	$\bar{2}$,80195
15	$\bar{1}$,96996	34	$\bar{1}$,94730	53	$\bar{1}$,88990	72	$\bar{1}$,52932	91	$\bar{2}$,82391
16	$\bar{1}$,96947	35	$\bar{1}$,94526	54	$\bar{1}$,88003	73	$\bar{1}$,49411	92	$\bar{2}$,82391
17	$\bar{1}$,96918	36	$\bar{1}$,94326	55	$\bar{1}$,86981	74	$\bar{1}$,45840	93	$\bar{2}$,74473
18	$\bar{1}$,96881	37	$\bar{1}$,94138	56	$\bar{1}$,85944	75	$\bar{1}$,42434		

TABLE VII.—*continued.*

Logarithm of the Carlisle chance of living 15 years for every age *a*.

a	λ chance.	*a*	λ chance.	*a*	λ chance.	*a*	λ chance.	*a*	λ chance.
0	$\bar{1}$,79934	18	$\bar{1}$,94744	36	$\bar{1}$,91244	54	$\bar{1}$,78495	72	$\bar{1}$,14027
1	$\bar{1}$,86922	19	$\bar{1}$,94609	37	$\bar{1}$,91080	55	$\bar{1}$,77048	73	$\bar{1}$,06511
2	$\bar{1}$,90280	20	$\bar{1}$,94471	38	$\bar{1}$,90888	56	$\bar{1}$,75530	74	$\bar{2}$,99263
3	$\bar{1}$,92893	21	$\bar{1}$,94331	39	$\bar{1}$,90669	57	$\bar{1}$,73729	75	$\bar{2}$,92827
4	$\bar{1}$,94270	22	$\bar{1}$,94173	40	$\bar{1}$,90448	58	$\bar{1}$,71582	76	$\bar{2}$,84078
5	$\bar{1}$,95230	23	$\bar{1}$,94004	41	$\bar{1}$,90231	59	$\bar{1}$,69114	77	$\bar{2}$,74184
6	$\bar{1}$,95702	24	$\bar{1}$,93823	42	$\bar{1}$,90000	60	$\bar{1}$,66256	78	$\bar{2}$,64853
7	$\bar{1}$,95936	25	$\bar{1}$,93613	43	$\bar{1}$,89712	61	$\bar{1}$,63375	79	$\bar{2}$,56823
8	$\bar{1}$,96015	26	$\bar{1}$,93364	44	$\bar{1}$,89284	62	$\bar{1}$,60238	80	$\bar{2}$,49803
9	$\bar{1}$,95995	27	$\bar{1}$ 93082	45	$\bar{1}$,88687	63	$\bar{1}$,56958	81	$\bar{2}$,43900
10	$\bar{1}$,95907	28	$\bar{1}$,92792	46	$\bar{1}$,87856	64	$\bar{1}$,53648	82	$\bar{2}$,39494
11	$\bar{1}$,95784	29	$\bar{1}$,92534	47	$\bar{1}$,86922	65	$\bar{1}$,49937	83	$\bar{2}$,35164
12	$\bar{1}$,95672	30	$\bar{1}$,92316	48	$\bar{1}$,85905	66	$\bar{1}$,46123	84	$\bar{2}$,31794
13	$\bar{1}$,95551	31	$\bar{1}$,92108	49	$\bar{1}$,84820	67	$\bar{1}$,41770	85	$\bar{2}$,30588
14	$\bar{1}$,95397	32	$\bar{1}$,91905	50	$\bar{1}$,83656	68	$\bar{1}$,37157	86	$\bar{2}$,28043
15	$\bar{1}$,95209	33	$\bar{1}$,91709	51	$\bar{1}$,82421	69	$\bar{1}$,32120	87	$\bar{2}$,22768
16	$\bar{1}$,95038	34	$\bar{1}$,91538	52	$\bar{1}$,81160	70	$\bar{1}$,26797	88	$\bar{2}$,11163
17	$\bar{1}$,94885	35	$\bar{1}$,91383	53	$\bar{1}$,79853	71	$\bar{1}$,20730		

Logarithm of the Carlisle chance of living 20 years for every age *a*.

0	$\bar{1}$,78462	17	$\bar{1}$,92652	34	$\bar{1}$,88356	51	$\bar{1}$,72007	68	$\bar{2}$,94257
1	$\bar{1}$,85412	18	$\bar{1}$,92479	35	$\bar{1}$,88059	52	$\bar{1}$,69998	69	$\bar{2}$,85542
2	$\bar{1}$,88759	19	$\bar{1}$,92295	36	$\bar{1}$,87721	53	$\bar{1}$,67599	70	$\bar{2}$,77190
3	$\bar{1}$,91369	20	$\bar{1}$,92082	37	$\bar{1}$,87349	54	$\bar{1}$,64744	71	$\bar{2}$,66383
4	$\bar{1}$,92742	21	$\bar{1}$,91821	38	$\bar{1}$,86906	55	$\bar{1}$,61410	72	$\bar{2}$,54404
5	$\bar{1}$,93699	22	$\bar{1}$,91521	39	$\bar{1}$,86329	56	$\bar{1}$,57835	73	$\bar{2}$,43202
6	$\bar{1}$,94160	23	$\bar{1}$,91197	40	$\bar{1}$,85602	57	$\bar{1}$,53949	74	$\bar{2}$,33701
7	$\bar{1}$,94376	24	$\bar{1}$,90866	41	$\bar{1}$,84692	58	$\bar{1}$,49930	75	$\bar{2}$,25311
8	$\bar{1}$,94421	25	$\bar{1}$,90528	42	$\bar{1}$,83711	59	$\bar{1}$,45991	76	$\bar{2}$,18132
9	$\bar{1}$,94328	26	$\bar{1}$,90199	43	$\bar{1}$,82684	60	$\bar{1}$,41763	77	$\bar{2}$,12205
10	$\bar{1}$,94120	27	$\bar{1}$,89872	45	$\bar{1}$,81628	61	$\bar{1}$,37606	78	$\bar{2}$,06227
11	$\bar{1}$,93874	28	$\bar{1}$,89572	45	$\bar{1}$,80513	62	$\bar{1}$,32950	79	$\bar{2}$,00757
12	$\bar{1}$,93639	29	$\bar{1}$,89342	46	$\bar{1}$,79339	63	$\bar{1}$,28021	80	$\bar{3}$,97515
13	$\bar{1}$,93414	30	$\bar{1}$,89172	45	$\bar{1}$,78101	64	$\bar{1}$,22611	81	$\bar{3}$,92237
14	$\bar{1}$,93201	31	$\bar{1}$,89027	48	$\bar{1}$,76768	65	$\bar{1}$,16864	82	$\bar{3}$,83863
15	$\bar{1}$,92999	32	$\bar{1}$,88847	49	$\bar{1}$,75312	66	$\bar{1}$,10317	83	$\bar{3}$,68263
16	$\bar{1}$,92821	33	$\bar{1}$,88624	50	$\bar{1}$,73723	67	$\bar{1}$,02866		

TABLE VII.—*continued.*

Logarithm of the Carlisle chance of living 25 years at every age *a*.

a	λ chance.	*a*	λ chance.	*a*	λ chance.	*a*	λ chance.	*a*	λ chance.
0	$\bar{1}$,76930	16	$\bar{1}$,90311	32	$\bar{1}$,85116	48	$\bar{1}$,64514	64	$\bar{2}$,76034
1	$\bar{1}$,83969	17	$\bar{1}$,90001	33	$\bar{1}$,84641	49	$\bar{1}$,61591	65	$\bar{2}$,67257
2	$\bar{1}$,87198	18	$\bar{1}$,89673	34	$\bar{1}$,84016	50	$\bar{1}$,58086	66	$\bar{2}$,55969
3	$\bar{1}$,89774	19	$\bar{1}$,89339	35	$\bar{1}$,83213	51	$\bar{1}$,5431	67	$\bar{2}$,43242
4	$\bar{1}$,91075	20	$\bar{1}$,88997	36	$\bar{1}$,82182	52	$\bar{1}$,50218	68	$\bar{2}$,30948
5	$\bar{1}$,91912	21	$\bar{1}$,88657	37	$\bar{1}$,81060	53	$\bar{1}$,45948	69	$\bar{2}$,19980
6	$\bar{1}$,92251	22	$\bar{1}$,88311	38	$\bar{1}$,79878	54	$\bar{1}$,41651	70	$\bar{2}$,09673
7	$\bar{1}$,92342	23	$\bar{1}$,87977	39	$\bar{1}$,78672	55	$\bar{1}$,36918	71	$\bar{2}$,00437
8	$\bar{1}$,92283	24	$\bar{1}$,87674	40	$\bar{1}$,77428	56	$\bar{1}$,32067	72	$\bar{3}$,92425
9	$\bar{1}$,92131	25	$\bar{1}$,87385	41	$\bar{1}$,76175	57	$\bar{1}$,26661	73	$\bar{3}$,84575
10	$\bar{1}$,91909	26	$\bar{1}$,87117	42	$\bar{1}$,74891	58	$\bar{1}$,20993	74	$\bar{3}$,77634
11	$\bar{1}$,91657	27	$\bar{1}$,86813	43	$\bar{1}$,73548	59	$\bar{1}$,14954	75	$\bar{3}$,73023
12	$\bar{1}$,91406	28	$\bar{1}$,86487	44	$\bar{1}$,72120	60	$\bar{1}$,08690	76	$\bar{3}$,66469
13	$\bar{1}$,91150	29	$\bar{1}$,86159	45	$\bar{1}$,70581	61	$\bar{1}$,01800	77	$\bar{3}$,56575
14	$\bar{1}$,90888	30	$\bar{1}$,85848	46	$\bar{1}$,68926	62	$\bar{2}$,94045	78	$\bar{3}$,39326
15	$\bar{1}$,90610	31	$\bar{1}$,85504	47	$\bar{1}$,66940	63	$\bar{2}$,85121		

Logarithm of the Carlisle chance of living 30 years at every age *a*.

0	$\bar{1}$,75143	15	$\bar{1}$,87525	30	$\bar{1}$,81003	45	$\bar{1}$,54943	60	$\bar{2}$,59083
1	$\bar{1}$,81960	16	$\bar{1}$,87146	31	$\bar{1}$,79964	46	$\bar{1}$,51231	61	$\bar{2}$,47452
2	$\bar{1}$,85164	17	$\bar{1}$,86790	32	$\bar{1}$,78827	47	$\bar{1}$,47160	62	$\bar{2}$,34422
3	$\bar{1}$,87637	18	$\bar{1}$,86453	33	$\bar{1}$,77614	48	$\bar{1}$,42863	63	$\bar{2}$,21811
4	$\bar{1}$,88879	19	$\bar{1}$,86147	34	$\bar{1}$,76358	49	$\bar{1}$,38467	64	$\bar{2}$,10472
5	$\bar{1}$,89701	20	$\bar{1}$,85854	35	$\bar{1}$,75039	50	$\bar{1}$,33594	65	$\bar{3}$,99740
6	$\bar{1}$,90033	21	$\bar{1}$,85575	36	$\bar{1}$,73665	51	$\bar{1}$,28544	66	$\bar{3}$,90023
7	$\bar{1}$,90109	22	$\bar{1}$,85253	37	$\bar{1}$,72240	52	$\bar{1}$,22930	67	$\bar{3}$,81264
8	$\bar{1}$,90019	23	$\bar{1}$,84892	38	$\bar{1}$,70742	53	$\bar{1}$,17010	68	$\bar{3}$,72321
9	$\bar{1}$,89818	24	$\bar{1}$,84492	39	$\bar{1}$,69164	54	$\bar{1}$,10614	69	$\bar{3}$,63913
10	$\bar{1}$,89520	25	$\bar{1}$,84061	40	$\bar{1}$,67496	55	$\bar{1}$,03845	70	$\bar{3}$,57385
11	$\bar{1}$,89147	26	$\bar{1}$,83594	41	$\bar{1}$,65761	56	$\bar{2}$,96261	71	$\bar{3}$,48774
12	$\bar{1}$,88755	27	$\bar{1}$,83083	42	$\bar{1}$,63729	57	$\bar{2}$,87756	72	$\bar{3}$,36795
13	$\bar{1}$,88344	28	$\bar{1}$,82504	43	$\bar{1}$,61294	58	$\bar{2}$,78093	73	$\bar{3}$,17674
14	$\bar{1}$,87931	29	$\bar{1}$,81819	44	$\bar{1}$,58399	59	$\bar{2}$,68376		

Logarithm of the Carlisle chance of living 35 years for every age *a*.

0	$\bar{1}$,72933	14	$\bar{1}$,84739	28	$\bar{1}$,75476	42	$\bar{1}$,43949	56	$\bar{2}$,41913
1	$\bar{1}$,79743	15	$\bar{1}$,84382	29	$\bar{1}$,74162	43	$\bar{1}$,39642	57	$\bar{2}$,28133
2	$\bar{1}$,82932	16	$\bar{1}$,84065	30	$\bar{1}$,72829	44	$\bar{1}$,35277	58	$\bar{2}$,14784
3	$\bar{1}$,85373	17	$\bar{1}$,83732	31	$\bar{1}$,71448	45	$\bar{1}$,30451	59	$\bar{2}$,02815
4	$\bar{1}$,86565	18	$\bar{1}$,83368	32	$\bar{1}$,70007	46	$\bar{1}$,25462	60	$\bar{3}$,91566
5	$\bar{1}$,87312	19	$\bar{1}$,82964	33	$\bar{1}$,68477	47	$\bar{1}$,19872	61	$\bar{3}$,81506
6	$\bar{1}$,87523	20	$\bar{1}$,82530	34	$\bar{1}$,66850	48	$\bar{1}$,13925	62	$\bar{3}$,72443
7	$\bar{1}$,87458	21	$\bar{1}$,82052	35	$\bar{1}$,65106	49	$\bar{1}$,07432	63	$\bar{3}$,63185
8	$\bar{1}$,87213	22	$\bar{1}$,81522	36	$\bar{1}$,63251	50	$\bar{1}$,00520	64	$\bar{3}$,54405
9	$\bar{1}$,86861	23	$\bar{1}$,80909	37	$\bar{1}$,61078	51	$\bar{2}$,92738	65	$\bar{3}$,47452
10	$\bar{1}$,86435	24	$\bar{1}$,80152	38	$\bar{1}$,58488	52	$\bar{2}$,84025	66	$\bar{3}$,38360
11	$\bar{1}$,85983	25	$\bar{1}$,79216	39	$\bar{1}$,55443	53	$\bar{2}$,74110	67	$\bar{3}$,25633
12	$\bar{1}$,85544	26	$\bar{1}$,78055	40	$\bar{1}$,51858	54	$\bar{2}$,64036	68	$\bar{3}$,05420
13	$\bar{1}$,85123	27	$\bar{1}$,76794	41	$\bar{1}$,48066	55	$\bar{2}$,54237		

TABLE VII.—*continued.*

Logarithm of the Carlisle chance of living 40 years for every age *a*.

a	λ chance.	*a*	λ chance.	*a*	λ chance.	*a*	λ chance.	*a*	λ chance.
0	$\bar{1},70544$	13	$\bar{1},82038$	26	$\bar{1},69538$	39	$\bar{1},32320$	52	$\bar{2},24402$
1	$\bar{1},77233$	14	$\bar{1},81557$	27	$\bar{1},67973$	40	$\bar{1},27366$	53	$\bar{2},10801$
2	$\bar{1},80280$	15	$\bar{1},81057$	28	$\bar{1},66340$	41	$\bar{1},22298$	54	$\bar{3},98474$
3	$\bar{1},82567$	16	$\bar{1},80542$	29	$\bar{1},64654$	42	$\bar{1},16661$	55	$\bar{3},86721$
4	$\bar{1},83609$	17	$\bar{1},80001$	30	$\bar{1},62896$	43	$\bar{1},10705$	56	$\bar{3},75967$
5	$\bar{1},84227$	18	$\bar{1},79385$	31	$\bar{1},61034$	44	$\bar{1},04240$	57	$\bar{3},66154$
6	$\bar{1},84359$	19	$\bar{1},78624$	32	$\bar{1},58845$	45	$\bar{2},97377$	58	$\bar{3},56157$
7	$\bar{1},84247$	20	$\bar{1},77684$	33	$\bar{1},56223$	46	$\bar{2},89656$	59	$\bar{3},46748$
8	$\bar{1},83992$	21	$\bar{1},76513$	34	$\bar{1},53130$	47	$\bar{2},80967$	60	$\bar{3},39278$
9	$\bar{1},83669$	22	$\bar{1},75233$	35	$\bar{1},49469$	48	$\bar{2},71025$	61	$\bar{3},29843$
10	$\bar{1},83292$	23	$\bar{1},73882$	36	$\bar{1},45556$	49	$\bar{2},60854$	62	$\bar{3},16813$
11	$\bar{1},82901$	24	$\bar{1},72494$	37	$\bar{1},41298$	50	$\bar{2},50913$	63	$\bar{4},96284$
12	$\bar{1},82486$	25	$\bar{1},71042$	38	$\bar{1},36836$	51	$\bar{2},38390$		

Logarithm of the Carlisle chance of living 45 years for every age *a*.

0	$\bar{1},67459$	12	$\bar{1},78755$	24	$\bar{1},62987$	36	$\bar{1},19787$	48	$\bar{2},07716$
1	$\bar{1},74068$	13	$\bar{1},78055$	25	$\bar{1},61109$	37	$\bar{1},14010$	49	$\bar{3},95292$
2	$\bar{1},77070$	14	$\bar{1},77217$	26	$\bar{1},59125$	38	$\bar{1},07899$	50	$\bar{3},83396$
3	$\bar{1},79346$	15	$\bar{1},76212$	27	$\bar{1},56812$	39	$\bar{1},01283$	51	$\bar{3},72444$
4	$\bar{1},80417$	16	$\bar{1},75002$	28	$\bar{1},54086$	40	$\bar{2},94292$	52	$\bar{3},62423$
5	$\bar{1},81084$	17	$\bar{1},73712$	29	$\bar{1},50933$	41	$\bar{2},86492$	53	$\bar{3},52174$
6	$\bar{1},81277$	18	$\bar{1},72357$	30	$\bar{1},47258$	42	$\bar{2},77756$	54	$\bar{3},42408$
7	$\bar{1},81190$	19	$\bar{1},70967$	31	$\bar{1},43339$	43	$\bar{2},67805$	55	$\bar{3},34433$
8	$\bar{1},80907$	20	$\bar{1},69510$	32	$\bar{1},39065$	44	$\bar{2},57662$	56	$\bar{3},24304$
9	$\bar{1},80487$	21	$\bar{1},67996$	33	$\bar{1},34571$	45	$\bar{2},47770$	57	$\bar{3},10524$
10	$\bar{1},79968$	22	$\bar{1},66412$	34	$\bar{1},30007$	46	$\bar{2},35308$	58	$\bar{4},89256$
11	$\bar{1},79378$	23	$\bar{1},64745$	35	$\bar{1},24977$	47	$\bar{2},21344$		

Logarithm of the Carlisle chance of living 50 years for every age *a*.

0	$\bar{1},64316$	11	$\bar{1},73839$	22	$\bar{1},55251$	33	$\bar{1},05634$	44	$\bar{3},92100$
1	$\bar{1},70987$	12	$\bar{1},72466$	23	$\bar{1},52491$	34	$\bar{2},98970$	45	$\bar{3},80253$
2	$\bar{1},74011$	13	$\bar{1},71028$	24	$\bar{1},49266$	35	$\bar{2},91903$	46	$\bar{3},69362$
3	$\bar{1},76261$	14	$\bar{1},69560$	25	$\bar{1},45471$	36	$\bar{2},83982$	47	$\bar{3},59369$
4	$\bar{1},77234$	15	$\bar{1},68038$	26	$\bar{1},41430$	37	$\bar{2},75105$	48	$\bar{3},49289$
5	$\bar{1},77760$	16	$\bar{1},66486$	27	$\bar{1},37032$	38	$\bar{2},65000$	49	$\bar{3},39225$
6	$\bar{1},77754$	17	$\bar{1},64892$	28	$\bar{1},32434$	39	$\bar{2},54705$	50	$\bar{3},31109$
7	$\bar{1},77458$	18	$\bar{1},63222$	29	$\bar{1},27810$	40	$\bar{2},44685$	51	$\bar{3},20781$
8	$\bar{1},76925$	19	$\bar{1},61459$	30	$\bar{1},22766$	41	$\bar{2},32144$	52	$\bar{3},06793$
9	$\bar{1},76147$	20	$\bar{1},59577$	31	$\bar{1},17570$	42	$\bar{2},18133$	53	$\bar{4},85274$
10	$\bar{1},75123$	21	$\bar{1},57582$	32	$\bar{1},11777$	43	$\bar{2},04495$		

TABLE VII.—*continued.*

Logarithm of the Carlisle chance of living 55 years for every age *a*.

a	λ chance.	*a*	λ chance.	*a*	λ chance.	*a*	λ chance.	*a*	λ chance.
0	$\bar{1}$,60991	10	$\bar{1}$,66949	20	$\bar{1}$,43940	30	$\bar{2}$,89693	40	$\bar{3}$,77168
1	$\bar{1}$,67464	11	$\bar{1}$,65322	21	$\bar{1}$,39887	31	$\bar{2}$,81764	41	$\bar{3}$,66198
2	$\bar{1}$,70281	12	$\bar{1}$,63646	22	$\bar{1}$,35471	32	$\bar{2}$,72872	42	$\bar{3}$,56155
3	$\bar{1}$,72278	13	$\bar{1}$,61892	23	$\bar{1}$,30840	33	$\bar{2}$,62734	43	$\bar{3}$,45869
4	$\bar{1}$,72894	14	$\bar{1}$,60051	24	$\bar{1}$,26143	34	$\bar{2}$,52392	44	$\bar{3}$,36033
5	$\bar{1}$,72914	15	$\bar{1}$,58105	25	$\bar{1}$,20979	35	$\bar{2}$,42296	45	$\bar{3}$,27966
6	$\bar{1}$,72215	16	$\bar{1}$,56072	26	$\bar{1}$,15661	36	$\bar{2}$,29634	46	$\bar{3}$,17099
7	$\bar{1}$,71169	17	$\bar{1}$,53730	27	$\bar{1}$,09743	37	$\bar{2}$,15482	47	$\bar{3}$,03735
8	$\bar{1}$,69897	18	$\bar{1}$,50967	28	$\bar{1}$,03497	38	$\bar{2}$,01689	48	$\bar{4}$,82189
9	$\bar{1}$,68490	19	$\bar{1}$,47738	29	$\bar{2}$,96773	39	$\bar{3}$,89144		

Logarithm of the Carlisle chance of living 60 years for every age *a*.

0	$\bar{1}$,56146	9	$\bar{1}$,58982	18	$\bar{1}$,29315	27	$\bar{2}$,70839	36	$\bar{3}$,63689
1	$\bar{1}$,61925	10	$\bar{1}$,57016	19	$\bar{1}$,24615	28	$\bar{2}$,60597	37	$\bar{3}$,53593
2	$\bar{1}$,63992	11	$\bar{1}$,54908	20	$\bar{1}$,19448	29	$\bar{2}$,50196	38	$\bar{3}$,43063
3	$\bar{1}$,65251	12	$\bar{1}$,52484	21	$\bar{1}$,14119	39	$\bar{2}$,40086	39	$\bar{3}$,33077
4	$\bar{1}$,65237	13	$\bar{1}$,49638	22	$\bar{1}$,08183	31	$\bar{2}$,27417	40	$\bar{3}$,24881
5	$\bar{1}$,64740	14	$\bar{1}$,46331	23	$\bar{1}$,01902	32	$\bar{2}$,13249	41	$\bar{3}$,14535
6	$\bar{1}$,63698	15	$\bar{1}$,42467	24	$\bar{2}$,95106	33	$\bar{3}$,99425	42	$\bar{3}$,00524
7	$\bar{1}$,62349	16	$\bar{1}$,38377	25	$\bar{2}$,87906	34	$\bar{3}$,86830	43	$\bar{4}$,78968
8	$\bar{1}$,60761	17	$\bar{1}$,33950	26	$\bar{2}$,79855	35	$\bar{3}$,74779		

Logarithm of the Carlisle chance of living 65 years for every age *a*.

0	$\bar{1}$,47972	8	$\bar{1}$,48507	16	$\bar{1}$,12608	24	$\bar{2}$,48528	32	$\bar{3}$,51270
1	$\bar{1}$,53408	9	$\bar{1}$,45261	17	$\bar{1}$,06562	25	$\bar{2}$,38298	33	$\bar{3}$,40798
2	$\bar{1}$,55171	10	$\bar{1}$,41378	18	$\bar{1}$,00378	26	$\bar{2}$,25507	34	$\bar{3}$,30763
3	$\bar{1}$,56115	11	$\bar{1}$,37213	19	$\bar{2}$,93578	27	$\bar{2}$,11216	35	$\bar{3}$,22492
4	$\bar{1}$,55729	12	$\bar{1}$,32704	20	$\bar{2}$,86374	28	$\bar{3}$,97288	36	$\bar{3}$,12025
5	$\bar{1}$,54807	13	$\bar{1}$,27986	21	$\bar{2}$,78313	29	$\bar{3}$,84634	37	$\bar{4}$,97873
6	$\bar{1}$,53285	14	$\bar{1}$,23208	22	$\bar{2}$,69278	30	$\bar{3}$,72569	38	$\bar{4}$,76162
7	$\bar{1}$,51187	15	$\bar{1}$,17975	23	$\bar{2}$,59002	31	$\bar{3}$,61471		

Logarithm of the Carlisle chance of living 70 years for every age *a*.

0	$\bar{1}$,38039	7	$\bar{1}$,31407	14	$\bar{2}$,92171	21	$\bar{2}$,23965	28	$\bar{3}$,38661
1	$\bar{1}$,42994	8	$\bar{1}$,26855	15	$\bar{2}$,84302	22	$\bar{2}$,09655	29	$\bar{3}$,28567
2	$\bar{1}$,44010	9	$\bar{1}$,22138	16	$\bar{2}$,76802	23	$\bar{3}$,95693	30	$\bar{3}$,20281
3	$\bar{1}$,43861	10	$\bar{1}$,16886	17	$\bar{2}$,67757	24	$\bar{3}$,82967	31	$\bar{3}$,09808
4	$\bar{1}$,42008	11	$\bar{1}$,11445	18	$\bar{2}$,57478	25	$\bar{3}$,70782	32	$\bar{4}$,95640
5	$\bar{1}$,39170	12	$\bar{1}$,05416	19	$\bar{2}$,47001	26	$\bar{3}$,59561	33	$\bar{4}$,73897
6	$\bar{1}$,35590	13	$\bar{2}$,99049	20	$\bar{2}$,36767	27	$\bar{3}$,49237		

TABLE VII.—*continued.*

Logarithm of the Carlisle chance of living 75 years for every age *a*.

a	λ chance.	*a*	λ chance.	*a*	λ chance.	*a*	λ chance.	*a*	λ chance.
0	$\bar{1}$,22402	6	$\bar{1}$,09821	12	$\bar{2}$,66511	18	$\bar{3}$,94169	24	$\bar{3}$,26900
1	$\bar{1}$,25299	7	$\bar{1}$,04119	13	$\bar{2}$,56149	19	$\bar{3}$,81439	25	$\bar{3}$,18494
2	$\bar{1}$,24130	8	$\bar{2}$,97918	14	$\bar{2}$,45593	20	$\bar{3}$,69250	26	$\bar{3}$,07898
3	$\bar{1}$,22209	9	$\bar{2}$,91101	15	$\bar{2}$,35295	21	$\bar{3}$,58019	27	$\bar{4}$,93607
4	$\bar{1}$,18885	10	$\bar{2}$,83813	16	$\bar{2}$,22455	22	$\bar{3}$,47676	28	$\bar{4}$,71760
5	$\bar{1}$,14678	11	$\bar{2}$,75639	17	$\bar{2}$,08134	23	$\bar{3}$,37066		

Logarithm of the Carlisle chance of living 80 years for every age *a*.

0	$\bar{2}$,97909	5	$\bar{2}$,81604	10	$\bar{2}$,34206	15	$\bar{3}$,67778	20	$\bar{3}$,16963
1	$\bar{2}$,99530	6	$\bar{2}$,74015	11	$\bar{2}$,21291	16	$\bar{3}$,56508	21	$\bar{3}$,06356
2	$\bar{2}$,96941	7	$\bar{2}$,65214	12	$\bar{2}$,06888	17	$\bar{3}$,46155	22	$\bar{4}$,92046
3	$\bar{2}$,93272	8	$\bar{2}$,55018	13	$\bar{3}$,92839	18	$\bar{3}$,35542	23	$\bar{4}$,70166
4	$\bar{2}$,87848	9	$\bar{2}$,44523	14	$\bar{3}$,80031	19	$\bar{3}$,25372		

Logarithm of the Carlisle chance of living 85 years for every age *a*.

0	$\bar{2}$,64836	4	$\bar{2}$,41271	8	$\bar{3}$,91708	12	$\bar{3}$,44909	16	$\bar{3}$,04845
1	$\bar{2}$,63724	5	$\bar{2}$,31997	9	$\bar{3}$,78962	13	$\bar{3}$,34213	17	$\bar{4}$,90525
2	$\bar{2}$,58037	6	$\bar{2}$,19667	10	$\bar{3}$,66689	14	$\bar{3}$,23965	18	$\bar{4}$,68641
3	$\bar{2}$,50372	7	$\bar{2}$,05591	11	$\bar{3}$,55345	15	$\bar{3}$,15490		

Logarithm of the Carlisle chance of living 90 years for every age *a*.

0	$\bar{2}$,15229	3	$\bar{3}$,87062	6	$\bar{3}$,53721	9	$\bar{3}$,22895	12	$\bar{4}$,89279
1	$\bar{2}$,09377	4	$\bar{3}$,75709	7	$\bar{3}$,43612	10	$\bar{3}$,14401	13	$\bar{4}$,67312
2	$\bar{3}$,98414	5	$\bar{3}$,64480	8	$\bar{3}$,33082	11	$\bar{3}$,03682		

Logarithm of the Carlisle chance of living 95 years for every age *a*.

0	$\bar{3}$,47712	2	$\bar{3}$,36435	4	$\bar{3}$,19642	6	$\bar{3}$,02058	8	$\bar{4}$,66181
1	$\bar{3}$,43431	3	$\bar{3}$,28436	5	$\bar{3}$,12193	7	$\bar{4}$,87982		

Logarithm of the Carlisle chance of living 100 years for every age *a*.

0	$\bar{4}$,95424	1	$\bar{4}$,91768	2	$\bar{4}$,80805	3	$\bar{4}$,61535

TABLE VIII.—Logarithm of Deparcieux chance of living for every age *a*.

a	10 years.	20 years.	30 years.	40 years.	50 years.	60 years.	70 years.	80 years.	90 years.
0									
1									
2									
3	$\bar{1}$,93450	$\bar{1}$,89763	$\bar{1}$,85126	$\bar{1}$,80346	$\bar{1}$,73957	$\bar{1}$,62634	$\bar{1}$,39967	$\bar{2}$,85126	$\bar{3}$,90107
4	$\bar{1}$,94469	$\bar{1}$,90644	$\bar{1}$,85957	$\bar{1}$,81188	$\bar{1}$,74401	$\bar{1}$,62495	$\bar{1}$,37684	$\bar{2}$,78408	$\bar{3}$,01323
5	$\bar{1}$,95159	$\bar{1}$,91193	$\bar{1}$,86455	$\bar{1}$,81698	$\bar{1}$,74418	$\bar{1}$,61979	$\bar{1}$,34747	$\bar{2}$,70443	
6	$\bar{1}$,95683	$\bar{1}$,91575	$\bar{1}$,86784	$\bar{1}$,82040	$\bar{1}$,74248	$\bar{1}$,61130	$\bar{1}$,31482	$\bar{2}$,61130	
7	$\bar{1}$,96027	$\bar{1}$,91825	$\bar{1}$,86981	$\bar{1}$,82177	$\bar{1}$,73928	$\bar{1}$,59968	$\bar{1}$,27663	$\bar{2}$,50098	
8	$\bar{1}$,96282	$\bar{1}$,91985	$\bar{1}$,87151	$\bar{1}$,82222	$\bar{1}$,73410	$\bar{1}$,58512	$\bar{1}$,23231	$\bar{2}$,38721	
9	$\bar{1}$,96495	$\bar{1}$,92101	$\bar{1}$,87278	$\bar{1}$,82146	$\bar{1}$,72821	$\bar{1}$,56781	$\bar{1}$,18415	$\bar{2}$,25473	
10	$\bar{1}$,96614	$\bar{1}$,92122	$\bar{1}$,87309	$\bar{1}$,81970	$\bar{1}$,72110	$\bar{1}$,54688	$\bar{1}$,12740	$\bar{2}$,09691	
11	$\bar{1}$,96582	$\bar{1}$,92042	$\bar{1}$,87239	$\bar{1}$,81612	$\bar{1}$,71269	$\bar{1}$,52337	$\bar{1}$,06380	$\bar{3}$,90458	
12	$\bar{1}$,96448	$\bar{1}$,91860	$\bar{1}$,87069	$\bar{1}$,81067	$\bar{1}$,70296	$\bar{1}$,49545	$\bar{2}$,99190	$\bar{3}$,66454	
13	$\bar{1}$,96313	$\bar{1}$,91676	$\bar{1}$,86896	$\bar{1}$,80507	$\bar{1}$,69184	$\bar{1}$,46517	$\bar{2}$,91676	$\bar{3}$,36653	
14	$\bar{1}$,96175	$\bar{1}$,91488	$\bar{1}$,86719	$\bar{1}$,79932	$\bar{1}$,68026	$\bar{1}$,43215	$\bar{2}$,83939	$\bar{3}$,06854	
15	$\bar{1}$,96034	$\bar{1}$,91296	$\bar{1}$,86539	$\bar{1}$,79259	$\bar{1}$,66820	$\bar{1}$,39588	$\bar{2}$,75284		
16	$\bar{1}$,95892	$\bar{1}$,91101	$\bar{1}$,86357	$\bar{1}$,78565	$\bar{1}$,65447	$\bar{1}$,35799	$\bar{2}$,65447		
17	$\bar{1}$,95798	$\bar{1}$,90954	$\bar{1}$,86150	$\bar{1}$,77901	$\bar{1}$,63941	$\bar{1}$,31636	$\bar{2}$,54071		
18	$\bar{1}$,95703	$\bar{1}$,90869	$\bar{1}$,85940	$\bar{1}$,77128	$\bar{1}$,62230	$\bar{1}$,26949	$\bar{2}$,42439		
19	$\bar{1}$,95606	$\bar{1}$,90783	$\bar{1}$,85651	$\bar{1}$,76326	$\bar{1}$,60286	$\bar{1}$,21920	$\bar{2}$,28978		
20	$\bar{1}$,95508	$\bar{1}$,90695	$\bar{1}$,85356	$\bar{1}$,75496	$\bar{1}$,58074	$\bar{1}$,16126	$\bar{2}$,13077		
21	$\bar{1}$,95460	$\bar{1}$,90657	$\bar{1}$,85030	$\bar{1}$,74687	$\bar{1}$,55755	$\bar{1}$,09798	$\bar{3}$,93876		
22	$\bar{1}$,95412	$\bar{1}$,90621	$\bar{1}$,84619	$\bar{1}$,73848	$\bar{1}$,53097	$\bar{1}$,02742	$\bar{3}$,70006		
23	$\bar{1}$,95363	$\bar{1}$,90583	$\bar{1}$,84194	$\bar{1}$,72871	$\bar{1}$,50204	$\bar{2}$,95363	$\bar{3}$,40340		
24	$\bar{1}$,95313	$\bar{1}$,90544	$\bar{1}$,83757	$\bar{1}$,71851	$\bar{1}$,47040	$\bar{2}$,87764	$\bar{3}$,10679		
25	$\bar{1}$,95262	$\bar{1}$,90505	$\bar{1}$,83225	$\bar{1}$,70786	$\bar{1}$,43554	$\bar{2}$,79250			
26	$\bar{1}$,95209	$\bar{1}$,90465	$\bar{1}$,82673	$\bar{1}$,69555	$\bar{1}$,39907	$\bar{2}$,69555			
27	$\bar{1}$,95156	$\bar{1}$,90352	$\bar{1}$,82103	$\bar{1}$,68143	$\bar{1}$,35838	$\bar{2}$,58273			
28	$\bar{1}$,95166	$\bar{1}$,90237	$\bar{1}$,81425	$\bar{1}$,66527	$\bar{1}$,31246	$\bar{2}$,46736			
29	$\bar{1}$,95177	$\bar{1}$,90045	$\bar{1}$,80720	$\bar{1}$,64680	$\bar{1}$,26314	$\bar{2}$,33372			
30	$\bar{1}$,95187	$\bar{1}$,89848	$\bar{1}$,79988	$\bar{1}$,62566	$\bar{1}$,20618	$\bar{2}$,17569			
31	$\bar{1}$,95197	$\bar{1}$,89570	$\bar{1}$,79227	$\bar{1}$,60295	$\bar{1}$,14338	$\bar{3}$,98416			
32	$\bar{1}$,95209	$\bar{1}$,89207	$\bar{1}$,78436	$\bar{1}$,57685	$\bar{1}$,07330	$\bar{3}$,74594			
33	$\bar{1}$,95220	$\bar{1}$,88831	$\bar{1}$,77508	$\bar{1}$,54841	$\bar{1}$,00000	$\bar{3}$,44977			
34	$\bar{1}$,95231	$\bar{1}$,88444	$\bar{1}$,76538	$\bar{1}$,51727	$\bar{2}$,92451	$\bar{3}$,15366			
35	$\bar{1}$,95243	$\bar{1}$,87963	$\bar{1}$,75524	$\bar{1}$,48292	$\bar{2}$,83988				
36	$\bar{1}$,95256	$\bar{1}$,87464	$\bar{1}$,74346	$\bar{1}$,44698	$\bar{2}$,74346				
37	$\bar{1}$,95196	$\bar{1}$,86947	$\bar{1}$,72987	$\bar{1}$,40682	$\bar{2}$,63117				
38	$\bar{1}$,95071	$\bar{1}$,86259	$\bar{1}$,71361	$\bar{1}$,36080	$\bar{2}$,51570				
39	$\bar{1}$,94868	$\bar{1}$,85543	$\bar{1}$,69503	$\bar{1}$,31137	$\bar{2}$,38195				
40	$\bar{1}$,94661	$\bar{1}$,84801	$\bar{1}$,67379	$\bar{1}$,25431	$\bar{2}$,22382				
41	$\bar{1}$,94373	$\bar{1}$,84030	$\bar{1}$,65098	$\bar{1}$,19141	$\bar{2}$,03219				
42	$\bar{1}$,93998	$\bar{1}$,83227	$\bar{1}$,62476	$\bar{1}$,12121	$\bar{3}$,79385				
43	$\bar{1}$,93611	$\bar{1}$,82288	$\bar{1}$,59621	$\bar{1}$,04780	$\bar{3}$,49757				
44	$\bar{1}$,93213	$\bar{1}$,81307	$\bar{1}$,56496	$\bar{2}$,97220	$\bar{3}$,20135				
45	$\bar{1}$,92720	$\bar{1}$,80281	$\bar{1}$,53049	$\bar{2}$,88745					
46	$\bar{1}$,92208	$\bar{1}$,79090	$\bar{1}$,49442	$\bar{2}$,79090					
47	$\bar{1}$,91751	$\bar{1}$,77791	$\bar{1}$,45486	$\bar{2}$,67921					
48	$\bar{1}$,91188	$\bar{1}$,76290	$\bar{1}$,41009	$\bar{2}$,56499					
49	$\bar{1}$,90675	$\bar{1}$,74635	$\bar{1}$,36269	$\bar{2}$,43327					
50	$\bar{1}$,90140	$\bar{1}$,72718	$\bar{1}$,30770	$\bar{2}$,27721					
51	$\bar{1}$,89657	$\bar{1}$,70725	$\bar{1}$,24768	$\bar{2}$,08846					
52	$\bar{1}$,89229	$\bar{1}$,68478	$\bar{1}$,18123	$\bar{3}$,85387					

[*continued.*

TABLE VIII. *continued.*—Logarithm of Deparcieux chance of living for every age *a*.

a.	10 years.	20 years.	30 years.	40 years.	50 years.	60 years.	70 years.	80 years.	90 years
53	$\bar{1}$,88677	$\bar{1}$,66010	$\bar{1}$,11169	$\bar{3}$,56146					
54	$\bar{1}$,88094	$\bar{1}$,63283	$\bar{1}$,04007	$\bar{3}$,26922					
55	$\bar{1}$,87561	$\bar{1}$,60329	$\bar{2}$,96025						
56	$\bar{1}$,86882	$\bar{1}$,57234	$\bar{2}$,86882						
57	$\bar{1}$,86040	$\bar{1}$,53735	$\bar{2}$,76170						
58	$\bar{1}$,85102	$\bar{1}$,49821	$\bar{2}$,65311						
59	$\bar{1}$,83960	$\bar{1}$,45594	$\bar{2}$,52652						
60	$\bar{1}$,82578	$\bar{1}$,40630	$\bar{2}$,37581						
61	$\bar{1}$,81068	$\bar{1}$,35111	$\bar{2}$,19189						
62	$\bar{1}$,79249	$\bar{1}$,28894	$\bar{3}$,96158						
63	$\bar{1}$,77333	$\bar{1}$,22492	$\bar{3}$,67469						
64	$\bar{1}$,75189	$\bar{1}$,15913	$\bar{3}$,38828						
65	$\bar{1}$,72768	$\bar{1}$,08464							
66	$\bar{1}$,70352	$\bar{1}$,00000							
67	$\bar{1}$,67695	$\bar{2}$,90130							
68	$\bar{1}$,64719	$\bar{2}$,80209							
69	$\bar{1}$,61634	$\bar{2}$,68692							
70	$\bar{1}$,58052	$\bar{2}$,55003							
71	$\bar{1}$,54043	$\bar{2}$,38121							
72	$\bar{1}$,49645	$\bar{2}$,16909							
73	$\bar{1}$,45159	$\bar{3}$,90136							
74	$\bar{1}$,40724	$\bar{3}$,63639							
75	$\bar{1}$,35696								
76	$\bar{1}$,29648								
77	$\bar{1}$,22435								
78	$\bar{1}$,15490								
79	$\bar{1}$,07058								
80	$\bar{2}$,96951								
81	$\bar{2}$,84078								
82	$\bar{2}$,67264								
83	$\bar{2}$,44977								
84	$\bar{2}$,22915								

TABLE IX.—Logarithm of the Northampton chance for living at every age *a*

a	10 years.	20 years.	30 years.	40 years.	50 years.	60 years.	70 years.	80 years.	90 years.
0	$\bar{1},68764$	$\bar{1},64396$	$\bar{1},57564$	$\bar{1},49417$	$\bar{1},38958$	$\bar{1},24287$	$\bar{1},02428$	$\bar{2},60484$	$\bar{3},59643$
1	$\bar{1},81295$	$\bar{1},76713$	$\bar{1},69740$	$\bar{1},61431$	$\bar{1},50640$	$\bar{1},35435$	$\bar{1},12443$	$\bar{2},69151$	$\bar{3},59446$
2	$\bar{1},88378$	$\bar{1},83536$	$\bar{1},76454$	$\bar{1},67952$	$\bar{1},56809$	$\bar{1},41646$	$\bar{1},16788$	$\bar{2},69677$	$\bar{3},51790$
3	$\bar{1},91089$	$\bar{1},85979$	$\bar{1},78780$	$\bar{1},70070$	$\bar{1},58568$	$\bar{1},42229$	$\bar{1},16522$	$\bar{2},62961$	$\bar{3},37283$
4	$\bar{1},92894$	$\bar{1},87511$	$\bar{1},80190$	$\bar{1},71263$	$\bar{1},59383$	$\bar{1},42421$	$\bar{1},15070$	$\bar{2},55993$	$\bar{3},14495$
5	$\bar{1},93843$	$\bar{1},88180$	$\bar{1},80733$	$\bar{1},71581$	$\bar{1},59300$	$\bar{1},41691$	$\bar{1},12431$	$\bar{2},47370$	$\bar{4},80625$
6	$\bar{1},94739$	$\bar{1},88788$	$\bar{1},81211$	$\bar{1},71823$	$\bar{1},59118$	$\bar{1},40806$	$\bar{1},09539$	$\bar{2},37854$	
7	$\bar{1},95322$	$\bar{1},89101$	$\bar{1},81390$	$\bar{1},71755$	$\bar{1},58601$	$\bar{1},39522$	$\bar{1},05661$	$\bar{2},27263$	
8	$\bar{1},95660$	$\bar{1},89203$	$\bar{1},81352$	$\bar{1},71459$	$\bar{1},57827$	$\bar{1},37909$	$\bar{1},01505$	$\bar{2},15453$	
9	$\bar{1},95759$	$\bar{1},89080$	$\bar{1},81084$	$\bar{1},70923$	$\bar{1},56781$	$\bar{1},35940$	$\bar{2},96901$	$\bar{2},03386$	
10	$\bar{1},95632$	$\bar{1},88800$	$\bar{1},80653$	$\bar{1},70194$	$\bar{1},55523$	$\bar{1},33664$	$\bar{2},91720$	$\bar{3},90879$	
11	$\bar{1},95418$	$\bar{1},88451$	$\bar{1},80136$	$\bar{1},69345$	$\bar{1},54140$	$\bar{1},31148$	$\bar{2},85856$	$\bar{3},78151$	
12	$\bar{1},95158$	$\bar{1},88076$	$\bar{1},79574$	$\bar{1},68431$	$\bar{1},52668$	$\bar{1},28410$	$\bar{2},79299$	$\bar{3},63412$	
13	$\bar{1},94890$	$\bar{1},87691$	$\bar{1},78982$	$\bar{1},67479$	$\bar{1},51140$	$\bar{1},25433$	$\bar{2},71872$	$\bar{3},46194$	
14	$\bar{1},94617$	$\bar{1},87296$	$\bar{1},78369$	$\bar{1},66489$	$\bar{1},49527$	$\bar{1},22176$	$\bar{2},63099$	$\bar{3},21602$	
15	$\bar{1},94337$	$\bar{1},86890$	$\bar{1},77738$	$\bar{1},65457$	$\bar{1},47848$	$\bar{1},18588$	$\bar{2},53527$	$\bar{4},86782$	
16	$\bar{1},94049$	$\bar{1},86472$	$\bar{1},77084$	$\bar{1},64379$	$\bar{1},46067$	$\bar{1},14600$	$\bar{2},43115$		
17	$\bar{1},93779$	$\bar{1},86068$	$\bar{1},76433$	$\bar{1},63279$	$\bar{1},44200$	$\bar{1},10339$	$\bar{2},31948$		
18	$\bar{1},93543$	$\bar{1},85692$	$\bar{1},75799$	$\bar{1},62167$	$\bar{1},42249$	$\bar{1},05845$	$\bar{2},19793$		
19	$\bar{1},93341$	$\bar{1},85345$	$\bar{1},75184$	$\bar{1},61042$	$\bar{1},40201$	$\bar{1},01162$	$\bar{2},07647$		
20	$\bar{1},93168$	$\bar{1},85021$	$\bar{1},74562$	$\bar{1},59891$	$\bar{1},38032$	$\bar{2},96088$	$\bar{3},95247$		
21	$\bar{1},93033$	$\bar{1},84718$	$\bar{1},73927$	$\bar{1},58722$	$\bar{1},35730$	$\bar{2},90438$	$\bar{3},82733$		
22	$\bar{1},92918$	$\bar{1},84417$	$\bar{1},73274$	$\bar{1},57511$	$\bar{1},33253$	$\bar{2},84141$	$\bar{3},68255$		
23	$\bar{1},92801$	$\bar{1},84091$	$\bar{1},72589$	$\bar{1},56250$	$\bar{1},30543$	$\bar{2},76982$	$\bar{3},51304$		
24	$\bar{1},92679$	$\bar{1},83752$	$\bar{1},71872$	$\bar{1},54910$	$\bar{1},27559$	$\bar{2},68482$	$\bar{3},26984$		
25	$\bar{1},92553$	$\bar{1},83401$	$\bar{1},71120$	$\bar{1},53511$	$\bar{1},24251$	$\bar{2},59191$	$\bar{4},92445$		
26	$\bar{1},92423$	$\bar{1},83035$	$\bar{1},70330$	$\bar{1},52018$	$\bar{1},20551$	$\bar{2},49066$			
27	$\bar{1},92289$	$\bar{1},82654$	$\bar{1},69500$	$\bar{1},50421$	$\bar{1},16560$	$\bar{2},38162$			
28	$\bar{1},92149$	$\bar{1},82256$	$\bar{1},68624$	$\bar{1},48706$	$\bar{1},12302$	$\bar{2},26250$			
29	$\bar{1},92004$	$\bar{1},81843$	$\bar{1},67701$	$\bar{1},46860$	$\bar{1},07821$	$\bar{2},14306$			
30	$\bar{1},91853$	$\bar{1},81394$	$\bar{1},66723$	$\bar{1},44864$	$\bar{1},02920$	$\bar{2},02079$			
31	$\bar{1},91685$	$\bar{1},80894$	$\bar{1},65689$	$\bar{1},42697$	$\bar{2},97405$	$\bar{3},89700$			
32	$\bar{1},91498$	$\bar{1},80355$	$\bar{1},64592$	$\bar{1},40334$	$\bar{2},91223$	$\bar{3},75336$			
33	$\bar{1},91290$	$\bar{1},79788$	$\bar{1},63449$	$\bar{1},37742$	$\bar{2},84181$	$\bar{3},58503$			
34	$\bar{1},91073$	$\bar{1},79193$	$\bar{1},62231$	$\bar{1},34880$	$\bar{2},75802$	$\bar{3},34305$			
35	$\bar{1},90848$	$\bar{1},78567$	$\bar{1},60958$	$\bar{1},31698$	$\bar{2},66637$	$\bar{4},99892$			
36	$\bar{1},90612$	$\bar{1},77907$	$\bar{1},59595$	$\bar{1},28128$	$\bar{2},56643$				
37	$\bar{1},90365$	$\bar{1},77211$	$\bar{1},58132$	$\bar{1},24271$	$\bar{2},45873$				
38	$\bar{1},90107$	$\bar{1},76475$	$\bar{1},56557$	$\bar{1},20153$	$\bar{2},34101$				
39	$\bar{1},89839$	$\bar{1},75697$	$\bar{1},54856$	$\bar{1},15817$	$\bar{2},22302$				
40	$\bar{1},89541$	$\bar{1},74870$	$\bar{1},53011$	$\bar{1},11067$	$\bar{2},10226$				
41	$\bar{1},89209$	$\bar{1},74004$	$\bar{1},51012$	$\bar{1},05720$	$\bar{3},98015$				
42	$\bar{1},88857$	$\bar{1},73094$	$\bar{1},48836$	$\bar{2},99725$	$\bar{3},83838$				
43	$\bar{1},88498$	$\bar{1},72159$	$\bar{1},46452$	$\bar{2},92891$	$\bar{3},67213$				
44	$\bar{1},88120$	$\bar{1},71158$	$\bar{1},43807$	$\bar{2},84730$	$\bar{3},43232$				
45	$\bar{1},87719$	$\bar{1},70110$	$\bar{1},40850$	$\bar{2},75789$	$\bar{3},09044$				
46	$\bar{1},87295$	$\bar{1},68983$	$\bar{1},37516$	$\bar{2},66031$					
47	$\bar{1},86846$	$\bar{1},67767$	$\bar{1},33906$	$\bar{2},55508$					
48	$\bar{1},86368$	$\bar{1},66450$	$\bar{1},30046$	$\bar{2},43994$					
49	$\bar{1},85858$	$\bar{1},65017$	$\bar{1},25978$	$\bar{2},32463$					
50	$\bar{1},85329$	$\bar{1},63470$	$\bar{1},21526$	$\bar{2},20685$					
51	$\bar{1},84795$	$\bar{1},61803$	$\bar{1},16511$	$\bar{2},08806$					
52	$\bar{1},84237$	$\bar{1},59979$	$\bar{1},10868$	$\bar{3},94981$					

TABLE IX. continued.—Logarithm of the Northampton chance for living at every age *a*.

a.	10 years.	20 years.	30 years.	40 years.	50 years.	60 years.	70 years.	80 years.	90 years.
53	$\bar{1}$,83661	$\bar{1}$,57954	$\bar{1}$,04393	$\bar{3}$,78715					
54	$\bar{1}$,83038	$\bar{1}$,55687	$\bar{2}$,96610	$\bar{3}$,55112					
55	$\bar{1}$,82391	$\bar{1}$,53131	$\bar{2}$,88070	$\bar{3}$,21325					
56	$\bar{1}$,81688	$\bar{1}$,50221	$\bar{2}$,78736						
57	$\bar{1}$,80921	$\bar{1}$,47060	$\bar{2}$,68662						
58	$\bar{1}$,80082	$\bar{1}$,43678	$\bar{2}$,57626						
59	$\bar{1}$,79159	$\bar{1}$,40120	$\bar{2}$,46605						
60	$\bar{1}$,78141	$\bar{1}$,36197	$\bar{2}$,35356						
61	$\bar{1}$,77008	$\bar{1}$,31716	$\bar{2}$,24011						
62	$\bar{1}$,75742	$\bar{1}$,26631	$\bar{2}$,10744						
63	$\bar{1}$,74293	$\bar{1}$,20732	$\bar{3}$,95054						
64	$\bar{1}$,72649	$\bar{1}$,13572	$\bar{3}$,72074						
65	$\bar{1}$,70740	$\bar{1}$,05979	$\bar{3}$,38934						
66	$\bar{1}$,68553	$\bar{2}$,97048							
67	$\bar{1}$,66139	$\bar{2}$,87741							
68	$\bar{1}$,63596	$\bar{2}$,77544							
69	$\bar{1}$,60961	$\bar{2}$,67446							
70	$\bar{1}$,58056	$\bar{2}$,57215							
71	$\bar{1}$,54708	$\bar{2}$,47003							
72	$\bar{1}$,50889	$\bar{2}$,35002							
73	$\bar{1}$,46439	$\bar{2}$,20761							
74	$\bar{1}$,40923	$\bar{3}$,99425							
75	$\bar{1}$,34939	$\bar{3}$,68194							
76	$\bar{1}$,28515								
77	$\bar{1}$,21602								
78	$\bar{1}$,13948								
79	$\bar{1}$,06485								
80	$\bar{2}$,99159								
81	$\bar{2}$,92295								
82	$\bar{2}$,84113								
83	$\bar{2}$,74322								
84	$\bar{2}$,58502								
85	$\bar{2}$,33255								

How the value of particular assurances may be determined from the value of annuities, is shown in my Paper in the Philosophical Transactions for the year 1820, many of the cases of which are solved by methods essentially the same as those which have been long adopted; but when such assurances are but for terms, which are not of great extension, very near approximations may be had by using a geometrical progression, without confining the arithmetical operations to the same route, since the chance of extinction of the joint lives of the present age a, b, c, &c. taking place between the period commencing with the time $n+t-1$, and finishing with the time $n+t$, from the present, is $=(L_{n+t-1:a,b,c,\&c.} - L_{n+t:a,b,c,\&c.}) \div L_{a,b,c,\&c.}$; it follows that if r be the present value of unity, to be received certain in the time 1, and $L_{n+t-1:a,b,c,\&c.} = L_{n-1:a,b,c,} \times \pi^{t}$, whatever t may be, that $\substack{r\\1\\n\\m}\underline{\big| a,b,c,\&c.}$ or the assurance of unity to be received at the first of the equal periods 1, from the commencement of the time $n-1$ to the expiration of the time m, which shall happen after the extinction of the joint lives, is equal to $\frac{L_{n-1:a,b,c,\&c.}}{L_{a,b,c,\&c.}} \times \Big\{r^{n} \times (1-\pi) + r^{n+1} \times (\pi-\pi^{2}) + r^{n+2}(\pi^{2}-\pi^{3}) \ldots\ldots r^{m} \times (\pi^{m-n-1} - \pi^{m-n})\Big\} = (1-\pi) \times \frac{L_{n-1:a,b,c,\&c.}}{L_{a,b,c,\&c.}} \times \Big\{r^{n} + \pi r^{n+1} + \pi^{2} r^{n+2} + \pi^{3} r^{n+3} \ldots . r^{m} \pi^{m-n-1}\Big\} = \frac{(1-\pi)\,r}{L_{a,b,c,\&c.}} \times \Big\{r^{n-1} L_{n-1:a,b,c,\&c.} + r^{n} . L_{n:a,b,c,\&c.} + \ldots . r^{m-1} . L_{m-1:a,b,c,\&c.}\Big\} = (1-\pi) . r \times \substack{r\\1\\n-1\\m-1}\underline{\big| a,b,c,\&c.}$

If the assurance be not deferred, n will be equal to 1, and we shall have, according to the hypothesis, $\substack{r\\1\\1\\m}\underline{\big| a,b,c,\&c.} = (1-\pi) . r \times \substack{r\\1\\0\\m}\underline{\big| a,b,c,\&c.}$; and also $= \frac{1-\pi}{\pi} . \substack{r\\1\\1\\m}\underline{\big| a,b,c,\&c.}$. If t be

taken equal to 1, we shall have from the equation $L_{n+t-1:a,b,c,\&c.} = L_{n-1:a,b,c,\&c.} \times \pi^{t}$, $\pi = \frac{L_{n:a,b,c,\&c.}}{L_{n-1:a,b,c,\&c.}}$, and this would be the real value which should be taken for π, if the geometrical progression coincided perfectly with the fact; and it would be indifferent whether we made it equal to $\frac{L_{n+t:a,b,c,\&c}}{L_{n-1+t:a,b,c,\&c.}}$, or $\frac{L_{n:a,b,c,\&c.}}{L_{n-1:a,b,c,\&c.}}$, as the two would be the same; but this not being the case, there will be a preference; and generally, if not always, π should be taken an intermediate value between the two; and when the term is not very long, it will answer a good purpose to take it about the middle between them, inclining generally, though perhaps not always, rather nearer the last than the first, as the first terms are generally of more consequence than the last. If the said assurance be not deferred, and instead of being paid for immediately, be to be paid for by equal periodic payments, at an unite of time from each other, up to the time $m-1$ inclusive, and the first payment be to be made immediately, then will the present value of such periodic payment be $\overset{r}{\overbrace{\;}}\,{}^{1}_{0}{}_{m-1}\rfloor\underline{a, b, c, \&c.}$, and consequently each payment, from what is shown above, is equal to $\overset{r}{\overbrace{\;}}\,{}^{1}_{1}{}_{m}\rfloor\underline{a, b, c, \&c.} \div \overset{r}{\overbrace{\;}}\,{}^{1}_{0}{}_{m-1}\rfloor\underline{a, b, c, \&c.} = (1-\pi).\,r.$ From whence we may draw an inference worthy of remark, namely; when an assurance of joint lives is meant to commence immediately, and to continue for a term of t years, which is not large, and to be paid for by t annual payments, that those payments will not differ much with the increase of the time t, provided, as I have said, that t be not large, and the ages

be not at the extremes of life, a consequence which follows from the near agreement to a geometrical progression which takes place in the number of living at each small equal increment of time; that is to say, from the near coincidence of $\frac{L_{n:a,b,c,\&c.}}{L_{n-1:a,b,c,\&c.}}$ with $\frac{L_{n+t:a,b,c,\&c.}}{L_{n-1:a,b,c,\&c.}}$, or the small variation of π for the different values of t: and also, that when the number of years for which an assurance continues be not very long, and the ages be not at the extremes of life, the annual premiums will not differ widely from the premiums to be paid for an assurance of one year of a life older than the proposed life by about half the term: thus, according to the Northampton table, at three per cent. to assure 100 *l.* at the

Age	15	20	30	40	50	60	64
For 7 years, the annual premium by the common modes of calculation	£1..2..11	1.. 9.. 5	1..14..11	2.. 4.. 1	3.. 0.. 8	4.. 7.. 1	5.. 4..10
And the premium for one year assurance for an age 3 years older . . .	1..3.. 3	1.. 9.. 8	1..15.. 0	2. 4.. 6	3.. 1.. 0	4.. 7.. 8	5 5.. 6

the difference of which is very small.—As another example, let

Age	10	20	30	40	50	60
For 10 years, the annual premium will be, by common modes of calculation	£0..19.. 2	1..9.. 1	1..15.. 8	2.. 5.. 8	3.. 3.. 4	4..12.. 6
Premium for one year assurance, age 5 years older	0..17..11	1..10.. 7	1..16.. 4	2.. 6.. 8	3.. 5.. 1	4..15.. 2

Here, except at the age 10, the excess is rather more in the approximation than in the first set of examples; but it should be recollected, that we took the exact middle, instead of inclining to the early age.

According to the Carlisle table of mortality at 3 per cent. to assure 100*l.* at the

Age	10			20			30			40			50			60		
For 7 years, the annual premium, by common modes of calculation .	£0	10	5	0	13	10	0	19	10	1	7	8	1	11	0	3	13	8
For one year, the premium	0	10	5	0	13	9	0	19	2	1	8	6	1	12	1	3	15	9
For 10 years, the annual premium, by common modes of calculation .	0	11	3	0	14	7	1	0	4	1	7	7	1	14	11	3	17	8
For one year, at an age 5 years older	0	12	0	0	14	2	0	19	11	1	9	0	1	14	10	3	19	9

Moreover, because $\overset{r}{\underset{\substack{1\\1\\m}}{\urcorner}}\underline{a, b, c, \&c.}$, or the single premium for the assurance of unity, on the joint lives a, b, c, &c. for m years, is $= \overset{r}{\underset{\substack{1\\\bullet\\m-1}}{\urcorner}}\underline{a, b, c, \&c.}\,.\,r - \overset{r}{\underset{\substack{1\\1\\m}}{\urcorner}}\underline{a, b, c, \&c.} = \overset{r}{\underset{\substack{1\\\bullet\\m-1}}{\urcorner}}\underline{a, b, c, \&c.}\,.\,r + 1 -$ $\frac{L_{m:a, b, c, \&c.}}{L_{a, b, c, \&c.}}\,r^m - \overset{r}{\underset{\substack{1\\\bullet\\m-1}}{\urcorner}}\underline{a, b, c, \&c.} = 1 - \frac{L_{m:a, b, c, \&c.}}{L_{a, b, c, \&c.}}\cdot r^m - (1-r)\cdot\overset{r}{\underset{\substack{1\\\bullet\\m-1}}{\urcorner}}\underline{a, b, c, \&c.}$; if this be divided by $\overset{r}{\underset{\substack{1\\\bullet\\m-1}}{\urcorner}}\underline{a, b, c, \&c.}$, we shall have the annual premium for such assurance; that is, $\overset{r}{\underset{\substack{1\\1\\m}}{\urcorner}}\underline{a, b, c, \&c} \div \overset{r}{\underset{\substack{1\\0\\m-1}}{\urcorner}}\underline{a, b, c, \&c.} =$ $\dfrac{1-\frac{L_{m:a, b, c, \&c.}}{L_{a, b, c, \&c.}}.r^m}{\overset{r}{\underset{\substack{1\\\bullet\\m-1}}{\urcorner}}\underline{a, b, c, \&c.}} - 1 + r.$ The said annual premium may be expressed by

$$\left(1-\frac{L_{m, a, b, c, \&c.}}{L_{a, b, c, \&c.}}.r^m\right) \div \left(\overset{r}{\underset{\substack{1\\1\\m}}{\urcorner}}\underline{a-1, b-1, c-1, \&c.} \times \frac{L_{a-1, b-1, c-1, \&c.}}{r\,L_{a, b, c, \&c.}}\right) - 1 + r.$$

This last mode is well adapted to logarithms in the use of our general tables; and this method, supposing the annuities were accurately determinable by our general tables, would be accurate. The last formula is derived from that immediately before, in consequence of $\overset{r}{\underset{\substack{1\\\bullet\\m-1}}{\urcorner}}\underline{a, b, c, \&c.}$ being identical with $\overset{r}{\underset{\substack{1\\1\\m}}{\urcorner}}\underline{a-1, b-1, c-1, \&c.} \times \frac{L_{a-1, b-1, c-1, \&c.}}{r\,L_{a, b, c, \&c.}}$

Example. To find the annual premium to assure a life, at the age *a* years, for 10 years, according to the Carlisle mortality, and three per cent. interest.

$a =$	20	30	40	50	60	70
Log. of the accommoda. chance for living 10 yrs. at the age $a-1$, Tab. V.	$\bar{1}.9690$	$\bar{1}.9556$	$\bar{1}.9406$	$\bar{1}.9328$	$\bar{1}.8435$	$\bar{1}.7005$
$\lambda\, 1{,}03^{-1} =$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$
Sum . . . =	$\bar{1}.8406$	$\bar{1}.8272$	$\bar{1}.8122$	$\bar{1}.8044$	$\bar{1}.7151$	$\bar{1}.5721$
Corresponding . . .	.91443	.90407	.89892	.89379	.84846	.78092
To this we get from Ta. I.	31	362 10	103 10	205 21	248 5	94 5
$\lambda \overset{r}{\underset{10}{1}} \rfloor \overline{a-1}$ . . . . . =	.91474	.90779	.90005	.89605	.85099	.78191
Therefore, $\lambda \frac{L_{a+10}}{L_a}$ (T. VII.)	$\bar{1}.96682$	$\bar{1}.95400$	$\bar{1}.93772$	$\bar{1}.91830$	$\bar{1}.81893$	$\bar{1}.59873$
$\lambda\, 1{,}03^{-10} =$	$\bar{1}.87163$	$\bar{1}.87163$	$\bar{1}.87163$	$\bar{1}.87163$	$\bar{1}.87163$	$\bar{1}.87163$
Sum = the log. . . .	$\bar{1}.83845$	$\bar{1}.82563$	$\bar{1}.80935$	$\bar{1}.78993$	$\bar{1}.69056$	$\bar{1}.47036$
The N° corresponding =	.68937	.66932	.64469	.61650	.49041	.29536
Its complement to unity	.31063	.33068	.35531	.38350	.50959	.70464
The log. of the last . =	$\bar{1}.49224$	$\bar{1}.51941$	$\bar{1}.55061$	$\bar{1}.58377$	$\bar{1}.70722$	$\bar{1}.84797$
Complement of $\lambda \overset{r}{\underset{10}{1}} \rfloor \overline{a-1}$ =	$\bar{1}.08526$	$\bar{1}.09221$	$\bar{1}.09995$	$\bar{1}.10395$	$\bar{1}.14901$	$\bar{1}.21809$
$\lambda \frac{L_a}{L_{a-1}}$ . . . . =	$\bar{1}.99694$	$\bar{1}.99571$	$\bar{1}.99481$	$\bar{1}.99402$	$\bar{1}.98754$	$\bar{1}.97813$
$\lambda\, r^{-1}$ . . . . . . =	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$
Sum = logarithm . . .	$\bar{2}.56160$	$\bar{2}.59449$	$\bar{2}.63253$	$\bar{2}.66890$	$\bar{2}.83093$	$\bar{1}.03135$
Number corresponding =	.03644	.03931	.04291	.04666	.06775	.10749
$1{,}03^{-1} - 1$ . . . =	−.02913	−.02913	−.02913	−.02913	−.02913	−.02913
Ann. premium for an assurance of 1*l.* =	.00732	.01018	.01378	.01753	.03862	.07836
Ditto for 100*l.* . . .	£0..14..8	1..0..4	1..7..7	1..15..1	3..1..3	7..16..9

for annual premiums.

The reader has here an opportunity of comparing the results from my tables, with those above calculated by Mr. Milne's Carlisle tables.—I may probably be able at a future period to add examples, which I regret time will not at present permit.

Errata in Mr. Gompertz's *Paper in Part* II. *of Philosophical Transactions for* 1820.

Page Line

220 4 *for* '$a+y$' *put* '$a+x$'—lines 11 and 20, *dele* 'n' in the denominator.

221 21 the second symbol should be $\overbrace{\begin{smallmatrix}p\\n\\m\end{smallmatrix}}^{r}\Big|\underline{a,b,c}$.

223 4 *before* 'chance' *insert* 'value of the.'

224 2 and 5 in the symbol, before the second a, *insert* a comma—line 15, *insert* ':' before a''—five lines from the bottom, *for* 'c' *put* 'C'—line 2 from the bottom, *for* 'proved' *read* 'provided.'

226 13 in the 2d and 3d symbol, *put* 'n' *for* 'm'—and in the 4th, *put* 'p' *for* 'y'—in the first symbol in the bottom line, *put* 'n' *for* 'm'— and for the 3d symbol $\overbrace{\begin{smallmatrix}p\\n\\m\end{smallmatrix}}^{r}\Big|\overline{C}$

227 6 *for* 'will' *read* 'will be'—line 7, in symbol, *write* '$\frac{n}{m}$' *for* '$\frac{m}{n}$', and *for* '$\dot{r}$' *write* 'r'—line 14, *put* 's' in the lower angle in the right of the symbol, thus $\overbrace{\begin{smallmatrix}p\\n\\m\end{smallmatrix}}^{r}\Big|\frac{a,b,c}{\underline{s}}$

228 1, 3, 4, and 11, *for* $\overbrace{\;}\rceil$— where there is nothing in the lower angle, *write* $\overbrace{\;}\underline{\underline{\rfloor}}$

229 9, under the first Δ *put* 'b' *for* 'a'—line 2 from the bottom, between ': and $\frac{a}{s}$' *put* '+'.

231 2 from bottom, *put* a dash over the second T.

232 12 *for* 'n' *put* 'm'—line 13, *for* '$\acute{M}x$' *put* '$\acute{M}_x$'.

233 8 *for* '9457' *put* '4597'.

236 7 *for* 'F' *put* 'E'—line 11, *for* 'a & c' *put* 'b'.

240 8 in the second formula, *for* '$r+1$' *put* '$r+x$'.

243 19 *for* 'form' *put* 'from'.

244 11 *for* '$q-x$' *read* '$q+x$.'

246 12 *for* last 'p' *put* 'q'.

247 16 include the last 'L' with the expression of line 17 in '( )'.

251 11 *for* 'K' *write* 'k'.

255 14 and 15, *for* '+' *write* '-'.

256 2 *for* 'L_b' *write* 'L_a'.

258 4 from bottom, dash over the first 'K'.

260 7 from bottom, *insert* '—' before 'the.'

Page Line

262 1 *insert* ‘ × ’ before the last ‘ *n* ’—line 9, *for* ‘ of ’ *read* ‘ if ’.

265 1 *dele* ‘ *or last* ’—line 4, for the last ‘ L ’ *write* ‘ L̇.’

269 13 after ‘ there is ’ *insert* ‘ *only.*’

274 5 *for* ‘ A ’ *read* ‘ B ’—line 8, for the second ‘ *a* ’ at top and bottom *write* ‘ *b* ’ —line 10, *for* ‘ A ’ *write* ‘ B ’.

277 1 and 2, *dele* last ‘ *s* ’ in ‘ survives ’.

282 3 transpose the ‘ 3 ’ and ‘ 2 ’—line 7, in denominator *dele* ‘ *π*,’ in numerator *dele* ‘ 1 —.’

289 1 after ‘ $\mathrm{N}_x =$,’ *insert* ‘ 1 — ,’—line 3, after ‘ become ’ *insert* ‘ — ,’—line 5, at the commencement *insert* ‘ — ,’—line 8, after ‘ *b* ’ *insert* ‘ — , ’.

290 1 at the commencement *insert* ‘ — ,’—line 2, before ‘ *r* ’ *insert* ‘ — ,’—line 3, to $\frac{\mathrm{L}_c - \frac{1}{2}p}{\mathrm{L}_c}$ prefix ‘ + ’ instead of ‘ — ,’—line 6, *for* ‘ with ’ *put* ‘without’ —line 3 from bottom, in symbol, *for* ‘ o ’ *write* ‘ 80 ’—line 2 from bottom, *for* ‘ *n* ’ *put* ‘ *π* ’.

291 1 and 2 from bottom, *dele* dash above ‘ N.’

294 5 *insert* ‘ *t* ’ before the semicolon.

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——— Results of Meteorological Observations taken at the Madras Observatory under the superintendence of John Goldingham. Esq. MS. fol. *Read to the Society 24th March,* 1825.	John Goldingham, Esq.
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TRIGONOMETER (*vide* BOLLES) A brass Trigonometer, the beam 1 foot long, and semicircles $4\frac{1}{2}$ inches diameter.	Mr. William Bolles.
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WIGGINS (JOHN) South of Ireland.—Hints to Irish Landlords on the best means of obtaining and increasing their rents, improving their estates, and bettering the condition of the people, by a Land-agent, with an Appendix exemplifying the measures recommended. 8° *London*, 1824.	John Wiggins, Esq.
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From the Press of
W. NICOL,
Cleveland-row, St. James's.

METEOROLOGICAL JOURNAL,

KEPT BY THE ASSISTANT SECRETARY

AT THE APARTMENTS OF THE

ROYAL SOCIETY,

BY ORDER OF THE

PRESIDENT AND COUNCIL.

METEOROLOGICAL JOURNAL

for January, 1824.

1824 January.		Time. H.	Time. M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm. °	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
♃	1	9	0	29,265	47	703	40		NW	1,2	Cloudy.
		3	0	29,265	47	703	48		NW	1,2	Cloudy.
♀	2	9	0	29,313	46	773	45		W	2	Cloudy.
		3	0	29,520	47	769	47		NNW	1,2	Cloudy.
♄	3	9	0	30,224	41	822	41		NNW	1	Fine.
		3	0	30,300	41	767	41		W by N	1	Fine.
☉	4	9	0	30,535	36	686	36		NW	1	Foggy.
		3	0	30,519	43	734	43		W	1	Foggy.
☾	5	9	0	30,516	37	875	37		WNW	1	Foggy.
		3	0	30,443	40	771	40				Cloudy.
♂	6	9	0	30,214	37		35		E	1	Cloudy.
		3	0	30,125	40	650	40		W	1	Cloudy.
☿	7	9	0	30,194	34	810	34		SSW	1	Fine.
		3	0	30,326	40	800	40		N	1	Cloudy and hazy.
♃	8	9	0	30,326	35	833	35		S	1	Cloudy.
		3	0	30,282	39	794	39		S	1	Fine.
♀	9	9	0	30,221	40	857	40		W	1	Fine.
		3	0	30,192	44	707	44		W	1	Cloudy.
♄	10	9	0	30,150	43	835	43		SSW	1	Rain.
		3	0	30,084	44	781	44		SW	1	Cloudy.
☉	11	9	0	30,260	40	800	39		N	1	Cloudy.
		3	0	30,335	41	743	42	0.105	N	1	Fine.
☾	12	9	0	30,479	31	817	30		NNW	1	Hazy.
		3	0	30,456	35	833	35		W	1	Hazy.
♂	13	9	0	30,514	29	696	27		W	1	Hazy.
		2	30	30,459	31	904	32		W	1	Fine, rather hazy.
☿	14	9	0	30,435	27	769	25		W	1	Foggy.
		3	0	30,378	30	860	31		WNW	1	Hazy.
♃	15	9	0	30,370	34	785	29		NE	1	Foggy.
		3	0	30,340	36	758	37		N	1	Cloudy.
♀	16	9	0	30,586	34	813	31		N	1	Fine.
		3	0	30,588	35	683	36		NW	1	Fine.

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1824 January.	Time. H.	Time. M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm.	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
♄ 17	9	0	30,554	29	784	$28\frac{1}{2}$		W	1	Hazy.
	3	0	30,452	35	658	35		NW	1	Fine, rather hazy.
☉ 18	9	0	30,427	37	844	33		NW	1	Hazy.
	3	0	30,360	39	853	40		NNW	1	Fine.
☾ 19	9	0	30,339	39	735	33		NNW	1	Thick fog.
	3	0	30,180	42	842	42		NW	1	Cloudy.
♂ 20	8	30	30,126	38	909	38		N	1	Thick fog.
	3	0	30,046	41	849	41		W	1	Fog.
☿ 21	9	0	29,719	39	912	39		S	1,2	Fine.
	3	0	29,587	44	756	44				Cloudy.
♃ 22	9	0	29,279	41	904	41		W	1	Cloudy.
	3	0	29,051	45	824	45	0.232	S	1	Rain.
♀ 23	9	0	28,802	44	805	43		W	1	Cloudy.
	3	0	28,799	46	705	46		NW	1	Cloudy.
♄ 24	9	0	29,728	39	912	36		NW	1	Fine.
	3	0	29,854	44	781	44		NW	1	Fine.
☉ 25	9	0	29,964	50	790	41		W	1	Fine.
	3	0	30,016	52	738	$52\frac{1}{2}$		W	1	Fine.
☾ 26	9	0	30,142	50	790	49		W by S	1	Cloudy.
	2	30	30,112	50	850	51		W	1	Cloudy.
♂ 27	9	0	29,821	47	868	43		W	1,2	Cloudy.
	3	0	29,721	51	792	51		W	1,2	Rain.
☿ 28	9	0	29,565	40	800	$38\frac{1}{3}$	0.185	S by W	1	Hazy.
	3	0	29,517	41	849	41		S	1	Fine.
♃ 29	8	30	29,820	37	844	36		W	1	Fine.
	3	0	29,865	42	763	42		W by N	1	Fine.
♀ 30	9	0	30,127	33	786	32		W	1	Hazy.
	3	0	30,063	38	758	38		S by E	1	Fine.
♄ 31	8	0	30,025	39	875	35		S	1	Fine.
	2	30	29,985	42	711	42		S	1	Fine.

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for February, 1824.

1824 February.	Time.		Barom. corrected.	Therm. without.	Degree of Moisture by Daniell's Hygrom.	Six's Therm.	Rain.	Winds.		Weather.
	H.	M.	Inches.	°	°	°	Inches.	Points.	Str.	
☉ 1	9	0	29,956	35	783	34		SE	1	Fine and clear.
	3	0	29,924	42	658	42		SSE	1	Fine.
☽ 2	9	0	30,061	33	893	33		E	1	Hazy.
	3	0	30,097	40½	800	40½		E	1	Fine, rather hazy.
♂ 3	8	30	30,134	34	896	33		E	1	Hazy.
	3	0	30,007	43	785	43½		S	1	Fine.
☿ 4	9	0	29,719	44	862	41	0.020	W	1	Cloudy and hazy.
	3	0	29,718	47		48		E	1	Fine.
♃ 5	9	0	29,871	39	882	37	0.110	W	1	Fine.
	2	30	29,912	44	707	44		W	1	Fine.
♀ 6	9	0	30,070	36	871	35		W	1	Fine.
	3	0	30,134	41	795	43		SW	1	Cloudy.
♄ 7	9	0	30,160	49	777	49		SW	1	Cloudy.
	2	30	30,152	49	783	49		SW	1	Cloudy.
☉ 8	8	30	30,229	49	783	47		W	1,2	Cloudy.
	3	0	30,297	51	792	52		W	1,2	Cloudy.
☽ 9	9	0	30,403	49	907	48		W	1,2	Cloudy.
	2	30	30,402	52	766	52		SW	1	Cloudy.
♂ 10	9	0	30,390	46½	824	42	0.041	SW	1	Cloudy.
	3	0	30,197	51½	879	52		SW	1	Cloudy.
☿ 11	9	0	30,357	41	740	40		W	1	Fine.
	3	0	30,317	46	637	47		NW	1	Fine.
♃ 12	9	0	30,018	42	868	36½		W	1	Cloudy.
	3	0	29,733	47	868	47		SSW	1	Rain.
♀ 13	9	0	29,245	39	882	37		W	1	Fine.
	3	0	29,087	45	889	46		W	1	Fine.
♄ 14	9	0	28,906	39	853	38	0.930	SSW	1	Cloudy.
	3	0	29,018	42	842	45		E	1	Fine.
☉ 15	9	0	29,449	38	849	37		N	1	Fine.
	2	30	29,503	41	644	41		NNE	1	Fine.

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for February, 1824.

1824 February.		Time.	Barom. corrected.	Therm. without.	Degree of Moisture by Daniell's Hygrom.	Six's Therm.	Rain.	Winds.		Weather.
		H. M.	Inches.	°	°	°	Inches.	Points.	Str.	
☾	16	8 30	29,603	32	963	28		E	1	Foggy.
		3 0	29,539	38	818	38		E	1	Fine.
♂	17	8 30	29,425	34	862	33		E	1	Fine.
		2 30	29,323	39	713	39		E	1	Cloudy.
☿	18	9 0	29,294	37	875	33		E	1	Cloudy and hazy.
		3 0	29,332	43	785	44		S	1	Fine.
♃	19	9 0	29,393	40	857	36	0.070	E	1,2	Cloudy.
		2 30	29,362	47	681	47		E	1,2	Cloudy.
♀	20	9 0	29,505	41	904	40		E	1,2	Rain.
		3 0	29,579	42	790	43		E	1	Rain.
♄	21	9 0	29,771	39	854	37	0.250	W	1	Cloudy.
		3 0	29,808	40	857	41				Cloudy.
☉	22	9 0	29,903	39	971	37		E	1	Fog.
		3 0	29,883	40	864	41				Fine, rather hazy.
☾	23	9 0	29,970	41	931	40		E	2	Cloudy.
		3 0	29,995	41	1,000	42		E	1	Cloudy.
♂	24	8 30	29,978	40	829	38		E	1,2	Cloudy.
		3 0	29,898	42	816	43		E	1	Cloudy.
☿	25	9 0	29,800	37	875	37		NNE	1	Cloudy.
		3 0	29,782	38	818	39		E	1	Cloudy.
♃	26	9 0	29,775	37	875	33		N	1	Cloudy.
		2 30	29,702	39	765	40		N	1	Cloudy.
♀	27	8 30	29,617	35	900	35		N	1	Rain and sleet.
		3 0	29,611	39	941	39	0.060	NE	1	Rain.
♄	28	8 30	29,813	37	906	34		E	1	Cloudy, thick fog.
		3 0	29,858	41	712	41		E	1	Cloudy.
☉	29	9 0	29,912	38	909	36		N	1	Cloudy and foggy.
		2 30	29,906	39	941	39		E	1	Cloudy.

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for March, 1824.

1824 March.	Time. H.	Time. M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm. °	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
☾ 1	9	0	29,756	39	794	36		NNW	1	Fine.
	3	0	29,659	43	732	44		N	1	Rain.
♂ 2	8	30	29,627	30	635	29		N	2	Fine.
	2	30	29,635	35		35		N	1	Fine.
☿ 3	9	0	28,987	33	806	31		W	1	Cloudy.
	3	0	28,938	37	735	40		N	2	Rain and snow.
♃ 4	9	0	29,857	29	730	28		N	1	Fine.
	3	0	29,883	38	620	38		W	1	Fine.
♀ 5	9	0	29,554	44½	777	35		W	1	Cloudy.
	3	0	29,587	48	872	48		N	1	Cloudy.
♄ 6	9	0	29,820	43	836	41		W	1	Fine.
	3	0	29,801	51	705	51		W	1	Cloudy.
☉ 7	9	0	29,502	49	907	45	0.020	SW	2	Squally; violent gusts of wind and rain.
	3	0	29,495	52	851	52		W	2	Cloudy.
☾ 8	9	0	29,153	51	850	47		SSW variable.	3	Gale of wind and rain.
	3	0	29,197	52	682	52		SW	2	Cloudy.
♂ 9	9	0	29,728	43	760	37	0.303	S	1	Fine; somewhat hazy.
	3	0	29,649	48	777	48		SSW	1	Cloudy.
☿ 10	9	0	29,666	41	861	40		N	1	Cloudy.
	3	0	29,751	42	961	44		N	1	Cloudy.
♃ 11	9	0	30,044	38	758	33		SW	1	Cloudy, thick weather.
	3	0	29,801	43	734	46		SW	1	Rain.
♀ 12	9	0	29,520	42	709	36		W	1	Fine.
	3	0	29,450	41	813	43		W var. from W to N	1,2	Fine.
♄ 13	9	0	29,333	39	853	35		W	1	Cloudy.
	3	0	29,291	39	765	43	0.170	NW	1	Hail, with thunder.
☉ 14	9	0	29,750	38	849	31		N	1	Fine.
	3	0	29,971	44	683	45		N	1	Fine.
☾ 15	9	0		42	737	33		W	1	Cloudy.
	3	0	30,025	45	753	47		SW	1	Cloudy.

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1824 March.	Time. H.	Time. M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm. °	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
♅ 16	9	0	29,968	48	840	43	0.016	SW	1	Cloudy.
☿ 17	9	0	30,147	46	830	42		W	1	Cloudy and hazy.
	3	0	30,172	51	676	53		W	1	Fine.
♃ 18	9	0	30,238	48	745	41		W	1	Cloudy.
	3	0	30,208	56	691	56		W	1	Fine.
♀ 19	9	0	30,286	51	792	48		W	1	Cloudy and hazy.
	3	0	30,252	55	765	55		W	1	Cloudy.
♄ 20	9	0	30,274	43½	886	43		W	1	Thick and hazy.
	3	0	30,226	52	636	52		SW	1	Cloudy.
☉ 21	9	0	29,938	47	868	45		S	1,2	Cloudy.
	2	30	29,787	49	814	49		S	1	Rain.
☾ 22	9	0	29,580	44	781	39	0.144	SSW	1	Rain.
	3	0	29,554	41	877	44		SE	2	Rain.
♅ 23	9	0	29,794	38½	939	35	0.128	E	1	Cloudy; sleet and snow in the morning.
	3	0	29,894	43	711	43		E	1	Fine.
☿ 24	9	0	29,952	41	904	38	0.045	NE	1	Cloudy.
	3	0	29,944	44	732	45		N	1	Cloudy.
♃ 25	9	0	30,072	42	763	40		NE	2	Cloudy.
	3	0	30,073	44	659	46		NE	2	Cloudy.
♀ 26	9	0	29,990	43	734	39		NE	2	Cloudy.
	3	0	29,917	45½	753	46		N	2	Cloudy.
♄ 27	9	0	29,824	41	849	36½		N	1,2	Cloudy.
	3	0	29,910	41½	623	41½		N	1,2	Cloudy.
☉ 28	9	0	29,884	40	771	35		NE	1	Cloudy.
	3	0	29,909	41½	623	41½		N	1,2	Cloudy; snow in the forenoon.
☾ 29	9	0	29,982	37	664	30		W	1	Cloudy.
	3	0	29,917	45	482	45		NE	1	Cloudy.
♅ 30	9	0	29,707	45	777	38		W	1	Cloudy.
	3	0	29,630	50	574	44		N	1	Cloudy.
☿ 31	9	0	29,707	34½	966	31	0.027	N	1	Fine.
	3	0	29,721	39	537	39		NNE	1	Cloudy.

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for April, 1824.

1824 April.		Time. H.	Time. M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm. °	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
♃	1	9	0	29,905	38	599	28		N	1	Fine.
		3	0	29,774	45	535	45		W	1,2	Cloudy.
♀	2	9	0	29,108	44½	805	38	0.248	W	2	Rain.
		3	30	29,526	41½	795	34		N	1	Cloudy.
♄	3	9	0	30,022	38	712	31½		NNW	1	Fine.
		3	0	30,057	45	659	45½		NW	1	Cloudy.
☉	4	9	0	30,249	41	740	37		N	1	Fine,
		3	0	30,300	46	534	47		N	1	Cloudy.
☽	5	9	0	30,458	41	685	34		NE	1	Cloudy.
		3	0	30,425	46	483	46		N	1	Fine.
♂	6	9	0	30,456	42	711	37		N	1	Cloudy.
		3	0	30,416	48	515	48		N	1	Cloudy.
☿	7	9	0	30,162	40	943	38	0.018	NNW	1	Cloudy.
		3	0	30,120	46	659	47		N	2	Cloudy.
♃	8	9	0	30,264	43	759	38		NE	2	Cloudy.
		3	0	30.189	50	540	51		N	1,2	Fine.
♀	9	9	0	30,049	41½	904	41		NE	1,2	Cloudy.
		3	0	29,958	48	681	47		NNE	1	Cloudy.
♄	10	9	0	29,356	41	822	40	0.015	W	2	Rain.
		3	0	29,412	39	853	44		N	2,3	Squally.
☉	11	9	0	29,346	34	931	34		N	1,2	Snow.
		3	0	29,331	46	551	45		N	1	Fine.
☽	12	9	0	29,404	41	740	32	0.080	NNW	1	Fine.
		3	0	29,488	46	551	47		W	1	Fine.
♂	13	9	0	29,655	42½	710	32½		W	1	Fine.
		3	0	29,714	49	536	50		W	1	Fine.
☿	14	9	0	29,871	44	707	33		W	1	Fine.
		3	0	29,856	51	502	53½		NW	1	Cloudy.
♃	15	9	0	29,837	45	635	36		NE	1	Fine.
		3	0	29,698	50	470	52		E	1	Fine.

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for April, 1824.

1824 April.		Time. H.	M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm.	Rain. Inches.	Winds. Points.	Str.	Weather.
♀	16	9	0	29,292	42	868	40		E	3	Rain.
		2	30	29,241	42	921	44		E	2	Rain.
♄	17	9	0	29,456	43	924	40½	0.495	NE	2	Rain.
		3	0	29,578	46	897	46	0 108	NE	1,2	Rain.
☉	18	9	0	30,047	48	723	38		NE	1	Fine.
		3	0	30,132	51½	599	52		NE	1	Fine.
☾	19	9	0	30,263	51	580	37		E	1	Fine.
		3	0	30,245	55	538	56		E	1	Fine.
♂	20	9	0	30,316	53	451	42		E	1	Fine.
		3	0	30,264	61	444	60		SE	1	Fine.
☿	21	9	0	30,093	55	572	43		E	1	Fine.
		3	0	29,959	60	586	60		SE	1	Fine.
♃	22	9	0	29,900	57	669	51	0.053	W	2,3	Cloudy.
		3	0	29,927	61	589	63		W	2	Fine.
♀	23	9	0	29,471	56	764	50	0.038	S	2	Showery.
		3	0	29,127	53	874	56		S	1	Rain.
♄	24	9	0	29,955	53	820	48	0.122	W	1	Fine.
☉	25	9	0	30,128	53	793	45		SW	1	Cloudy.
		3	30	30,030	59	693	60		S	1	Cloudy.
☾	26	9	0	29,668	57	693	52		S	1	Cloudy.
		3	0	29,519	60	672	59		S	1	Cloudy.
♂	27	9	0	29,791	57	646	46½		W	1	Fine.
		3	0	29,867	61	569	62		W	1	Fine.
☿	28	9	0	29,916	55	765	53		S	2	Cloudy.
		3	0	29,867	56	813	57		S	1	Cloudy.
♃	29	9	0	29,753	62	673	51	0.005	SW	1	Fine.
		3	0	29,669	67	555	66½		SW	1	Fine.
♀	30	9	0	29,632	64	610	55		S	1	Fine.
		3	0	29,676	64	535	66		W	2	Fine.

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1824 May.		Time. H.	Time. M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm. °	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
♄	1	9	0	29,925	60	650	52		SW	2	Cloudy.
		3	0	29,914	63	611	64		SSW	1	Fine.
☉	2	9	0	29,860	55	840	53		N	1	Rain.
		3	0	29,763	57	806	58	0.150	E	1	Rain.
☾	3	9	0	29,532	49	946	48		W	1	Cloudy.
		3	0	29,483	59	626	60		N	1	Rain.
♂	4	9	0	29,632	50	700	43	0.180	W	2	Cloudy.
		3	0	29,714	56½	618	57		W	1	Cloudy.
☿	5	9	0	29,849	59	737	45		S	1	Cloudy.
		3	0	29,860	60	586	62		W	1	Cloudy.
♃	6	9	0	29,891	55	815	50	0.032	W	1	Cloudy.
		3	0	29,839	62	673	63		W	1	Fine.
♀	7	8	0	29,862	53	901	48½		E	1	Cloudy.
		3	0	29,918	64	572	64		W	1	Fine.
♄	8	9	0	30,160	59	737	49	0.023	W	1	Cloudy.
		3	0	30,211	62	572	64		W	1	Cloudy.
☉	9	9	0	30,308	54	817	49		E	1	Cloudy.
		5	0	30,206	58	692	61		E	1	Fine.
☾	10	9	0	30,001	60	629	46		NE	1	Fine.
		3	0	29,923	66	519	66		ENE	1	Fine.
♂	11	9	0	29,971	51	908	48		E	1	Cloudy and hazy.
		3	0	29,929	53	820	53		E	2	Cloudy.
☿	12	9	0	29,876	48	814	45		E	1	Rain.
		3	0	29,847	51	712	52		E	1	Cloudy.
♃	13	9	0	29,815	45½	965	43	0.056	N	1,2	Rain.
		3	0	29,728	48	745	49		E	2	Rain.
♀	14	9	0	29,517	47	934	44	0.245	E	1	Rain.
		3	0	29,518	47½	967	48		N	2	Rain.
♄	15	9	0	29,503	44	963	43		N	2	Rain.
		3	0	29,453	44½	982	45	0.321	N	2	Rain.
☉	16	9	0	29,751	45½	987	42	1.355	N	2	Rain.
		3	0	29,881	52	738	53		NNE	1,2	Showery.

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1824 May.		Time. H.	Time. M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm.	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
☾	17	9	0	30,019	52½	682	39	0.007	W	1	Fine.
		3	0	29,955	55	639	56		W	1	Cloudy.
♂	18	9	0	29,934	54	713	47		W	1,2	Cloudy.
		3	0	29,858	56	642	56		W	1	Cloudy.
☿	19	9	0	29,811	51	821	46		W	1	Cloudy.
		3	0	29,809	56	642	56		W	1	Cloudy.
♃	20	9	0	29,719	49	516	44	0.018	NW	1	Cloudy.
		3	0	29,752	51	580	54		W	1	Cloudy.
♀	21	9	0	29,878	50½	580	37		N	1	Cloudy.
		3	0	29,876	55½	420	56		E	1	Cloudy.
♄	22	9	0	29,958	47	769	38		N by E	1	Cloudy and showery.
		3	0	29,930	54	539	53½		E	2	Cloudy.
☉	23	9	0	29,905	50	790	43		N	1	Cloudy.
		3	0	29,864	56	569	57		N	1	Cloudy.
☾	24	9	0	29,914	52	879	49		NW	1	Rain.
		3	0	29,929	53	901	54		N by E	1	Rain.
♂	25	9	0	30,143	52	738	42		N	1	Cloudy.
☿	26	9	0	30,413	57	717	50	0.004	N	1	Cloudy.
		3	0	30,455	63½	572	64		N	1	Fine.
♃	27	9	0	30,578	62	673	49½		NW	1	Fine.
		3	0	30,573	69½	575	70		N	1	Fine.
♀	28	9	0	30,544	64	629	50		W	1	Fine.
		3	0	30,447	71	538	73		S	1	Fine blue sky.
♄	29	9	0	30,279	64	651	53		E	1	Cloudy.
☉	30	9	0	29,824	61	805	54		E	1	Cloudy.
		3	0	29,792	61½	742	59		E	1	Cloudy.
☾	31	9	0	29,852	60	739	51		W	1	Cloudy.
		3	0	29,865	66	655	67		W	1	Cloudy.

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1824 June.		Time.		Barometer corrected.	Therm. without.	Degree of Moisture by Daniell's Hygrom.	Six's Therm.	Rain.	Winds.		Weather.
		H.	M.	Inches.	°	°	°	Inches.	Points.	Str.	
♂	1	9	0	30,129	59	847	54½		N	1,2	Cloudy.
		3	0	30,175	66	678	67		N	2	Fine.
☿	2	9	0	30,335	58	905	49		N	1	Cloudy.
		3	0	30,276	70	556	70		N	1	Fine.
♃	3	9	0	30,338	53	901	50		N by E	2	Cloudy.
		3	0	30,259	58	578	57		N by E	2	Cloudy.
♀	4	9	0	30,329	54	739	50		E	1	Cloudy.
		5	0	30,192	65	572	66		NW	1	Fine.
♄	5	9	0	30,232	57	669	47		N	2	Cloudy.
		3	0	30,176	57	788	60½		N	1	Cloudy.
☉	6	9	0	30,189	54	870	51		E	1	Cloudy.
		3	0	30,143	67	612	67		N	1	Fine and clear.
☽	7	9	0	30,150	60	764	52½		E	1	Cloudy.
		3	0	30,098	71	638	71		N	1	Fine and clear.
♂	8	9	0	30,097	59	847	53		E	1	Fine, rather hazy.
		2	0	30,063	71	598	72		E	[illegible]	Fine and clear.
☿	9	9	0	29,986	57	874	49		E	1	Fine blue sky.
		3	0	29,907	68	637	68		E	2	Rain.
♃	10	9	0	29,905	52	1,000	52		E	2	Rain. Rain from 4 A. M.
		3	0	29,927	56	935	55½	0.378	N	1	Cloudy.
♀	11	9	0	30,093	53	793	48	0.023	N	1	Cloudy.
		3	0	30,101	57	599	58		N	1	Cloudy.
♄	12	9	0	30,179	57	504	43		N by W	1	Fine.
		3	0	30,136	60	607	61		N	1	Cloudy.
☉	13	9	0	30,100	59	737	45		W	1	Cloudy.
		3	0	29,986	63	475	66		S	2	Cloudy.
☽	14	9	0	29,445	58	814	51½		S	2	Cloudy.
		3	0	29,345	65	749	65	0.525	S	1	Fine, though somewhat [cloudy.
♂	15	9	0	29,220	56½	935	55		S	2	Cloudy.
		3	0	29,319	58	844	59	0.037	S	1	Cloudy.

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1824 June.	Time. H.	Time. M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm. °	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
☿ 16	9	0	29,530	57	815	50	0.005			
	3	0	29,563	63	650	64		N	1	Cloudy.
♃ 17	8	0	29,796	54	870	52		N	1	Fine.
	3	0	29,853	60	714	63		N	1	Fine and clear.
♀ 18	9	0	30,051	58	517	46		N by E	1	Fine.
	3	0	30,012	60	583	60		S	1	Rain.
♄ 19	9	0	29,681	60	800	49		S	1	Rain.
	3	0	29,642	58	794	59		S	1	Cloudy.
☉ 20	9	0	29,391	63	837	51		S	1	Rain.
	3	0	29,403	58		60		S	1	Cloudy.
☽ 21	9	0	29,489	65	713	51		W	1	Cloudy.
	3	0	29,505	65	616	66½	0.505	W	1	Fine.
♂ 22	9	0	29,576	65	676	50		S	1	Fine.
	3	0	29,533	66	611	67		E	2	Rain.
☿ 23	9	0	29,377	57	937	54		E	1	Cloudy.
	3	0	29,358	63	696	63	0.148	N	2	Cloudy.
♃ 24	9	0	29,365	55	966	53	0.373	N by W	1	Rain.
	2	0	29,425	55	935	57		W	1	Cloudy.
♀ 25	9	0	29,687	56	870	52	0.102	W	1	Cloudy.
	3	30	29,778	60	793	62		W	1	Fine.
♄ 26	9	0	29,998	65	651	52		W	1	Fine.
	3	0	30,003	69	537	70		E	1	Fine.
☉ 27	9	0	30,001	67	612	52	0.015	N	1	Fine.
	3	0	29,943	70	556	70½		S	1	Cloudy and showery.
☽ 28	9	0	29,922	67	753	59		S	1	Fine.
	2	30	29,894	70	597	71		S	1	Cloudy and showery.
♂ 29	9	0	29,671	67	826	58	0.072	S	1	Cloudy.
	3	0	29,631	72	640	74		S	1	Cloudy.
☿ 30	9	0	29,871	66	749	55		S	2	Cloudy.
	3	0	29,856	68	615	70				

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1824 July.		Time. H.	M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm. °	Rain. Inches.	Winds. Points.	Str.	Weather.
♃	1	9	0	29,848	67	612	54		W	1	Cloudy.
		3	0	29,772	69½	618	71		W	1	Cloudy.
♀	2	8	0	29,557	64	799	58	0.053	W	2	Rain.
		1	30	29,562	70	727	70		W	2	Cloudy.
♄	3	9	0	29,592	65	676	60	0.024	W	1	Cloudy.
		3	0	29,572	64	774	69		W	1	Rain.
☉	4	9	0	29,661	64	723	56	0.293	W	1	Cloudy.
		3	0	29,779	66	726	68		W	1	Showery.
☽	5	9	0	29,980	67	658	51		W	1	Fair, somewhat cloudy.
		3	0	29,944	69	660	70	0.070	W	1	Cloudy.
♂	6	9	0	29,874	62	748	56		SE	1	Cloudy.
		3	0	29,813	63	800	65	0.023	S	1	Cloudy.
☿	7	9	0	29,865	63	855	58		SW	1	Cloudy.
		3	0	29,797	65	852	65		S	2	Rain.
♃	8	9	0	29,994	68	729	59		W	1	Cloudy.
		3	0	30,003	74	639	74	0.025	W	1	Fine.
♀	9	9	0	29,947	66	801	62		W	1	Cloudy.
		3	0	29,865	73	724	75		S	1	Showery.
♄	10	9	0	29,911	69	682	58		N	2	Fine.
		3	0	29,930	73	598	74		W	1	Fine.
☉	11	9	30	30,073	71	598	55		W	1	Fine.
		3	0	30,016	74	600	75		W	1,2	Cloudy.
☽	12	9	0	30,061	65	878	59		W	1	Cloudy and thick haze.
		3	0	30,006	77	580	78		W	1	Fine, blue sky.
♂	13	9	0	30,047	71	618	57		W	1	Fine, rather hazy.
		3	0	30,080	80	528	81		W	1	Fine, blue sky.
☿	14	9	0	29,906	77	637	65		W	1	Fine.
		3	0*	29,907	79	660	81	0.460	W	1	Cloudy, thunder at a distance.
♃	15	9	0	29,899	67	727	74		W	1	Cloudy.
		3	0	29,928	74	639			W	1	Cloudy.

* 9 PM 29,884 73 }
10 27 29,832 69 } . . . . { A storm of thunder and lightning from 9 till 10.

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♀ 16	9 0	30,082	71	704	60		W	1	Cloudy and hazy.
	3 0	30,063	72	681	74		W	1	Cloudy.
♄ 17	9 0	30,233	69	537	59		E	1	Fine.
	3 0	30,232	74	442	74½		N	1	Fine.
☉ 18	9 0	30,252	68	593	58		N	2	Fine.
	3 0	30,286	67	592	71		N by E	1	Fine, with light showers.
☾ 19	9 0	30,452	66	631	57		NNE	1	Fine.
	3 0	30,447	70	473	73		NNE	2	Fine.
♂ 20	9 0	30,394	66	749	56		W	1	Cloudy.
	3 0	30,324	73	523	74		N	1	Fine.
☿ 21	9 0	30,250	69	706	58		N	1	Fine, rather hazy.
	3 0	30,234	72	566	74		N by E	2	Cloudy.
♃ 22	9 0	30,260	68	799	61		E	1	Cloudy and hazy.
	3 0	30,228	71	598	73		E	1	Fine.
♀ 23	9 0	30,177	70	577	59		SE	1	Fine, but hazy.
	3 0	30,099	71		76		SE	1	Fine.
♄ 24	9 0	29,839	70	618	58½		W	1	Fine.
	3 0	29,822	74	561	77		NW	1	Fine.
☉ 25	9 0	29,859	70	577	57		W	1	Cloudy.
	3 0	29,855	72	540	73		W	1	Cloudy.
☾ 26	9 0	29,854	71	618	60		E	1	Cloudy.
	3 0	29,795	68	637	71		E	1	Cloudy.
♂ 27	9 0	30,013	61	769	55	0.082	N	1	Cloudy.
	3 0	30,051	65	484	66		W	1	Cloudy.
☿ 28	9 0	30,290	68	554	53		W	1	Fine.
	3 0	30,274	72	504	74		S	1	Cloudy.
♃ 29	9 0	30,104	68	637	54		E	1	Cloudy.
	3 0	29,970	69	639	72		E	1	Fine.
♀ 30	9 0	29,681	62	637	53		N	1	Cloudy.
	3 0	29,604	71	578	71		N	1	Cloudy.
♄ 31	9 0	29,709	63	800	54		NE	1	Fine.
	3 0	29,654	70	727	72		W	1	Cloudy.

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1824 August.		Time. H.	Time. M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm. °	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
☉	1	9	0	29,687	60	739	58	0.118	N	1	Rain.
		3	0	29,787	63	800	64		N	1	Cloudy.
☽	2	9	0	30,085	63	696	51		W	1	Fine.
		3	0	30,076	70	556	70		S	1	Fine.
♂	3	9	0	30,003	66	801	59	0.176	SSW	1	Cloudy.
		3	0	29,944	70	577	71		S by E	1	Fine.
☿	4	9	0	29,877	65	801	60	0.060	S	1	Cloudy.
		3	0	29,807	69	660	70		W	1	Fine.
♃	5	9	0	29,745	65	749	60		W	1	Cloudy.
		3	0	29,735	68	573	70		W	2	Cloudy.
♀	6	8	30	29,659	62	828	58		W	1	Cloudy.
		3	0	29,679	65	749	65		W	1	Showery.
♄	7	9	0	29,890	61	880	54	0.052	NNW	1	Cloudy.
		3	0	29,920	68	681	69		N	1	Cloudy.
☉	8	9	0	29,900	63	774	58	0.016	WNW	1	Cloudy.
		3	0	29,834	65	801	65		SW	2	Cloudy and hazy.
☽	9	9	0	29,725	68	681	62	0.019	W	1	Fine.
		3	0	29,728	72	599	73		W	1	Fine.
♂	10	9	0	29,873	66	631	55		W	1	Fine, rather hazy.
		3	0	29,853	71	558	72		SW	1	Fine.
☿	11	9	0	29,695	68	752	61½	0.019	W	2	Cloudy.
		3	0	29,740	72	661	73		W	1	Fine.
♃	12	9	0	29,812	69	660	60		SSW	1	Cloudy.
		3	0	29,792	69	682	72		N	1	Cloudy.
♀	13	9	0	29,884	67	658	56		W	1	Cloudy.
		3	0	29,853	65	700	70	0.775	W	1	Cloudy; thunder at 2 P.M. A violent storm of rain, thunder, and lightning, at 5 P. M.
♄	14	9	0	30,012	63	673	51	0.091	W by N	1	Fine.
		3	0	29,998	68	573	67		W	1	Fine.
☉	15	9	0	29,759	63	855	57	0.006	S	2	Rain.
		3	0	29,492	63	855	64		NW	1	Rain.

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1824 August.	Time. H.	Time. M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm. °	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
☾ 16	9	0	29,738	63	748	52	0.407	W	2	Fine.
	3	0	29,771	67	635	68		W	2	Cloudy.
♂ 17	9	0	29,825	66	655	54	0.130	W	1	Cloudy.
	3	0	29,712	65	773	68		S	1	Cloudy.
☿ 18	9	0	29,674	65	725	56		W	1	Fine.
	3	0	29,573	68	615	69		W	1	Fine.
♃ 19	9	0	29,808	63	722	51		W	1	Fine, rather hazy.
	3	0	29,830	64	591	66		W	1	Fine.
♀ 20	9	0	29,850	60	939	58	0.018	E	1	Dark rainy weather.
	3	0	29,863	66	776	67		W	1	Cloudy and dark.
♄ 21	9	0	29,785	64	881	60		W	1	Rain.
	3	0	29,793	66	801	67		W	1	Cloudy.
☉ 22	8	30	30,022	60	850	55½	0.200	N	1	Cloudy and hazy.
	3	0	30,057	63	722	65		NNW	1	Fine.
☾ 23	8	30	30,132	55	1,000	50½		N	1	Cloudy and hazy.
	3	0	30,117	68	554	70		NE	1	Fine.
♂ 24	9	0	30,224	56	902	53		N	1	Cloudy and hazy.
	3	0	30,210	65	902	65		NE	1	Fine, somewhat hazy.
☿ 25	9	0	30,330	59	969	55		E	1	Cloudy and hazy.
	3	0	30,298	69½	706	70		E	1	Fine.
♃ 26	9	0	30,370	64	854	60		N by E	2	Cloudy.
	3	0	30,309	68	799	70		E	2	Cloudy.
♀ 27	9	0	30,290	63	826	55½	0.005	N	1	Cloudy.
	3	0	30,184	68	729	69½		E	1	Cloudy.
♄ 28	9	0	30,036	66	776	57		E	1	Fine.
	3	0	29,949	69	682	71		E	1	Fine and clear.
☉ 29	9	0	29,925	68	823	53		E	1	Cloudy and thick haze.
	3	0	29,878	75½	702	77		SE	2	Fine and clear.
☾ 30	9	0	29,906	63	1,000	69		ENE	1	Cloudy.
	3	0	29,862	74	773	75		E	1	Cloudy.
♂ 31	9	0	29,912	65	852	63		N	1	Hazy and cloudy.
	3	0	29,940	71	773	71		NNW	1	Fine.

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1824 September.		Time. H.	M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm. °	Rain. Inches.	Winds. Points.	Str.	Weather.
☿	1	9	0	30,018	69	798	61		E	1	Fine.
		3	0	29,995	80	528	81		S	1	Fine.
♃	2	9	0	30,025	73	749	63		E by S	1	Fine, somewhat hazy.
		3	0	29,989	79	598	80		E by S	1	Fine and clear.
♀	3	9	0	29,966	69	797	65		W	1	Cloudy and hazy.
		3	0	29,924	77	658	78		W	1	Cloudy, with some drops of rain.
♄	4	9	0	29,877	68	776	66	0,188	N	1	Cloudy.
		3	0	29,826	74	543	75		W	2	Fine.
☉	5	9	0	29,768	67	635	57		W	1	Fine.
☽	6	9	0	29,496	64	854	56		S	2	Light rain.
		3	0	29,409	69	596	70		S	2	Fine.
♂	7	9	0	29,508	67	777	58½	0,101	SSE	2	Cloudy.
		3	0	29,488	66	776	68		SW	1	Cloudy and showery.
☿	8	9	0	29,380	64	827	58	0,720	W	2	Showery.
		3	0	29,464	64	673	65		W	1,2	Fine.
♃	9	9	0	29,667	60	793	52	0,125	W	1	Cloudy.
		3	0	29,687	63	774	71		W	1	Cloudy.
♀	10	9	0	29,747	58	875	54		W	1	Cloudy.
		3	0	29,771	64	844	66		S	1	Cloudy.
♄	11	9	0	29,656	62	828	56	0,430	S	1	Cloudy.
		3	0	29,698	64	774	68		W	1	Cloudy.
☉	12	9	0	29,617	60	821	58	0,227	WSW	3	Rain.
		3	0	29,705	64	748	65		W	1,2	Cloudy.
☽	13	9	0	30,069	61	825	52	0,040	W	2	Fine, but hazy.
		3	0	30,090	66	749	67		SW	1	Fine.
♂	14	9	0	30,030	65	773	57		S	2	Cloudy.
		3	0	30,016	67	777	67		S	2	Fine, with thin clouds.
☿	15	9	0	30,065	67	826	64		W	1	Cloudy.
		3	0	30,097	71	725	72	0,014	W	1	Fine.

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1824 September.		Time. H.	M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm. °	Rain. Inches.	Winds. Points.	Str.	Weather.
♃	16	9	0	30,253	60	821	54		N	1	Hazy.
		3	0	30,228	67	727	68		E	1	Cloudy.
♀	17	9	0	30,166	63	883	59		E	1	Cloudy and very hazy.
		3	0	30,120	70	853	70		E	1	Fine.
♄	18	9	0	30,067	63	910	60			1	Fine.
		3	0	29,923	65	904	72		W	1	Hazy.
☉	19	9	0	29,918	62	943	60		W	1	Fine.
		3	0	29,876	67	826	67		W	1	Cloudy.
☾	20	9	0	29,840	58	935	56	0.140	W	1	Cloudy.
		3	0	29,846	60	850	61		W	1	Rain.
♂	21	9	0	29,841	54	965	51	0.064	W	1	Rain.
		3	0	29,832	58	844	58		W	1	Rain.
☿	22	9	0	30,044	61	853	53		S	1	Fine.
		3	0	30,048	63	800	65		N	1	Fine.
♃	23	9	0	29,925	58	935	55	0.093	N	1	Rain.
		3	0	29,916	63	774	63		N	1	Fine.
♀	24	9	0	30,032	60	850	56	0.148	N	1	Cloudy.
		3	0	30,029	62	805	64		N	1	Cloudy.
♄	25	9	0	30,014	58	814	56	0.008	W	1	Cloudy.
		3	0	29,898	59	737	62		W	1	Cloudy.
☉	26	9	0	30,042	45	612	41½		NNW	1	Fine.
		3	0	29,983	50	730	51		NNW	1	Fine.
☾	27	9	0	29,590	47	868	43		W	1	Rain.
		3	0	29,507	54	635	54	0.075	W	1	Fine.
♂	28	9	0	29,812	41	795	37		W	1	Fine.
		3	0	29,881	51	580	51		W	1	Fine.
☿	29	9	0	29,905	46	750	38		E	1	Hazy.
		4	0	29,852	56	715	58		E	1	Fine.
♃	30	9	0	29,616	57	784	49		S	1	Hazy.
		3	0	29,395	58	966	50		E	1	Hazy.

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♀	1	9	0	29,169	59	788	59		E	3	Rain.
		3	0	29,074	61	825	63	0.310	SSW	2	Fine.
♄	2	9	0	29,345	57	843	50		S	3	Rain.
		3	0	29,432	57	874	62	0.045	S	2	Rain.
☉	3	9	0	29,757	56	764	50		S	1	Fine.
		3	0	29,798	61	672	62		W	1	Fine.
☾	4	9	0	29,796	56	813	51		E	1	Fine.
		3	0	29,696	61	742	63		E	1	Cloudy.
♂	5	9	0	29,613	57	935	56		E	1	Fog and rain.
		3	0	29,545	58	884	59	0.069	E	1	Rain.
☿	6	9	0	29,430	59	762	58		E	1	Hazy.
		3	0	29,331	60	939	62	0.093	E	1	Rain.
♃	7	9	0	29,268	62	747	58	0.137	SSW	2	Cloudy.
		3	0	29,267	63	800	63		S	1	Cloudy.
♀	8	9	0	29,319	61	825	57	0.082	SSW	1	Fine.
		3	0	29,317	64	707	65		SSW	1	Fine, rather hazy.
♄	9	9	0	29,539	59	818	54		W	1	Cloudy.
		3	0	29,566	63	722	63		W	1	Cloudy.
☉	10	9	0	29,538	50	940	49		E	1	Cloudy.
☾	11	9	0	28,890	55	815	52	0.413	E	1	Rain.
		3	0	28,923	56	813	57		S	1	Fine.
♂	12	9	0	28,839	52	967	47	0.085	N	2	Rain.
		3	0	28,924	55	790	55		SE	3	Rain.
☿	13	9	0	29,377	40	886	37		NW	1,2	Cloudy.
		3	0	29,449	45	588	45		NNE	1	Fine.
♃	14	9	0	29,578	41	822	34	0.030	W	1	Hazy.
		3	0	29,583	49	639	50		W	1	Hazy, with clouds.
♂	15	9	0	29,660	40	943	36		WNW	1	Cloudy and foggy.
		3	0	29,668	44	805	49		N	1	Rain.
♄	16	9	0	29,896	37	758	34	0.011	NW	1	Fine.
		3	0	29,932	42	619	44		NNE	1	Fine.

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☉ 17	9	0	30,030	35	867	31		NW	1	Fine.
	3	0	29,985	49	660	49½		W	1	Cloudy.
☾ 18	9	0	30,154	39	794	31	0.024	W	1	Cloudy.
	3	0	30,116	40	1,000	41		W	1	Fine.
♂ 19	9	0	30,126	48½	872	40		W	1	Cloudy.
	3	0	30,049	54	739	55		W	1,2	Cloudy.
☿ 20	9	0	30,024	53	766	50		W	1,2	Cloudy.
	3	0	30,029	55	739	57	0.003	W	1	Cloudy.
♃ 21	9	0	30,008	50	850	46		E	1	Hazy.
	3	0	29,967	55	870	56		S	1	Fine.
♀ 22	9	0	29,935	53	874	48		ESE	1	Cloudy.
	3	0	29,856	58	814	59		E	1	Cloudy.
♄ 23	9	0	29,924	57	843	54		W	1	Cloudy and hazy.
	3	0	29,930	60	821	61		WNW	1	Fine.
☉ 24	9	0	29,840	55	966	51½		E	1	Cloudy and hazy.
	3	0	29,718	60	793	61		SE	1	Fine.
☾ 25	9	0	29,528	58	875	56		S by E	3	Cloudy.
	3	0	29,518	59	847	60	0.040	SW	2	Fine.
♂ 26	9	0	29,243	57	874	52		W	2	Cloudy.
	3	0	29,278	54	765	58		W	2	Fine.
☿ 27	9	0	29,504	51	908	48		W	1	Fine.
	3	0	29,571	56	740	58		W	1	Fine.
♃ 28	9	0	29,657	53	932	48		W	1	Fine.
	3	0	29,652	55	933	56		W	1	Cloudy.
♀ ● 29	9	0	29,671	52	935	50	0.023	S	1	Cloudy.
	3	0	29,525	53	932	53	0.097	W	2	Cloudy.
♄ 30	9	0	29,899	46½	750	45		NNW	1	Cloudy.
	3	0	30,041	58	529	59		N	1	Fine.
☉ 31	9	0	30,110	43	868	39		W	1	Cloudy.
	3	0	29,847	50	850	50		W	2	Rain.

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☾	1	9	0	29,640	51	821	48	0.200	W	1	Cloudy.
		3	0	29,602	55	765	55		W	1	Cloudy.
♂	2	9	0	29,709	54	930	53		W	1	Rain.
		3	0	29,519	58	787	59	0.018	WNW	1	Cloudy.
☿	3	9	0	29,809	46	830	43		W	1	Fine.
		3	0	29,788	52	682	52		W	1	Fine.
♃	4	9	0	29,809	43	835	41		NNW	1	Fine.
		3	0	29,796	46	636	47		W by N	1	Fine.
♀	5	9	0	29,751	42	921	39		W	1	Fine, rather hazy.
		3	0	29,810	45	729	46		NW	1	Fine.
♄	6	9	0	30,135	36	903	29		W by N	1	Fine.
		2	0	30,095	43	633	43		WSW	1	Fine.
☉	7	9	0	29,947	52	822	42		W	1	Rain.
		3	0	29,907	55	739	56		W by S	1,2	Fine.
☾	8	9	0	29,683	54	817	50		S	2	Cloudy.
		3	0	29,585	56	841	57		S	1	Cloudy.
♂	9	9	0	29,971	44	805	42		W	1	Fine.
		3	0	30,007	59	471	60		W	1	Thin clouds.
☿	10	9	0	29,885	55	840	46	0.089	W	1	Cloudy.
		3	0	29,784	55	739	56		WSW	1	Cloudy.
♃	11	9	0	29,824	57	787	57		WNW	1	Rain.
		3	0	29,768	57	899	57		W	1	Rain.
♀	12	9	0	29,980	47	854	45	0.705	W	1	Fine.
		4	0	30,069	48	745	54		W	1	Fine.
♄	13	9	0	30,056	46	830	42		W	1	Cloudy.
		4	0	29,749	54	870	54		W	2	Cloudy.
☉	14	9	0	29,518	53½	874	46		W	2	Rain.
		3	0	29,511	51	792	53	0.177	W	2	Cloudy.
☾	15	9	0	29,900	43	835	39		W	1	Cloudy.
		3	0	29,967	43	759	47	0.007	W	1	Fine.

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1824 November.		Time. H.	Time. M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm. °	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
♂	16	9	0	30,246	40	771	35		W	1	Cloudy.
		3	0	30,045	49	784	49		SW	2	Cloudy.
☿	17	9	0	29,785	51	763	49		SW		Cloudy.
		3	0	29,636	52	822	52		SSW	2	Cloudy.
♃	18	9	0	29,354	58	814	51		SW	2	Rain.
		3	0	29,618	58	823	52		SW	3	Rain.
♀	19	9	0	29,764	49	969	46		SSW	1,2	Cloudy.
		3	0	29,681	49	969	50		S	1	Rain.
♄	20	9	0	29,456	47	901	44	0.220	E	2	Rain.
		3	0	29,409	49	876	52	0.392	E	1	Cloudy.
☉	21	9	0	29,402	50	940	47	0.253	S	1	Fine.
		3	0	29,386	52	940	52		WSW	1	Fine.
☾	22	9	0	29,409	43	962	41	0.315	S	1	Fine.
		3	0	29,351	47	659	48		SSE	1	Fine.
♂	23	9	0	28,531	53	712	45		S & SSE	3	Cloudy.
		3	0	28,512	50	880	53	0.165	SW		A most violent gale.
☿	24	9	0	28,989	47	769	45		W	1	Fine.
		3	0	29,042	48	777	50		W	1,2	Fine.
♃	25	9	0	29,305	44	927	41		W	1	Cloudy and foggy.
		3	0	29,396	48	809	48		W	1	Cloudy.
♀	26	9	0	29,678	36	936	35	0.005	W	1	Foggy.
		3	0	29,680	40	857	40		W	1	Cloudy.
♄	27	9	0	29,741	41	795	38		E	1	Cloudy and hazy.
		3	0	29,783	43	785	44		E	1	Cloudy.
☉	28	9	0	29,669	49	814	40		N	1	Cloudy and hazy.
		3	0	29,124	52	850	52½		SW	1	Cloudy.
☾	29	9	0	29,190	49	907	47	0.198	W	2	Cloudy.
		3	0	29,255	51	676	52		W	1	Fine.
♂	30	9	0	29,473	46	795	41		SW	1	Cloudy.
		3	0	29,243	52	804	52	0.410	SSW	3	Rain.

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1824 December.		Time. H.	Time. M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm. °	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
☿	1	9	0	29,309	44	805	44		SW	2	Cloudy.
		3	0	29,431	40	743	44		W	1	Fine.
♃	2	9	0	29,631	36	839	34		W	1	Fine.
		3	0	29,425	45	753	45		S	2	Rain.
♀	3	9	0	29,447	37	758	37		W	1	Fine, rather hazy.
		3	0	29,662	39	794	47		W	1	Fine.
♄	4	9	0	29,522	37	734	36	0.255	N	1	Rain.
		3	0	29,344	41	932	41		E	1	Cloudy.
☉	5	9	0	29,516	39	971	38	0.333	N	1	Rain.
		3	0	29,725	39	941	40		N	1,2	Fine.
☾	6	9	0	29,828	32	926	31		W	1	Cloudy and hazy.
		3	0	29,643	41	904	41		SW	1	Fine.
♂	7	9	0	29,500	38	848	36	0.145	W	1	Fine.
		2	30	29,555	42	842	42		W	1	Fine.
☿	8	9	0	29,863	40	886	35	0.003	W	1	Cloudy and hazy.
		3	0	29,781	45	824	45		W by S		Cloudy.
♃	9	9	0	29,707	40	971	39	0.023	W	1	Cloudy and hazy.
		3	0	29,639	43	886	44		W	1	Fine.
♀	10	9	0	29,921	33	929	32	0.180	W	1	Hazy.
		3	0	29,990	47	571	47		W	1	Fine.
♄	11	9	0	30,106	41	959	32	0.005	W	1	Hazy, thick weather.
		3	0	30,110	48	777	48		N by W	1	Rain.
☉	12	9	0	30,245	45	824	44		W	1	Fine.
		3	0	30,292	49	826	49		W	1	Fine.
☾	13	9	0	30,385	46	864	44		W	1	Cloudy.
		3	0	30,393	49	876	49		W	1	Hazy.
♂	14	9	0	30,459	45	929	44		W	1	Cloudy and hazy.
		3	0	30,363	46	898	46½		SW	2	Cloudy.
☿	15	9	0	30,072	44	963	43		W	1	Fine.
		3	0	29,963	49	845	49½		W	1	Cloudy.
♃	16	9	0	29,925	42	763	41	0.039	N	1	Fine, rather hazy.
		3	0	29,937	43	962	44		NW	1	Hazy.

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1824 December.		Time. H.	Time. M.	Barometer corrected. Inches.	Therm. without. °	Degree of Moisture by Daniell's Hygrom. °	Six's Therm. °	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
♀	17	9	0	30,030	39	971	37		W	1	Cloudy and hazy.
		3	0	30,011	43	962	43		W	1	Rain.
♄	18	9	0	30,170	43	924	42	0.070	S	1	Rain.
		3	0	30,125	48	808	48		W	1	Fine.
☉	19	9	0	30,078	48	968	47	0.010	W	1	Cloudy.
		3	0	30,024	51	908	51		W	1,2	Cloudy.
☾	20	9	0	29,409	51	821	47		W	2,3	Rain.
		3	0	29,491	45	776	50		W	1	Fine.
♂	21	9	0	29,416	45	706	40	0.008	SW	3	Cloudy.
		3	0	29,548	51	879	51		W	2	Cloudy.
☿	22	9	0	29,004	51	734	46		W	3	Cloudy.
		3	0	28,811	48	702	52		W	3	Fine.
♃	23	9	0	29,773	34	655	34	0.128	W by N	1	Fine.
		3	0	29,926	37	641	37		W by N	1	Fine.
♀	24	9	0	29,477	44	805	33	0.160		3	Rain.
		3	0	29,601	44	805	50	0.140			Rain.
♄	25	9	0	29,530	51	792	41		W	2	Cloudy.
		3	0	29,495	54	791	54		WNW		
☉	26	9	0	29,816	43	684	43		W	1	Fine.
		3	0	29,973	45	682	45		W by N	1	Fine.
☾	27	9	0	29,916	47	901	39		W by S	3	Cloudy.
		3	0	29,812	52	822	53		W	2	Cloudy.
♂	28	9	30	29,770	50	820	49	0.005	W	2	Fine.
☿	29	9	0	30,061	39	853	39	0.310		1	Cloudy.
		3	0	30,158	39	882	40		NNE	2	Cloudy.
♃	30	9	0	30,142	46	864	35	0.040	N	2	Rain.
		5	0	30,155	46		51		W		Fine.
♀	31	9	0	30,184	48	840	44		W	1	Cloudy.
		3	0	30,127	50	880	51	0.008	W	2	Rain.

1824.	Height of Barometer,* corrected.			Height of Thermometer without.			Degrees of Moisture by Daniell's Hygrometer.			Six's thermometer.			Rain.†
	Greatest height.	Least height.	Mean height.	Greatest height.	Least height.	Mean height.	Greatest height.	Least height.	Mean height.	Greatest height.	Least height.	Mean height.	
	Inches.	Inches.	Inches.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Inches.
January	30,588	28,799	30,055	52	27	40,0	912	650	794	52½	25	39,1	0,522
February	30,403	28,906	29,807	52	32	41,2	1000	637	834	52	28	46,0	1,481
March	30,286	28,938	29,293	55	29	43,4	961	537	763	55	28	41,6	0,853
April	30,458	29,208	29,851	67	34	49,4	943	444	680	66½	28	46,3	1,182
May	30,578	29,453	29,925	71	44½	55,4	987	519	722	73	42	52,2	2,391
June	30,338	29,220	29,861	72	52	61,2	1000	475	712	74	45	57,9	2,183
July	30,452	29,557	29,977	80	61	69,2	878	442	654	81	51	64,1	1,030
August	30,370	29,492	29,903	75½	55	65,7	1000	554	746	77	50½	63,0	2,092
September	30,253	29,380	29,855	80	41	62,3	966	528	783	81	37	60,2	2,373
October	30,154	28,839	29,627	64	35	53,2	1000	529	817	65	31	51,9	1,462
November	30,246	29,042	29,629	59	36	49,1	969	471	815	60	29	47,5	3,154
December	30,496	28,870	29,812	54	32	43,9	971	571	837	54	31	43,0	1,972
Whole year			29,799			52,8			763			50,6	20,695

* The quicksilver in the bason of the barometer is 100 feet above the level of low water spring tides at Somerset-place.

† The Rain Gage is 114 feet above the same level, and 75 feet above the surrounding ground.